Tuning of Fuzzy Fractional PD$^{\beta}$ + I Controllers by Genetic Algorithm

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Abstract: In this paper we consider the development of an optimal fuzzy fractional PD + I controller in which the parameters are tuned by a Genetic algorithm (GA). Fuzzy control is an intelligent control methodology that mimics human thinking and reacting in order to improved the performance of systems. On the other hand, GA can reach exact or approximate solutions to optimization and search problems. In this line of thought, the performance of the proposed fuzzy fractional control is illustrated through two application examples.

1 INTRODUCTION

Fractional calculus (FC) is a generalization of integration and differentiation to a non-integer order $\alpha \in C$, being the fundamental operator ${}_aD_t^{\alpha}$, where a and t are the limits of the operation (Oldham and Spanier, 1974; Podlubny, 1999a). The FC concepts constitute a useful tool to describe several physical phenomena, such as heat, flow, electricity, magnetism, mechanics or fluid dynamics. Presently, the FC theory is applied in almost all areas of science and engineering, being recognized its ability in bettering the modelling and control of many dynamical systems. In fact, during the last years FC has been used increasingly to model the constitutive behavior of materials and physical systems exhibiting hereditary and memory properties. This is the main advantage of fractional-order derivatives in comparison with classical integer-order models, where these effects are simply neglected.

In this paper we investigate several control strategies based on fuzzy fractional-order algorithms. The fractional-order PID controller (PI$^{\alpha}$D$^{\beta}$ controller) involves an integrator of order $\alpha \in \Re^+$ and a differentiator of order $\beta \in \Re^+$. It was demonstrated the good performance of this type of controller, in comparison with the conventional PID algorithms. Recently, there have been a lot of researches in the application of fuzzy PID control (Mizumoto, 1995; Carvajal et al., 2000; Eker and Torun, 2006; Barbosa, 2010; Barbosa et al., 2010; Das et al., 2012; Delavari et al., 2010; Tian et al., 2010; Padula and Visioli, 2011). The fuzzy method offer a systematic procedure to design controllers for many kind of systems, that often leads to a better performance than that of the conventional PID controller. It is a methodology of intelligent control that mimics human thinking and reacting by using a multivalent fuzzy logic and elements of artificial intelligence.

Bearing these ideas in mind, the paper is organized as follows. Section 2 gives the fundamentals of fractional-order control systems. Section 3 presents the control and optimization strategies. Section 4 gives some simulations results assessing the effectiveness of the proposed methodology. Finally, section 5 draws the main conclusions.

2 FRACTIONAL – ORDER CONTROL SYSTEMS

Fractional-order control systems are characterized by differential equations that have, in the dynamical system and/or in the control algorithm, an integral and/or a derivative of fractional-order (Machado, 1997). Due to the fact that these operators are defined by irrational continuous transfer functions, in the Laplace domain, or infinite dimensional discrete transfer functions, in the Z domain, we often encounter evaluation problems in the simulations. Therefore, when analyzing fractional-order systems, we usually adopt continuous or discrete integer-order approximations of fractional-order operators (Podlubny, 1999b; Barbosa et al., 2006). The following two subsections provide a background for the remaining of the article by giving the fundamental aspects of the FC, and the discrete integer-order approximations of fractional-order op-

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erators.

2.1 Fundamentals of Fractional Calculus

The mathematical definition of a fractional-order derivative and integral has been the subject of several different approaches (Oldham and Spanier, 1974; Podlubny, 1999a). One commonly used definition for the fractional-order derivative is given by the Riemann-Liouville definition ($\alpha>0$):

$$ {}_aD_t^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \frac{d^n}{dt^n} \int_a^t \frac{f(\tau)}{(t-\tau)^{\alpha-n+1}} d\tau $$

$$ n-1 < \alpha < n \tag{1} $$

where $f(t)$ is the applied function, $\Gamma(x)$ is the Gamma function of x and $n \in \aleph$ (Mainardi and Gorenflo, 2000). Another widely used definition is given by the Grünwald-Letnikov approach ($\alpha \in \Re$):

$$ {}_aD_t^{\alpha} f(t) = \lim_{h \to 0} \frac{1}{h^{\alpha}} \sum_{k=0}^{\left[\frac{t-a}{h}\right]} (-1)^k \begin{pmatrix} \alpha \\ k \end{pmatrix} f(t-kh) \tag{2a} $$

$$ \begin{pmatrix} \alpha \\ k \end{pmatrix} = \frac{\Gamma(\alpha+1)}{\Gamma(k+1)\Gamma(\alpha-k+1)} \tag{2b} $$

where h is the time increment and $[x]$ means the integer part of x.

The "memory" effect of these operators is demonstrated by (1) and (2), where the convolution integral in (1) and the infinite series in (2), reveal the unlimited memory of these operators, ideal for modelling hereditary and memory properties in physical systems and materials.

An alternative definition to (1) and (2), which reveals useful for the analysis of fractional-order control systems, is given by the Laplace transform method. Considering vanishing initial conditions, the fractional *differintegration* is defined in the Laplace domain, $F(s) = L\{f(t)\}$, as:

$$ L\{{}_aD_t^{\alpha} f(t)\} = s^{\alpha} F(s), \quad \alpha \in \Re \tag{3} $$

The open-loop Bode diagrams of amplitude and phase of the operator s^{α} have correspondingly a slope of 20α dB/dec and a constant phase of $\alpha\pi/2$ rad over the entire frequency domain.

2.2 Approximations of Fractional – Order Operators

In this paper we adopt discrete integer-order approximations to the fundamental element s^{α} ($\alpha \in \Re$) of a fractional-order control (FOC) strategy. The usual approach for obtaining discrete equivalents of continuous operators of type s^{α} adopts the Euler, Tustin and Al-Alaoui generating functions (Chen et al., 2004; Barbosa et al., 2006).

It is well known that rational-type approximations frequently converge faster than polynomial-type approximations and have a wider domain of convergence in the complex domain (Chen et al., 2004). Thus, by using the Euler operator $w(z^{-1}) = (1-z^{-1})/T_c$, and performing a power series expansion of $[w(z^{-1})]^{\alpha} = [(1-z^{-1})/T_c]^{\alpha}$ gives the discretization formula corresponding to the Grünwald-Letnikov definition (2):

$$ D^{\alpha}(z^{-1}) = \left(\frac{1-z^{-1}}{T_c}\right)^{\alpha} $$

$$ = \sum_{k=0}^{\infty} \left(\frac{1}{T_c}\right)^{\alpha} (-1)^k \begin{pmatrix} \alpha \\ k \end{pmatrix} z^{-k} = \sum_{k=0}^{\infty} h^{\alpha}(k) z^{-k} \tag{4} $$

where T_c is the sampling period and $h^{\alpha}(k)$ is the impulse response sequence.

A rational-type approximation can be obtained through a Padé approximation to the impulse response sequence $h^{\alpha}(k)$, yielding the discrete transfer function:

$$ H(z^{-1}) = \frac{b_0 + b_1 z^{-1} + \ldots + b_m z^{-m}}{1 + a_1 z^{-1} + \ldots + a_n z^{-n}} = \sum_{k=0}^{\infty} h(k) z^{-k} \tag{5} $$

where $m \le n$ and the coefficients a_k and b_k are determined by fitting the first $m+n+1$ values of $h^{\alpha}(k)$ into the impulse response $h(k)$ of the desired approximation $H(z^{-1})$. Thus, we obtain an approximation that matchs the desired impulse response $h^{\alpha}(k)$ for the first $m+n+1$ values of k (Barbosa et al., 2006). Note that the above Padé approximation is obtained by considering the Euler operator but the determination process will be exactly the same for other types of discretization schemes.

3 CONTROL AND OPTIMIZATION STRATEGIES

3.1 Fractional PID control

The generalized PID controller $G_c(s)$ has a transfer function of the form (Podlubny, 1999b):

$$ G_c(s) = \frac{U(s)}{E(s)} = K_p + \frac{K_i}{s^{\alpha}} + K_d s^{\beta} \qquad \alpha, \beta > 0 \tag{6} $$

where α and β are the orders of the fractional integrator and differentiator, respectively. The parameters K_p, K_i and K_d are correspondingly the proportional, integral, and derivative gains of the controller. Clearly, taking $(\alpha,\beta) = \{(1,1),(1,0),(0,1),(0,0)\}$ we get the classical {PID, PI, PD, P} controllers, respectively (Jesus and Machado, 2008). Other PID controllers are possible, namely: PD$^\beta$ controller, PI$^\alpha$ controller, PID$^\beta$ controller, and so on. The fractional order controller is more flexible and gives the possibility of adjusting more carefully the closed-loop system characteristics (Podlubny, 1999a).

In the time domain the PI$^\alpha$D$^\beta$ is represented by:

$$u(t) = K_p e(t) + K_i\, {}_0D_t^{-\alpha} e(t) + K_d\, {}_0D_t^{\beta} e(t) \qquad (7)$$

where the fractional order differential operators may be implemented using the approximations (4) and (5).

3.2 Fuzzy Fractional PD+I Control

Fuzzy control emerged on the foundations of Zadeh's fuzzy set theory (Barbosa, 2010; Barbosa et al., 2010; Mizumoto, 1995). This kind of control is based on the ability of a human being to find solutions for particular problematic situations. It is well know from our experience, that humans have the ability to simultaneously process a large amount of information and make effective decisions, although neither input information nor consequent actions are precisely defined. Through multivalent fuzzy logic, linguistic expressions in antecedent and consequent parts of IF-THEN rules describing the operator's actions can be efficaciously converted into a fully-structured control algorithm.

In the system of Fig.1, we apply a fuzzy logic control (FLC) for the PD$^\beta$ actions and the integral of the error is added to the output in order to find a fuzzy PD$^\beta$ + I controller (Barbosa, 2010). The block diagram of Fig. 2 illustrates the configuration of the proposed fuzzy controller.

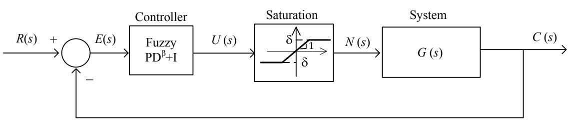

Figure 1: Block diagram of the fuzzy control system.

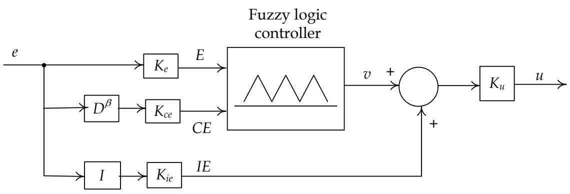

Figure 2: Fuzzy PD$^\beta$ + I controller.

In this controller, the control actions are the error e, the fractional derivative of e and the integral of e. The U represents the controller output. Also, the controller has four gains to be tuned, K_e, K_{ie}, K_{ce} corresponding to the inputs and K_u to the output.

The control action U is generally a nonlinear function of error E, fractional change of error CE, and integral of error IE:

$$U(k) = [f(E,CE) + IE]\,K_u =$$
$$\left[f\left(K_e e(k) + K_{ce} D^\beta e(k)\right) + K_{ie} I e(k)\right] K_u \qquad (8)$$

where D^β is the discrete fractional derivative implemented as rational approximation (5) using the Euler scheme (2.2); the integral of error is calculated by rectangular integration:

$$I\left(z^{-1}\right) = \frac{T_c}{1 - z^{-1}} \qquad (9)$$

To further illustrate the performance of the fuzzy PD$^\beta$+I a saturation nonlinearity is included in the closed-loop system of Fig.1, and inserted in series with the output of the fuzzy controller. The saturation element is defined as:

$$n(u) = \begin{cases} u, & |u| < \delta \\ \delta\, sign(u), & |u| \geq \delta \end{cases} \qquad (10)$$

where u and n are respectively the input and the output of the saturation block and $sign(u)$ is the signum function.

Here we give an emphasis of the proposed FLC presented in Fig. 2. The basic structure for FLC is illustrated in Fig. 3 (Passino and Yurkovich, 1998).

The fuzzy rule base, which reflects the collected knowledge about how a particular control problem must be treated, is one of the main components of a fuzzy controller. The other parts of the controller perform make up the tasks necessary for the controller to be efficient.

For the fuzzy PD$^\beta$+I controller illustrated in Fig.2, the rule-base can be constructed in the following form (see Table 1):

If E is NM and CE is NS Then v is NL

Table 1: Fuzzy control rules.

$E \setminus CE$	NL	NM	NS	ZR	PS	PM	PL
NL	NL	NL	NL	NL	NM	NS	ZR
NM	NL	NL	NL	NM	NS	ZR	PS
NS	NL	NL	NM	NS	ZR	PS	PM
ZR	NL	NM	NS	ZR	PS	PM	PL
PS	NM	NS	ZR	PS	PM	PL	PL
PM	NS	ZR	PS	PM	PL	PL	PL
PL	ZR	PS	PM	PL	PL	PL	PL

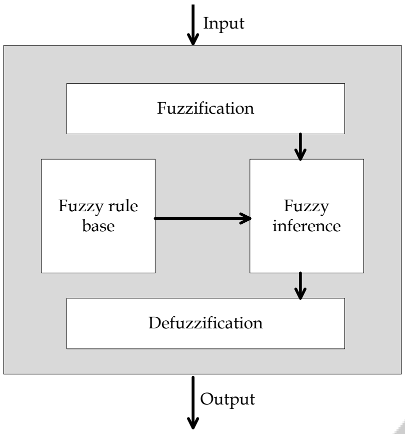

Figure 3: Structure for fuzzy logic controller.

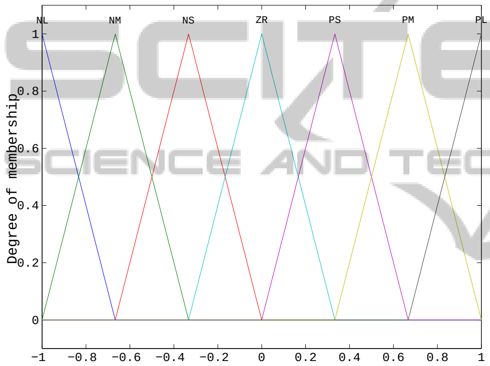

Figure 4: Membership functions for E, CE and v.

where NL, NM, NS, ZR, PS, PM, and PL are linguistic values representing ”negative low”, ”negative medium” and so on, E is the error, CE is the fractional derivative of error and v is the output of the fuzzy PD$^{\beta}$ controller. The membership functions for the premises and consequents of the rules are shown in Fig. 4.

With two inputs and one output the input-output mapping of the fuzzy logic controller is described by a non linear surface, presented in Fig.5.

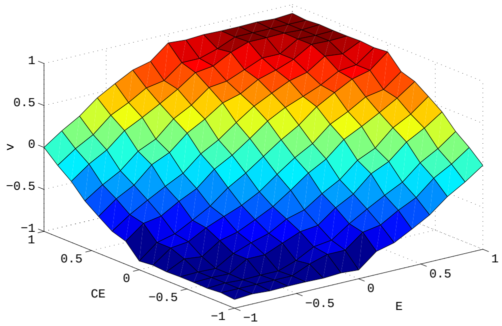

Figure 5: Control surface.

The fuzzy controller will be adjusted by changing the parameter values of K_e, K_{ce}, K_{ie} and K_u. The fuzzy inference mechanism operates by using the product to combine the conjunctions in the premise of the rules and in the representation of the fuzzy implication. For the defuzzification process we use the centroid method.

3.3 Genetic Optimization

A genetic algorithm (GA) is a search process for finding approximate solutions in optimization problems. GAs are a particular class of algorithms that use techniques inspired by evolutionary biology such as inheritance, mutation, natural selection, and crossover, established by the Darwin's theory of evolution. Some applications of GAs are in the field of robotics, nonlinear dynamical systems, data analysis, engineering and many others in the real world applications (Goldberg, 1989; Michalewicz, 1996; Jesus and Machado, 2009).

In this work we propose a fuzzy fractional PD$^{\beta}$+I controller, where the gains will be tuned through the application of a GA, in order to achieve a superior control performance of the control system of Fig. 1. The optimization fitness function corresponds to the minimization of the integral time absolute error (ITAE) criteria, that measure the response error as defined as (Jesus and Machado, 2009):

$$J(K_e, K_{ce}, K_{ie}, K_u) = \int_0^{\infty} t\,|r(t) - c(t)|\,dt \qquad (11)$$

where (K_e, K_{ce}, K_{ie}, K_u) are the PD$^{\beta}$+I controller parameters to be optimized.

4 SIMULATIONS

In this section we analyze the closed-loop system of Fig. 1 with a fuzzy fractional PD$^{\beta}$ + I controller (Fig. 2). In all the experiments, the fractional order derivative D$^{\beta}$ in scheme of Fig. 2 is implemented by using a 4th order Padé discrete rational transfer function ($m = n = 4$) of type (5). It is used a sampling period of $T_c = 0.01$ s. The PD$^{\beta}$+I controller is tuned through the minimization of the ITAE (11) using a GA. We use $\delta = 15.0$. We establish the following values for the GA parameters: population size $P = 20$, crossover probability $C = 0.8$, mutation probability $M = 0.05$ and number of generations $Ng = 50$.

In the first case, we compare a fuzzy fractional PD$^{\beta}$ controller which leads to the lower error ($\beta =$

$0.8, K_{ie} = 0$), with a fuzzy integer PD controller ($\beta = 1, K_{ie} = 0$). Figure 6 shows the unit step responses of both controllers. The plant system $G_1(s)$ used is represented by the transfer function:

$$G_1(s) = \frac{1}{s^2} \qquad (12)$$

The controller parameters, corresponding to the minimization of the ITAE index, lead to the values for the fuzzy integer PD controller: $\{K_e, K_{ce}, K_u\} \equiv \{0.8675, 0.5062, 4.5817\}$, with $J = 0.5416$, and for the fuzzy fractional PD$^\beta$ controller to the following values:$\{K_e, K_{ce}, K_u\} \equiv \{1.1459, 1.4110, 4.9945\}$, with $J = 0.3063$. These values lead us to conclude that the fuzzy fractional order controller produced better results than the integer one, since the transient response (namely, the overshoot and settling time) and the error J are smaller, as can be seen in Fig. 6.

In a second experiment, we consider a fuzzy PD$^\beta$+I controller which leads for lower error to $\beta = 0.5$, applied to a process $G_2(s)$ represented by the transfer function (13), where the time delay is $T = 1$ [s].

$$G_2(s) = \frac{e^{-sT}}{0.2s^2 + 1.2s + 1} \qquad (13)$$

Once more time, we consider for comparison the corresponding integer version ($\beta = 1$). Figure 7 shows the unit step responses of both controllers.

The controller parameters, corresponding to the minimization of the ITAE index, lead to the values for the fuzzy integer controller: $\{K_e, K_{ce}, K_{ie}, K_u\} \equiv \{1.1592, 0.2314, 1.2576, 0.3681\}$, with $J = 6.3940$, and for the fuzzy fractional controller: $\{K_e, K_{ce}, K_{ie}, K_u\} \equiv \{0.1247, 0.8682, 0.5906, 0.6945\}$, with $J = 3.7972$. These values lead us to remain the previously conclusions drawn for $G_1(s)$, namely that the fuzzy fractional order controller produced better results than the integer ones, since the transient response (in particular the rise time and overshoot) and the error J are smaller.

In conclusion, with the fuzzy fractional PD$^\beta$+I controller we get the best controller tuning, superior to the performance revealed by the integer-order scheme. Moreover, we prove the effectiveness of this control structure when used in systems with time delay. In fact, systems with time delay are more difficult to be controlled with the classical methodologies, however the proposed algorithm reveals that is very effective in the control of this type of systems.

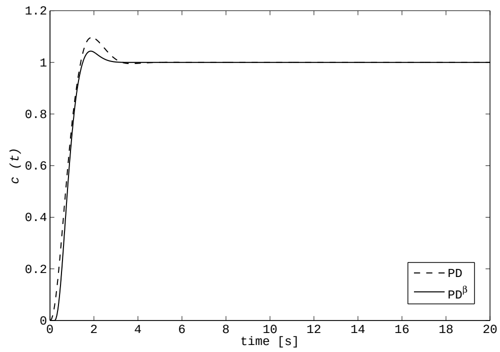

Figure 6: Step responses of the closed-loop system, with fuzzy PD and PD$^\beta$ ($\beta = 0.8$) controllers.

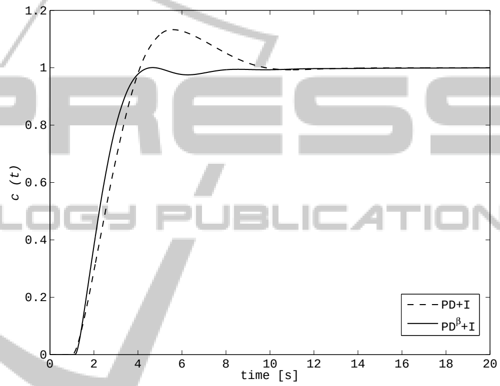

Figure 7: Step responses of the closed-loop system, with fuzzy PD+I and PD$^\beta$+I ($\beta = 0.5$) controllers.

5 CONCLUSIONS

This paper presented the fundamental aspects of application the FC theory in the control systems. In this line of thought, it were studied several systems. The dynamics of the systems were analyzed in the perspective of FC, with the use of a fuzzy PD$^\beta$+I controller in which the parameters were tuned through a GA algorithm.

In general, the control strategies presented, give better results than those obtained with conventional integer control structures, showing its effectiveness in the control of nonlinear systems.

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