A Case of the Container-Vessel Scheduling Problem

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Abstract: We study a difficult real life scheduling problem encountered in oil and petrochemical industry, involving inventory and distribution operations, which requires integrated scheduling. The problem itself is NP-complete, however we show some special cases, and propose polynomial time solution methods. These could be used as a starting point for a heuristic making use of these simplified cases. This study proposes two alternative approaches for the main problem, one of them making use of one of the special cases using minimum cost flow formulation, and the other one using Benders Decomposition once the problem is reformulated to make it easier to handle. Both results show promising results and computation time. Benders Decomposition approach allows exact solutions to be found in a much faster fashion.

1 INTRODUCTION

This study is focused on the demand-supply coordination problem encountered in petrochemical industry such as oil and gasoline. The cost to produce and deliver gasoline products to the market consists of three major components: the transportation cost of crude oil to refiners, the operation cost of refinery processing, and the cost of marketing and distribution. An oil company typically operates many tens of refineries, with several million barrels of crude oil per day and several billion dollars on crude transportation per year. As the retail gasoline prices continue to rapidly elevate around the world, effectively coordinating the demand and supply of gasoline products has therefore become even more crucial to oil companies. Particularly in this study, the company uses its own and chartered vessels to distribute the gasoline products to discharging/demand locations. Each discharging location carries its own inventories and serves as a depot of distribution for the local market. Since vessels are expensive in both variable and fixed costs, any inefficiency in the supply process could result in a substantial operating cost. The distribution scheduling problem encountered in this process is very complicated due to the involvement of heterogeneous vessels (e.g., in terms of their loading capacities, discharging and berthing times, and operating costs) and the fact that each vessel has multi-level of loading capacities such that a load beyond the normal/base capacity will result in an extra overload cost. Practical issues faced include which vessel should deliver to which depot in which time period, whether a particular vessel trip should carry an extra load and by how much, and what should be the ending inventory at a depot in a particular period, etc. Due to high distribution cost of gasoline products, an effectively distribution schedule could help the company to further improve the profit of its supply chain and to strengthen its competitive advantage in the market place.

Our goal is to minimize the operating costs related to shipping and handling of goods. The fleet size is not fixed, nor an initial amount is set, so one of the tasks we have at hand is to determine the number of vessels that will be used within the planning horizon. Shipping costs can be divided into two categories:1) The fixed cost related to either purchase or lease of a vessel, 2) the overloading cost which is incurred if the vessels carry above a certain capacity. There are two more costs that we need to watch out for. Each shipment made to a port may incur a holding or penalty cost based on the demand. If the demand is not met

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on time, it cannot be satisfied at a later time period, and therefore we need to pay penalty for each unit. Also, if the port is forced to hold some inventory, then a holding cost is charged. In addition to all these cost factors, we also need to consider the fact that each vessel is available for a certain amount of time within a period, and therefore even if a vessel has enough capacity, it may not have enough time to visit all the ports we desire.

Optimally solving distribution operations scheduling problem is not an easy task. Previous work related to industrial shipping varies a lot. Here, we focus on the existing results that are closely related to our work. A large summary of works related to various types of vessel scheduling and routing problem can be found in the literature survey by (Christiansen et al., 2004). Two more recent surveys can be found more specifically in the area of combined inventory management and routing ((Andersson et al., 2010)) and on fleet composition and routing ((Hoff et al., 2010)). The most recent literature survey is by (Christiansen et al., 2012), which takes a look at the publications in the last decade, and list possible research areas that could be pursued in this area.

(Xinlian et al., 2000) presents an algorithm which combines the linear programming technique with that of dynamic programming to improve the solution to linear model for fleet planning. Even though their approach is similar, the problem they are dealing with requires demand satisfaction and initial fleet is already given, and the decision is to whether add new vessels to the existing fleet or not.

(Cho and Perakis, 2001) presented a better formulation to the original fleet deployment problem proposed by (Ronen, 1986). In this formulation, just like we do, there is a single loading port, finite number of customer ports, and a finite planning horizon. However, they require the demand to be met, and the fleet size is constant. The costs incurred are due to routes chosen, shipping cargoes, and unloading time. They show that this formulation is better for computational efficiency.

(Cho and Perakis, 1996) present a study regarding fleet size and design of optimal liner routes for a container shipping company. The problem is solved by generating a number of candidate routes for the different ships first, and then, the problem is formulated and solved as a linear programming model, where the columns represent the candidate routes. They extend this model to a mixed integer programming model that also considers investment alternatives to expanding fleet capacity. (Bendall and Stent, 2001) also present a model for determining the optimal number of ships and fleet deployment plan.

On the other hand, (Nicholson and Pullen, 1971) were the first ones to propose dynamic programming application to ship fleet management. The problem they dealt with was to determine the sequence in which the currently owned ships should be sold and the extent to which charter ships should be taken on. They tackle the problem in two stages. The first stage determines a good priority ordering for selling the ships regardless of the rate at which charter ships are taken on. The second stage uses dynamic programming to determine an optimal level of chartering given the priority replacement order. This first stage priority ordering essentially reduces the dynamic programming calculation from a problem with as many as states as number of ships in fleet to a 1 state variable problem which is computationally manageable by dynamic programming methods. Several authors use benchmark instances to compare the results of different strategies and heuristics. (Gheysens et al., 1984) define 20 test instances with 12100 nodes for the standard fleet size and mix vehicle routing problem. (Wu et al., 2005) deals with trucks that vary in capacity and age are utilized over space and time to meet customer demand. Operational decisions (including demand allocation and empty truck repositioning) and tactical decisions (including asset procurements and sales) are explicitly examined in a linear programming model to determine the optimal fleet size and mix. The method uses a time-space network, common to fleet-management problems, but also includes capital cost decisions, wherein assets of different ages carry different costs, as is common to replacement analysis problems. A two-phase solution approach is developed to solve large-scale instances of the problem. Phase I allocates customer demand among assets through Benders decomposition with a demand-shifting algorithm assuring feasibility in each subproblem. Phase II uses the initial bounds and dual variables from Phase I and further improves the solution convergence through the use of Lagrangian relaxation.

A network optimization approach has been proposed by (Bookbinder and Reece, 1988), where they formulate a multi-commodity capacitated distribution-planning problem as a non-linear mixed integer programming model, and solve it as a generalized assignment problem within an algorithm for the overall distribution/routing problem based on a Bender's type decomposition.

(Lei et al., 2009) proposes an approach to a bidirectional flow problem where each iteration starts with a given planning horizon, which is then partitioned into three planning intervals, where each interval consists of consecutive time periods in the given planning horizon. Afterwards, some constraint relaxations are applied to the problem in which all the forward demand and all the backward demand of the time periods in the third planning interval are consolidated into a single forward demand and a single backward demand, which is an idea we use in one of our approaches.

(Choi et al., 2012) focuses on minimizing total tardiness, rather than the operating costs, and the routes for vessels are observed under three different cases, one of them being arbitrary, just like in our problem. Later on, they talk about the other problems in the literature and how their approach is related to them.

2 PROBLEM DEFINITION

This paper, brings together some of the ideas that were proposed in the literature before. We are given a fleet $|V|$ of container vessels, $v \in \mathbf{V}$ that distributes the goods from a main distribution center to a number of customer ports over a $|T|$-period planning horizon. Each vessel has two loading capacities: the regular loading capacity u_v^0, and the maximum loading capacity u_v^{max} so that carrying a load beyond u_v^0 will impose an over loading charge $g_v^0/unit$ and carrying a load beyond u_v^{max} violates the feasibility. In addition to this limitation, for every vessel there is total available time τ_v which is used up by the berthing time $b_{v,p}$ at ports which vary depending on vessel type. There are $|p|$ customer ports on the network, each port $p \in \mathbf{P}$ has a demand, $d_{p,t} \geq 0$ in period $t \in \mathbf{T}$. For every port, unsatisfied demand are penalized at $p_{p,t}/unit$ based on the unsatisfied demand and no backlogging is allowed. On the other hand, end of period inventory incurs a holding cost of $h_n/unit$. Let c_v^f denote the fixed cost if the vessel is being dispatched in a period. The problem is finding a feasible vessel dispatching schedule to minimize the total shortage and overage penalty plus the vessel overloading and fixed cost.The minimum cost flow network formulation proposed guarantees optimality when the number of vessels dispatched in every period is known. To define our problem more formally, we define the following set of variables:

$S_{p,t} \in \mathbb{Z}_+$: amount of shortage at port p in period t
$Q_{v,p,t} \in \mathbb{Z}_+$: amount of supply delivered to port p in period t via vessel v's regular capacity
$O_{v,p,t} \in \mathbb{Z}_+$: amount of supply delivered to port p in period t via vessel v's overloading capacity
$I_{p,t} \in \mathbb{Z}_+$: ending inventory at port p in period t
$Y_{v,p,t} \in \{0,1\}$: $Y_{v,n,t} = 1$ if vessel v delivers to port p in period t
$Z_{v,t} \in \{0,1\}$: $Z_{v,t} = 1$ if vessel v is dispatched in period t

Based on this, the constraints to the problem will include the following:

A vessel must not be carrying anything if it's not dispatched, nor visiting ports:

$$Q_{v,p,t} + O_{v,p,t} \leq u_v^{max} Y_{v,p,t} \quad \forall\, v \in \mathbf{V},\ p \in \mathbf{P},\ t \in \mathbf{T} \tag{1a}$$

$$Y_{v,p,t} \leq Z_{v,t} \quad \forall\, v \in \mathbf{V},\ p \in \mathbf{P},\ t \in \mathbf{T} \tag{1b}$$

Vessels dispatched must not be used over their time and regular/maximum capacity:

$$\sum_{p \in \mathbf{P}} b_{v,p} Y_{v,p,t} \leq \tau_v \quad \forall\, v \in \mathbf{V},\ t \in \mathbf{T} \tag{2a}$$

$$\sum_{p \in \mathbf{P}} Q_{v,p,t} \leq u_v^0 \quad \forall\, v \in \mathbf{V},\ t \in \mathbf{T} \tag{2b}$$

$$\sum_{p \in \mathbf{P}} (Q_{v,p,t} + O_{v,p,t}) \leq u_v^{max} \quad \forall\, v \in \mathbf{V},\ t \in \mathbf{T} \tag{2c}$$

The last group of constraints is to help to formulate our objective, which is a compositions of all expenses (penalties, etc.).
Vessel dispatching costs:

$$c^D = \sum_{v \in \mathbf{V}} \sum_{t \in \mathbf{T}} c_v^f Z_{v,t} \tag{3a}$$

Early arrival penalties:

$$c^H = \sum_{p \in \mathbf{P}} \sum_{t \in \mathbf{T}} h_p I_{p,t} \tag{3b}$$

Unsatisfied demands' penalties:

$$c^U = \sum_{p \in \mathbf{P}} \sum_{t \in \mathbf{T}} p_{p,t} S_{p,t} \tag{3c}$$

Overloading penalties:

$$c^O = \sum_{v \in \mathbf{V}} \sum_{p \in \mathbf{P}} \sum_{t \in \mathbf{T}} g_v^0 O_{v,p,t} \tag{3d}$$

Then our problem is to minimize $c^D + c^H + c^U + c^O$, subject to the constraints (1)-(3) and the sign and type restrictions in the definitions of the decision variables.

If the dispatching information is already available, i.e. $|V_1|$ vessels for t=1, $|V_2|$ for t=2, ..., $|V_T|$ for t=T, then there becomes no need for the binary variables. In addition, define new variables, $x_{v,k,n,t}$ and $r_{v,k,n,t}$, which are the normal and over flows shipped by vessel v dispatched in period k for port n to satisfy the demand on period t. Based on this definition, the following can be established:

$$Q_{v,p,t} = \sum_{k=t}^{T} x_{v,t,p,k} \quad \forall\, v \in \mathbf{V},\ p \in \mathbf{P},\ t \in \mathbf{T} \tag{4a}$$

$$O_{v,p,t} = \sum_{k=t}^{T} r_{v,t,p,k} \quad \forall\, v \in \mathbf{V},\ p \in \mathbf{P},\ t \in \mathbf{T} \tag{4b}$$

$$S_{p,t} = d_{p,t} - \sum_{k\in\mathbf{T}} \sum_{v\in\mathbf{V}_k} (x_{v,k,p,t} + r_{v,k,p,t}) \quad \forall\, p \in \mathbf{P},\ t \in \mathbf{T} \tag{4c}$$

$$I_{p,t} = \sum_{k\in\mathbf{T}} \sum_{w=t+1}^{T} \sum_{v\in\mathbf{V}_k} (x_{v,k,p,w} + r_{v,k,p,w}) \quad \forall\, p \in \mathbf{P},\ t \in \mathbf{T} \tag{4d}$$

Based on the above assumptions and definitions, we get the following model. Objective function is the same except that the last part is now a constant based on vessel dispatching information, i.e. $Z_{v,t}$ values are known. Constraints (5b) and (5c) assure that normal and over capacity are not exceeded, where as constraint (5d) prevents shipments for a specific demand to be more than the demand itself, therefore making the first part of the objective function always nonnegative.

$$\begin{aligned} min \sum_{p\in\mathbf{P}} \sum_{t\in\mathbf{T}} p_{p,t}\Big(d_{p,t} - \sum_{k=1}^{T} \sum_{v\in\mathbf{V}_k} (x_{v,k,p,t} + r_{v,k,p,t})\Big) \\ + \sum_{v\in\mathbf{V}} \sum_{p\in\mathbf{P}} \sum_{t\in\mathbf{T}} g_v^0 \sum_{k=t}^{T} r_{v,t,p,k} + \sum_{v\in\mathbf{V}} \sum_{t\in\mathbf{T}} c_v^f Z_{v,t} \\ + \sum_{p\in\mathbf{P}} \sum_{t\in\mathbf{T}} h_p \sum_{k=1}^{t} \sum_{w=t+1}^{T} \sum_{v\in\mathbf{V}_k} (x_{v,k,p,w} + r_{v,k,p,w}) \end{aligned} \tag{5a}$$

$$s.t. \sum_{p\in\mathbf{P}} b_{v,p} Y_{v,p,t} \le \tau_v \quad \forall\, v \in \mathbf{V},\ t \in \mathbf{T} \tag{5b}$$

$$\sum_{k=t}^{T} \sum_{n\in N} (x_{v,t,n,k} + r_{v,t,n,k}) \le u_v^{max} \quad \forall\, v \in \mathbf{V},\ t \in \mathbf{T} \tag{5c}$$

$$\sum_{k=t}^{T} x_{v,t,n,k} + r_{v,t,n,k} \le u_v^{max} Y_{v,n,t} \quad \forall\, v \in \mathbf{V},\ p \in \mathbf{P},\ t \in \mathbf{T} \tag{5d}$$

$$\sum_{n\in N} \sum_{k=t}^{T} x_{v,t,n,k} \le u_v^0 \quad \forall\, v \in \mathbf{V},\ t \in \mathbf{T} \tag{5e}$$

$$\sum_{k=1}^{t} \sum_{v\in V_k} (x_{v,k,n,t} + r_{v,k,n,t}) \quad \forall\, p \in \mathbf{P},\ t \in \mathbf{T} \tag{5f}$$

$$Y_{v,n,t} \le Z_{v,t} \quad \forall\, v \in \mathbf{V},\ p \in \mathbf{P},\ t \in \mathbf{T} \tag{5g}$$

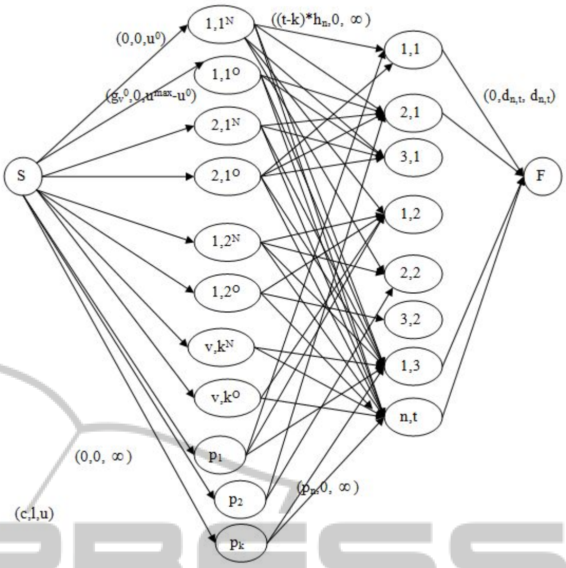

Figure 1: An example of the network for $|V_1| = 2$, $|V_2| = 1$, $|V_3| = 2$, $|N| = 3$, and $|T| = 3$.

Lemma 2.1. *The above problem can be reformulated without the berthing time constraint and solved as a minimum cost flow problem by assuming the knowledge of the number of vessels dispatched in each time period.*

Proof. First, we construct a dummy source node S, and a dummy sink node F. Associate to each vessel $v \in \mathbf{V}_k$, 2 nodes $(v,k)^P$ and $(v,k)^O$, one for normal and other for over capacity. These nodes are connected to the source node with 0 and g_v^0 costs, a lower bound of 0 and an upper bound u_v^0 and $u_v^{max} - u_v^0$ respectively. Add another set of $|P|$ nodes (p_p) for case of shortage at each port with 0 costs, 0 lower bounds and no upper bounds. Next, take care of the ports by adding $|P| * |T|$ nodes denoted (p,t) for each port n at every period t. The arcs between nodes corresponding to vessels and ports incur a holding cost of $h_p(t-k)$, has a lower bound of 0 and no upper bound. Also, there will be arcs between shortage nodes, (p_p), and ports, (p,t), where the shortage costs $p_{p,t}$ will be charged. Finally, add arcs between ports and the sink, with a lower and upper bound of $d_{p,t}$ and no cost.This network will have $2|V||T| + |P| + |P||T|$ many nodes, and $|V||P||T^2| + |P^2|$ many arcs, making minimum cost flow approach practical for problems of reasonable size. An example network is shown in Figure 1. □

Lemma 2.2. *The objective function values and constraints for both problems above are the same, assuming we guessed the right number of vessels.*

Proof. First of all, the fixed cost due to vessels for both problems will be the same. Next, assume $x^*_{v,k,n,t}$ and $r^*_{v,k,n,t}$ are the optimal flow vectors corresponding to the minimum cost flow problem. Then, using the equalities corresponding the variables of two problems, the objective function value of the original problem becomes:

$$\sum_{p\in\mathbf{P}}\sum_{t\in\mathbf{T}} p_{p,t}(d_{p,t}-\sum_{k\in\mathbf{T}}\sum_{v\in\mathbf{V}_k}(x_{v,k,p,t}+r_{v,k,p,t}))+ \sum_{v\in\mathbf{V}}\sum_{p\in\mathbf{P}}\sum_{t\in\mathbf{T}} g^0_v \sum_{k=t}^{T} r_{v,t,p,k}+\sum_{v\in\mathbf{V}}\sum_{t\in\mathbf{T}} c^f_v Z_{v,t}+ \sum_{p\in\mathbf{P}}\sum_{t\in\mathbf{T}} h_p \sum_{k\in\mathbf{T}}\sum_{w=t+1}^{T}\sum_{v\in\mathbf{V}_k}(x_{v,k,p,w}+r_{v,k,p,w}) = \sum_{p\in\mathbf{P}}\sum_{t\in\mathbf{T}} p_{p,t}S^*_{p,t}+\sum_{v\in\mathbf{V}}\sum_{p\in\mathbf{P}}\sum_{t\in\mathbf{T}} g^0_v O^*_{v,k,p,t}+ \sum_{v\in\mathbf{V}}\sum_{t\in\mathbf{T}} c^f_v Z^*_{v,t}+\sum_{p\in\mathbf{P}}\sum_{t\in\mathbf{T}} h_p I^*_{p,t} \tag{6}$$

The first 2 lines of this equality (6) and the objective function of the minimum cost flow are exactly the same, which only leaves us with the inventory part. The w index is for shipments that are on a future date than current period t, and the k index is taking into account all shipments that have been made up to period t. Therefore, a shipment made on period k for period t will appear in the summation $(t-k)$ many times, allowing us to replace index w with t, remove the summation regarding w, and charge the holding cost as many times as necessary. This shows that both objective function values are the same.

As far as the constraints are concerned, first realize that in the original problem, (2a) is no longer required while berthing times are large enough. Similarly, (1a) and (1b) were associated with the fact that dispatching information was not available, so now, they could be dropped as well. (2b) in the original problem is the same constraint as (5d) in the reduced problem, and they are both concerned with normal capacity of a vessel. (5c) and (5d) of the reduced problem, added together, imply the same restriction on maximum vessel capacity as (2c) of the original problem. On the other hand, the flow balance constraint in the original problem is taken care of by two means: 1) the new index k for the variables, tells us when shipment was made, so we now whether a shipment is held at inventory or used immediately, 2) in the reduced problem, shipment for a specific demand will not be more than the demand itself, therefore shortage never becomes negative according to the relation between $S_{n,t}$ and $x_{v,k,n,t}$, $r_{v,k,n,t}$. □

3 SPECIAL CASES

Figure 2 is a list of special cases deduced from the general problem, for which we propose efficient solution approaches. Case 7 makes use of the method proposed by (Detti, 2009), used for solving knapsack problems with divisible item sizes.

Based on our minimum cost network flow approach, we propose the following two heuristics for no berthing time case:

3.1 Backward Heuristic

1. Divide the planning horizon into two groups, primary and secondary, for each port, the new demand is equal to sum of the individual demands in each group, holding cost is the minimum and penalty cost is the maximum of individual penalties.
2. Start with $|P|*|T|$ vessels in that group in total, solve the minimum cost flow problem iterating through all vessel dispatching combinations available for a group.
3. Once the optimal number of vessels required for each group are determined, repeat the procedure of dividing into groups and solving as a minimum cost flow problem for the individual groups. Demand belonging to ports in the other individual group is also added to the demand of the ports in the secondary period of the group under consideration.
4. Once the primary group has only 1 period remaining, optimal number of vessels have been determined for that group, start over.

3.2 Greedy Heuristic

1. Start with no vessels assigned to each period.
2. Add a vessel to any period and solve the problem. Remove the vessel, and add to another period, and solve again. Once the best vessel addition has been determined, move on to next vessel addition.
3. Keep determining the best vessel to add until objective function no longer improves.

Going back to the original problem with berthing constraints, we propose modified greedy heuristics and a decomposition based exact method. We first introduce the algorithms and then compare them with state of the are integer programming solver, XpressMP.

3.3 Improved Greedy Heuristic

1. Start with no vessels assigned to each period.

Case	|V|	|N|	|T|	Other Assumptions	Method	Complexity
1	1	>1	1	- no berthing time	- sort ports based on *p* - solve greedily	$O(N \cdot \log(N))$
2	1	>1	1	$b_n = b, \forall n$	- case 1 where the first τ / b gets served	$O(N \cdot \log(N))$
3	1	>1	>1	- each customer has demand only once	- compute savings, $w_{nk} = p_n \cdot d_{nk} - h_n(t(n) - k)$ where *t(n)* is the period of demand - solve greedily - move unsatisfied demand to previous period	$O(N \cdot \log(N) \cdot T)$
4	>1	>1	>1	- each customer has demand only once - each vessel must visit two ports - each demand must be satisfied by one vessel	- precompute cost of visiting pairs of ports (i, j), where $p_i \geq p_j$ and buying a vessel - solve the matching problem	$O(N^3)$
5	1	1	>1	- orders nonsubstitutable - demand nonsplittable (member of team type demand) - shortage not allowed - a vessel must satisfy consecutive demands	- Let $TT = \{t \mid b \leq \tau_t, 1 \leq t \leq T\}$ - Precompute arc costs, based on holding, overloading and vessel fixed cost - Solve the shortest path problem	$O(N^2)$
6	>1	1	>1	- vessel dispatching information available	- minimum cost flow problem	$O(N^3)$
7	>1	>1	>1	$p_n = p, h_n = h, \forall n$ - demands are divisible amongst eachother - small |T| - no overload - no berthing time	- use Detti algorithm with different |V| - Let $\Phi_{v,t}$ be the set of ports not being visited by |V| vessels in period *t* - go to previous period and calculate penalties carrying over the unsatisfied demand in the last period - use dynamic programming	$O((NV + N\log N)N + N^T)$
8	>1	>1	>1	$p_n = p, h_n = h, \forall n$ - demands are divisible amongst eachother - small |T| - no overload - no berthing time	$u \geq d_{n,t} = \sum_{j=0}^{\log u} \alpha_{n,t,j} \cdot 2^j, \forall n, t$ such that $\alpha_{n,t,j} \in \{0,1\}$ $b_{t,j} = \sum_{\forall n} \alpha_{n,t,j}$, $s_{t,j} = 2^j$ $v_{t,j} = penalty - holding$ Max $\sum_{\forall t} \sum_{\forall j} v_{t,j} \cdot \alpha_{n,t,j}$ $\sum_{\forall n} \sum_{\forall j} s_{t,j} \cdot \alpha_{n,t,j} \leq u \quad \forall t$ $\sum_{\forall j} \alpha_{n,t,j} \leq b_{t,j} \quad \forall t, n$ - solve with different |V| for every *t*	At most N^T cases

Figure 2: Special cases deduced from the general problem under different assumptions.

2. For each vessel type, compute maximal subsets of ports such that no further port can be added to a set due to berthing time constraint.
3. For each vessel type, in every period, sort the subset of ports in decreasing order based on $\sum_{n \in Maximal_v} d_{n,t} p_{n,t}$

Table 1: $|N|$=3, $|T|$=4, vessel type same, number of vessels in each period shown, as well as the optimal objective function value for each case, indicating that even a much simpler version of the original problem is not submodular.

Case 1	Obj. Func.	Case 2	Obj. Func.	Case Int	Obj. Func.	Case Union	Obj. Func.	Comp.
(3,2,2)	170	(2,1,3)	390	(2,1,2)	540	(3,2,3)	50	Lower
(1,2,1)	840	(2,1,3)	390	(1,1,1)	1140	(2,2,3)	90	Equal
(2,2,1)	580	(2,1,3)	390	(2,1,1)	880	(2,2,3)	90	Equal
(1,2,3)	350	(2,1,3)	390	(1,1,3)	650	(2,2,3)	90	Equal
(1,3,2)	390	(2,1,3)	390	(1,1,2)	800	(2,3,3)	50	Lower

Table 2: $|N|$=10, $|T|$=10, 3 different vessel types, number of vessels each method solves for vary for XpressMP, Backward and Greedy as 10, 100, 1 in respect. Runs are terminated after 2 hours or when 0.1% gap from the best bound is reached.

Time (sec)			Objective Function			Gap (%)		
Xpress	Backward	Greedy	Xpress	Backward	Greedy	Xpress	Backward	Greedy
7200	412.50	435.8	7503	7799	7733	1.7384	5.47	4.66
7200	420.40	435.20	8141	8314	8298	2.1557	4.20	4.01
7200	393.30	420.60	8270	8483	8450	0.9988	3.48	3.11
7200	332.30	390.70	7759	7806	7800	0.3519	0.95	0.88
7200	289.90	316.80	7316	7395	7375	0.1414	1.20	0.94
7200	310.30	336.60	8412	8494	8487	3.2309	4.16	4.09
7200	345.2	347.6	8270	8356	8338	3.8278	4.82	4.62
7200	429.50	443.30	7918	8159	8112	1.1641	4.08	3.53
7200	421.8	425.8	7475	7825	7768	1.4574	5.87	5.17
7200	306.70	321.10	7448	7661	7600	2.4489	5.16	4.40

4. Next, add any vessel to any period allowing it to only serve the top ranked subset of ports and solve the problem. Try different vessel types, for different periods in the same manner. Determine the best vessel to add to which period.
5. Once a vessel assignment has been determined, update remaining demand and sort subsets of ports accordingly.
6. Keep determining the best vessel to add until objective function no longer improves.

3.4 Bender's Type Decomposition Approach

1. Choose a feasible assignment of ports to a vessel to start with.
2. Solve the master problem to obtain a new objective function value and new port assignments to other vessels.
3. Keeping port assignments fixed, solve the dual problem.
4. STOP, if master and dual objective values are close enough, otherwise go back to the master problem.

4 COMPUTATIONAL RESULTS

With the minimum cost flow network formulation proposed, one question that arose was whether it was worth investing more resources into finding a dispatching information, and maybe go from there. However, as can be seen at 1, the minimum cost flow problem is not submodular. *Case 1* and *Case 2* are random dispatches, where as *Case Int* refers to the scenario where minimum of number of vessels dispatched in each latter case is used, and *Case Union* refers to of *Case 1* and *Case 2* are random dispatches, where as *Case Int* refers to the scenario, where minimum of number of vessels dispatched in each latter case is used, and *Case Union* refers to the scenario, where maximum of number of vessels dispatched in each latter case is used.

Backward and Greedy Heuristic both performed well, however, as can be seen in Table 2, Backward Heuristic always takes shorter, where as Greedy Heuristic performs better by a slight margin.

However, it must be kept in mind that Table 2 reflects results for the version of the problem with no berthing time constraint. Improved Greedy Heuristic designed to deal with this issue performs a bit slower than the previously mentioned heuristics, but gives good bounds for the solution of the original problem as can be seen in Table 3.

Table 3: $|N|$=10, $|T|$=10, 3 different vessel types, number of vessels each method solves for vary for XpressMP and Improved Greedy as 10 and 1 in respect. All runs are terminated after 2 hours or when 0.1% gap from the best bound is reached.

Time (sec)		Objective Function		Gap (%)	
Xpress	I. Greedy	Xpress	I. Greedy	Xpress	I. Greedy
7200	593.9	7503	7533	1.74	2.13
7200	582.2	8141	8226	2.16	3.17
7200	635.3	8270	8381	1.00	2.31
7200	678.2	7759	7921	0.35	2.39
7200	668.6	7316	7436	0.14	1.75
7200	594.4	8412	8548	3.23	4.77
7200	649.3	8270	8353	3.83	4.78
7200	667.3	7918	8092	1.16	3.29
7200	641.1	7475	7513	1.46	1.96
7200	662.1	7448	7565	2.45	3.96

Table 4: $|N|$=10, $|T|$=10, 3 different vessel types, number of vessels each method solves for vary for XpressMP and Improved Greedy as 10 and 1 in respect. All runs are terminated after 2 hours or when 0.1% percent gap from the best bound is reached.

Time (sec)		Objective Function		Gap (%)	
Xpress	Decomposition	Xpress	Decomposition	Xpress	Decomposition
7200	767.1	7503	7380	1.74	0.10
7200	757.5	8141	7973	2.16	0.10
7200	905.8	8270	8196	1.00	0.10
7200	1291.4	7759	7739	0.35	0.10
7200	1420.4	7316	7313	0.14	0.10
7200	959	8412	8148	3.23	0.10
7200	1183.5	8270	7961	3.83	0.10
7200	907.4	7918	7834	1.16	0.10
7200	1032.8	7475	7373	1.46	0.10
7200	1349.4	7448	7273	2.45	0.10

The formulation proposed for Bender's type approach is computationally efficient, as can be on Tables 4 and 5. The running time is a bit longer, but we're able to get exact solutions.

5 CONCLUSIONS

In this study, we studied a difficult real life supply chain scheduling problem encountered in oil and petrochemical industry, which involves production, inventory, and distribution operations, and requires an integrated scheduling to minimize the total operation cost. We showed the hardness of this problem, and showed that some of its special cases are polynomial time solvable. A minimum cost flow based heuristic, motivated by the observations from one of the special cases, was proposed and demonstrated to have a promising performance under the set of test cases considered in this study. Also, a new formulation of the model was developed, which made Bender's type decomposition method computationally efficient. Therefore, we're now able to get really good(exact) results for big problems at a much faster fashion then solver XpressMP.

There are several interesting extensions of the work presented here. These include integrating the inland production with single or multiple refineries at different locations on the network, and multiple products needed by the same customer port. This integration would cause the supply chain to become bigger, and therefore more complex, however closer to reality, as inland production and demand satisfaction are activities that need synchronization. Furthermore, the involvement of multiple refineries and multiple products introduces the new optimization issues due to assigning refineries to customer ports and allocating vessel capacity for different products. This will make the modeling and the design of search procedures more interesting and challenging.

Also, for the simplicity of modeling, in this study, we assumed a linear penalty function for vessel overloading. However, this penalty cost is in reality very complex and is affected by many factors such as the level of overloading and navigation conditions. A nonlinear cost function would be more meaningful in this case.

Table 5: $|N|$=15, $|T|$=10, 3 different vessel types, number of vessels each method solves for vary for XpressMP and Improved Greedy as 10 and 1 in respect. All runs are terminated after 2 hours or when 0.1% percent gap from the best bound is reached.

Time (sec)		Objective Function		Gap (%)	
Xpress	Decomposition	Xpress	Decomposition	Xpress	Decomposition
7200	805	7764	7619	1.98	0.10
7200	777.9	8652	8448	2.46	0.10
7200	990.5	8326	8245	1.07	0.10
7200	1401.1	8001	7978	0.39	0.10
7200	1544.6	7464	7461	0.15	0.10
7200	1015.4	8789	8496	3.43	0.10
7200	1216.9	9423	9026	4.30	0.10
7200	984.8	8128	8035	1.24	0.10
7200	1114.2	7849	7727	1.66	0.10
7200	1413.5	7652	7455	2.67	0.10

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