Multiple External Representations in Remediation of Math Errors

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Abstract: The proposition of error remediation is a widely used feature in Intelligent Tutoring Systems, but the use of Multiple External Representations to assist it, is a research subject. This paper presents (or discuss) the use of Multiple External Representations contribution in error remediation in Learning Objects. To perform this study, we present an architectural model, a conceptual framework for mathematical error classification and Multiple External Representations, using a cognitive remediation for errors. Following is presented the application of contextual remediation of error based on Multiple External Representations in a Learning Object. And finally, we present the performance of students during the application of an experiment consisting of the following steps: pre-test, test and post-test.

1 INTRODUCTION

The analysis of mathematical errors is a great challenge, once specific knowledge of the content as well as the factors that originated them are required. The variety and the complexity of mathematical errors demand specific knowledge, fact that makes this task more difficult in regard of error classification (Peng and Luo, 2009). Nowadays, mathematical error is considered a natural stage on knowledge construction (Fiori and Zuccheri, 2005); (Peng and Luo, 2009), ie, it is a common phenomenon in the scholar trajectory of the student, wich is independent of age and/or performance level.ned the manuscript must be appropriately modified.

The classification of an error can become an enhancer agent in the acquisition of a concept by the student when applied in the proper way. Some studies applied to Learning Objects (LO) as (Marczal and Direne, 2011); (Bazzo et al., 2011); (Leite et al., 2011); (Leite et al., 2012) have already discussed this issue and presented some approaches, so that the error can assist the learner in the learning process.

The discussion presented in this study concerns the use of mathematical classification errors in order to provide its remediation through Multiple External Representations (MER) on LOs. The contribution is that the error made by the student, can provide a more appropriate remediation using a MER. This provides a better direction of learning, since the study focuses exclusively on the individual needs of each student.

The error remediation present in Intelligent Tutoring Systems (ITS) aims to provide students with the most appropriate feedback, which may be linked to the student profile or the path that they are following, interfering even before the learner makes a mistake. Moreover, the LOs tend to provide standard feedback to the learner regardless error itself. Just as educational games has the philosophy to become learning fun, but in reality usually provide less care explicit and harsher penalties than intelligent tutoring systems (Milk et al., 2010; (Easterday et al., 2011).

The MERs are providing relevant results and present themselves increasingly involved in educational materials, as well as the use of MERs have benefits when incorporated into STI, because they provide a systematic interaction (Rau et al., 2012). Other studies conducted in Brazil, also have shown results in this direction (Leite et al., 2010); (Leite et al., 2011); (Leite et al., 2012).

The aim of this proposal is to provide support to remediate student errors through MREs, which can be tables, lists, pictures, simulations, diagrams, maps, natural language text, among others. This

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support is obtained through the identification and classification of mathematical errors and the functions of the MREs, allowing the student to review and create new strategies during the learning process.

In this paper, we present the results of an experiment that used the error remediation from the errors classification and functions of MER applied to a LO involving mathematical concepts. The architectural model that supports this study is discussed as well as the modular composition.

2 METHODS

To discuss classification of errors present in the literature were chosen authors (Radatz, 1979); (Vergnaud, 1986); (Movshovitz-hadar and Zaslavsky, 1987); (Peng and Luo, 2009), which had in their research subject, the study of the mathematical error. From the cited authors, an error categorization was organized and presented by Leite, Pimentel and Oliveira (2011), with the following nomenclature: (1) misinterpretation of language, (2) directly identifiable, (3) indirectly identifiable and (4) non-categorizable solution. The rationale for the study of a mathematical classification of errors is the bond and the complexity required when proposing an MER as remediation.

The study presented by Ainsworth (2006) defined a MERs functional taxonomy, emphasizing the distinct functions of learning (and communication), used to illustrate the advantages of MERs. The External Representations (ERs) are divided in three key functions: complementary roles, to constrain interpretation and to construct deeper understanding. This classification was also incorporated into this study in order to choose an ER more related to a specific error.

Applying the approach in remediation of errors in LOs using MERs requires the LO to be implemented with a functionalist architecture (see Figure 1). The architecture consists of three main modules: the error classifier module (1), which aims to identify and classify the error by comparing the solution of the learner with the ideal solution, using production rules; the MER classifier (2), responsible for binding the proper MER function to the error, also explores the production rules; and the MRE Manager Module (3), to identify the most appropriate MER and present it to the learner in the learning session.

In order to validate this study two LOs with distinct profiles were consolidated. These LO were developed to be applied in Brazil, and this is the reason why their screenshots are presented in portuguese. The Pythagoras Max (sess Figure 2) and Pythagoras Mix (see Figure 3). The former had all the assumptions of the study to apply the MER-based remediation, and the latter had only the mathematical problem in structured form of statement, signaling that the answer was incorrect or not.

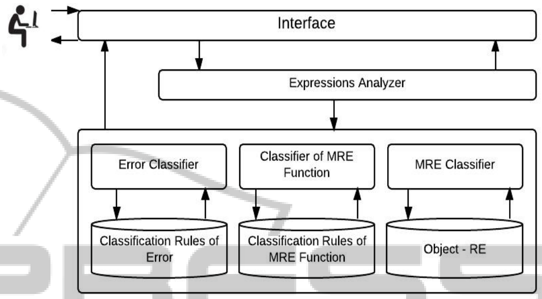

Figure 1: Functionalist architecture of the LO developed.

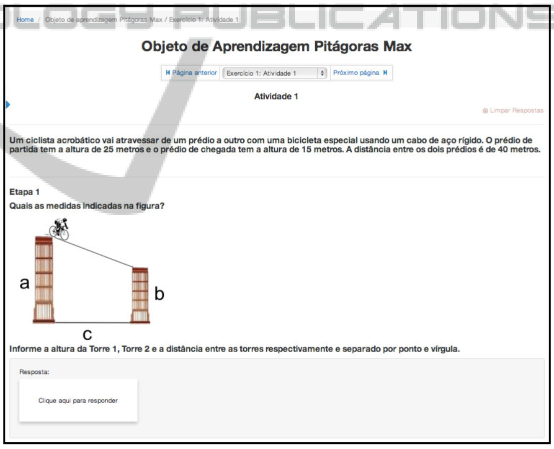

Figure 2: Pythagoras Max with MER Error Remedion.

Figure 3: Pythagoras Mix without MER Error Remedion.

3 EXPERIMENTAL DESIGN AND PROCEDURE

For the experiment 20 students from 9th grade of elementary school of a public school were taken as subjects, they were divided into two groups, the experimental group and the control group. The experimental group used the Pythagorean Max LO, which explore the process of remediation of error based on MER. The control group used the Pythagoras Mix LO, with identical questions, but without remediation of error process with MER.

The students were distributed according to the pre-test results, this test consisted of 6 questions involving concepts related to the Pythagorean Theorem, which were defined by the success percentage, and thus group they belong to. In order to make more precise and formal, pre-test grades were turned into hit percentage.

The experiment was conducted in 4 lessons, with 50 minutes long each. In the first lesson, we performed a pre-test in order to identify the prior knowledge about the content covered. In another day, participants were invited to interact with the LO in the proposed activities, to take the post-test, followed by the application of a satisfaction questionnaire. The questionnaire consisted of 12 questions regarding aspects related to the use of Learning Objects, 6 questions about ease of use, three questions regarding the feedback provided to the student and, finally, 3 issues covering the impact of using LO to learn a concept.

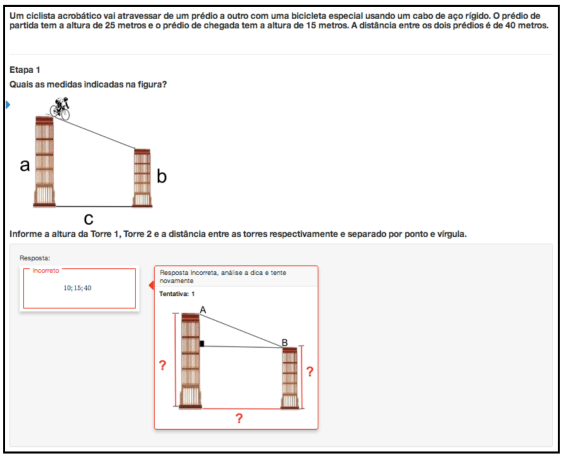

Figure 4: Pythagoras Max with MER Error Remediation: First question.

The paired t-test was used for data analysis in order to identify any significant gain in learning. Additionally, we applied a satisfaction questionnaire to the learners. Figure 4 shows an example of the Pythagoras Max problem and the respective MERs presented to remedy the error. Thus, for each type of error made (misinterpretation of language; directly identifiable; indirectly identifiable and non-solution categorizable), it is identified and classified by the Error Classification Module. Following the MRE classifier module selected the MRE function compatible with the classification error (complementary roles, barring in understanding construction and deep knowledge). Subsequent to the manager module MRE selected the most compatible representation.

4 EXPERIMENTAL DESIGN AND PROCEDURE

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The experiment was conducted during 4 (four) lessons, with 50 minutes long each. In the first lesson, we performed a pre-test in order to identify the prior knowledge about the content covered. In another day, participants were invited to interact with the LO in the proposed activities, to take the post-test, followed by the application of a satisfaction questionnaire. The questionnaire consisted of 12 questions regarding aspects related to the use of Learning Objects, 6 questions about ease of use, three questions regarding the feedback provided to the student and, finally, 3 issues covering the impact of using LO to learn a concept.

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The experiment aimed to find a positive confirmation on the use of MERs in the remediation of error. Thus, we expected to find significant results in the use of Max LO, by proposing remediation of error with MERs. Students' grades, the average and standard deviation of pre-test and post-test are shown in Tables 1.

The results confirm the hypothesis, the use of remediation of error supported in MERs from the classification error contributed to increased the student knowledge. The hypothesis of the experiment is that the Max LO helps the learner learn concepts providing a significant gain.

The performance of participants in the Pythagoras Max LO can say that it is possible to rule out the Null Hypothesis, which reached 0.05% of significance, concluding with 95% confidence that the LO brought gains to the acquisition of mathematical concepts.

Table 1: Results of Pythagoras MAX and MIX.

MAX Pythagoras			MIX Pythagoras		
Student	Pretest (%)	Postest (%)	Student	Pretest (%)	Postest (%)
A1	66,7	100,0	A1	93,3	100,0
A2	50,0	66,7	A2	56,7	96,7
A3	80,0	83,3	A3	66,7	66,7
A4	96,7	96,7	A4	93,3	100,0
A5	50,0	83,3	A5	96,7	100,0
A6	83,3	100,0	A6	93,3	100,0
A7	66,7	83,3	A7	83,3	83,3
A8	67,5	90,6	A8	100,0	66,7
A9	75,5	85,9	A9	96,7	100,0
A10	68,5	86,3	A10	79,2	93,7
Average	**70,5**	**87,6**	**Average**	**85,9**	**90,7**
Standard Deviation	**14,3**	**10,0**	**Standard Deviation**	**14,4**	**13,7**

The null hypothesis of Pythagoras Max LO is the average of the post-test is less than or equal to the average of the pretest. Furthermore, the claim whether the post-test average was significantly higher than the average pretest identifying a gain in student learning. For this purpose, we used a paired t-test, since the sample size is smaller than 30. With a confidence level of 95% ($\alpha = 0.05$), we obtain $p = 0.000412178$ ($t = 4.9202$, $df = 9$). Thus, as $p < \alpha$, we can deny the null hypothet hypothepants s in the acquisition of concepts.

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The null hypothesis of Pythagoras Mix LO is the average post-test is less than or equal to the average of the pre-test. With a confidence level, concluding that there is evidence to say with 95% confidence the Pythagoras Max LO helped the particiof 95% ($\alpha = 0.05$), we obtain $p = 0.20834$ ($t = 0.8527$, $df = 9$). Thus, as $p > \alpha$, it can not rule out the null hypothesis, concluding that there is no evidence to say that Pythagoras Mix LO assisted the participants in the acquisition of concepts. This perhaps is the fact it is a reproduction model of didactic classroom, composed solely of the problems statements.

This highlights the importance of paradigm shift when migrating from the traditional approach to computer-mediated. If LOs are not built with proper care can not help the learner and even more may end up hindering their learning.

As for satisfaction questionnaire applied to the end of the interaction with the LOs, Pythagoras Max and Pythagoras Mix: 48% of the group of questions regarding the ease of use found fully satisfactory aspects regarding the ease of use of the LO. In the other group, independently, analysing aspects related feedback, 42% manifested in a fully satisfactory as a form of feedback displayed. While the group solving tasks using the LO also analysed independently, 54% considered fully relevant using an LO for the acquisition of a concept.

5 DISCUSSION AND CONCLUSIONS

There are many advantages in using a diagnosis followed by an intervention, may be mentioned detection and remediation of errors in the same context, also is possible, in ITS, analyse partial solutions of the learner.

The MRE offered in LOs through the remedies are intended to provide further insight apprentice path to which it is following, from diagnostic error. The aim of the proposal was to provide support to students through an MRE, which can be a sentence of natural language, tables, lists, pictures, simulations, diagrams, maps, etc.

The use of remediation enables an intervention on the learner before progression on a particular error, thereby avoiding a complete solution, but misguided. As a consequence there is a reduction in the number of errors that can occur, this aspect was considered in the relevant proposal of the study.

The average found in MIX LO pre-test is bigger than the MAX LO pre-test, but in post-test we can see this difference between averages was decreased. This, is more one signal that indicating that the LO MAX positively influenced the learner acquisition of knowledge.

The architecture presented allowed facilitate the remediation of errors made by the learner, through a more specific categorization error, which is to be split in more categories, providing opportunities for a range of varieties that allows the learner to acquire mathematical knowledge. As a way to meet a higher level of granularity regarding appropriate presentation of MER, we used the classification of MERs functions.

In conclusion, the present study extends the concepts involving ITS and concept acquisition, classification of errors linked to MERs. Still want to do more experiments with a larger sample of students, expanding the validation study. This experiment was aimed to validate assumptions.

Future works, we intend to apply the architecture of remediation of errors based on a classification of errors, in the other areas of knowledge, in order to validate the modularity of the architecture and the use of MREs in remediation of errors in LO.

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REFERENCES

Ainsworth, S.: DeFT: A conceptual framework for considering learning with multiple representations. *In: Learning andInstruction*, 16, 183-198, (2006)

Bazzo, G., Direne, A., Marczal, D. (2011) "Classificação automática de erros de aprendizes humanos do processo de indução analítica" In XXII SBIE – *Simpósio Brasileiro de Informática na Educação* (SBIE-2011), p. 130-139, Aracaju, Sergipe, Brasil. SBC

Easterday, M. W., Aleven, V., Scheines, R., Carver, S. M.: Using Tutors to Improve Educational Games.; *In AIED* (2011), 63-71.

Fiori, C.; Zuccheri, L. An Experimental research on error patterns in written subtraction. *Journal Educational Studies in Mathematics*, v. 60, n. 3, pp. 323-331 (2005)

Leite, M. D.; Garcia, L. S.; Pimentel, A. R.; Sunye, M. S.; Castilho, M. A.; Silva, F. A user-interface environment as a support in maths teaching for deaf children. *In: 12th International Conference on Enterprise Information Systems* (ICEIS), 2010, Funchal. *12th International Conference on Enterprise Information Systems* (ICEIS).Funchal: Science and Technology Publications, 2010.

Leite, M. D.; Pimentel, A. R. Oliveira, F. D. Um estudo sobre classificação de erros: uma proposta aplicada a Objetos de Aprendizagem. *In: 22o. Simpósio Brasileiro de Informática na Educação*, 2011, Aracaju - SE. Anais do XXII SBIE - XVII WIE, 2011. p. 264-273. Brasil. SBC

Leite, M. D., Pimentel, A. R., M. H. Pietruchinski. Remediação de erros baseada em Múltiplas Representações Externas e classificação de erros aplicada a Objetos de Aprendizagem Inteligentes. In: *Congresso Brasileiro de Informática na Educação, 2012*, Rio de Janeiro - RJ. Anais do XXIII SBIE - XVIII WIE, 2012. Brasil. SBC

Marczal, D.; Direne, A. I. (2011) "Um arcabouço que enfatiza a retroação a contextos de erro na solução de problemas", *Revista Brasileira de Informática na Educação*, v. 19, n. 63, pp. 19-27, 2011. Brasil. SBC

Peng, A.; Luo, Z. A framework for examining mathematics teacher knowledge as used in error analysis . *For the Learning of Mathematics*, v. 29, n. 3, pp. 22-25, November (2009).

Rau, M. A., Aleven, V., Rummel, N.: Intelligent tutoring systems with multiple representations and self-explanation prompts support learning of fractions: In: *14th International Conference on Artificial Intelligence*, pp. 441-448. IOS Press, Amsterdam (2009).