Control of Mobile Manipulator with Skid-steering Platform Moving in Unknown Terrain in Presence of Disturbance

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Abstract: In this paper new approach to control of nonholonomic mobile manipulator with skid-steering platform has been presented. For mathematical model of such object, expressed in auxiliary coordinates, control law based on virtual force concept has been introduced. Skid-steering mobile platform is an underactuated control system with rectangular input matrix. In our approach it was assumed that there exists additional control input, giving additional column in input matrix and causing this matrix to be invertible. Because such actuator does not exists in reality, therefore this input was kept equal to zero equivalently. Simulations have proved that such method works properly in unknown terrain and in presence of disturbances.

1 INTRODUCTION

Mobile manipulator, which is the subject of considerations, consists of a mobile platform and an onboard rigid manipulating arm. Such robotic object can execute more complicated tasks than its components. Manipulating arm is fully controlled while mobile platform equipped with more than one axis of fixed wheels is an underactuated system.

Wheeled mobile platforms can be treated as independent robots or as an transportation part of complex robotic assemble, for instance mobile manipulators. Depending on wheels' type and way in which they are fixed to the cart, motion of wheeled mobile platforms can be realized with or without slipping effect. If no slippage effect between wheels and surface occurs, then there exists an equation describing forbidden directions for realized velocities of the system. Such equation is called nonholonomic constraint in platform's motion.

Special kind of wheeled mobile platforms are platforms with tracks. They can be modeled by cart with more than one axis equipped with fixed wheels, see Figure 1. These platforms are called skid-steering mobile platforms, due to skidding effect observed in theirs behavior.

Designing control algorithms for skid-steering platforms is a challenging task. First attempt to solve this problem can be found in the paper (Caracciolo et al., 1999), in which some artificial assumption about lateral slip during the platform's motion has been done. Such equation, although derived from slipping effect, can play a role of special nonholonomic constraint. Similar approach can be found in (Kozłowski and Pazderski, 2004) or in other papers.

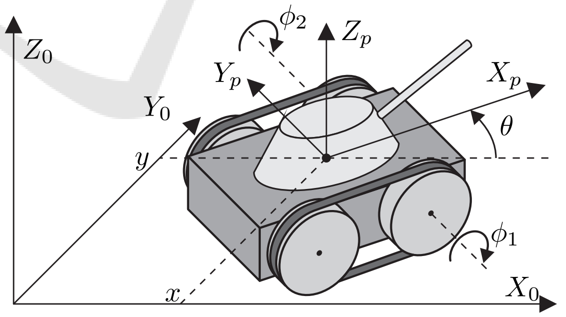

Figure 1: Scheme of mobile manipulator with skid-steering platform – tank with tracks.

Another approach to the control problem for skid-steering platform has been presented recently in (Pazderski and Kozłowski, 2008). Authors have treated skid-steering platform as an underactuated system on dynamic level with non-stationary kinematics (non-stationary velocity constraint). They have used tunable dynamic oscillator to get globally uniformly bounded stability of proposed control algorithm. The same idea can be found in (Mohammadpour et al., 2010) and (Maalouf et al., 2006).

In this paper a totally new concept for control problem of mobile manipulator's with skid-steering

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platform has been presented. Mobile manipulator with such platform can be considered as an underactuated system. Till now every control algorithm presented in literature tried to avoid problem of missing control inputs, e.g. assuming additional constraints, because underactuated mobile platform has got rectangular input matrix (which is non-invertible). Problem with inverting such matrix (standing before control signals) can be solved with the idea of so-called "virtual force". This is a essential novelty in solution of skid-steering mobile vehicles.

The paper is organized in the following way. In Section 2 concept of proposed virtual force approach to the control nonholonomic mobile manipulators is presented. Section 3 illustrates theoretical design of mathematical model of considered objects. In Section 4 the control problem is formulated. In Section 5 a new control algorithm is designed. Section 6 contains the simulation results. Section 7 presents some conclusions.

2 VIRTUAL FORCE CONCEPT

In the paper a mobile manipulator with skid-steering mobile platform has been considered. Such platform with nonholonomic constraint for longitudinal slippage can be treated as an underactuated system. It means that number of controls is smaller than number of state variables. This results in non-square, rectangular input matrix $B(q)$ that would be impossible to invert and would cause difficulties in control of mobile manipulator dynamics.

In order to solve this problem a new method to nonholonomic system control based on so-called "artificial force" is presented in this paper. Additional input, further in the text called "artificial force", is added to existing controls to make $B(q)$ square and invertible. In reality, such artificial forces do not exist with physical systems and therefore, those forces are assumed to be equal to zero equivalently.

$$u_{v,i} \equiv 0, \quad i = 1, \dots, k. \tag{1}$$

The above equations (1) describing artificial forces are equations with implicit functions. These functions depend on state variables, controller gains and desired trajectories. From these equations missing reference signals could be calculated. It is possible to solve those equations with different control algorithms. In this paper it was demonstrated how it can work for skid-steering platform.

Similar approach was used for inverse kinematic problem of redundant manipulators (non-square Jacobi matrix) with an extended Jacobian (Baillieul, 1985), (Baillieul, 1986) and to solve the control problem of underactuated system – for an inverted pendulum mounted on unicycle vehicle by (Mazur and Kędzierski, 2008).

3 MATHEMATICAL MODEL OF MOBILE MANIPULATOR

Lets consider rigid manipulator mounted on the skid-steering mobile platform with two axes of fixed wheels, presented in Figure 1. State of such object can be described by vector of platform's generalized variables q_m and manipulator's variables q_r, i.e. $q^T = (q_m, q_r)$

$$q_m^T = \begin{pmatrix} x & y & \theta & \phi_1 & \phi_2 \end{pmatrix},$$

$$q_r^T = \begin{pmatrix} q_1 & q_2 & \dots & q_p \end{pmatrix},$$

where (x, y) are position coordinates of mass center expressed in global frame X_0Y_0, θ is an orientation of skid-steering platform, ϕ_i is an angle of rotation of wheels located on the right or the left side, whereas q_r is a vector of manipulator joint variables. For generalized velocities of the skid-steering platform there exists a relationship (Kozłowski and Pazderski, 2004)

$$\dot{q}_m = \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{pmatrix} = Rot(z, \theta) \begin{pmatrix} v_x \\ v_y \\ \omega \end{pmatrix}.$$

Symbols v_x and v_y mean elements of linear velocity projected on local axes X_p and Y_p while ω is angular velocity of the skid-steering mobile platform.

In this paper an assumption only about wheels motion without longitudinal slippage was made. Moreover, it was assumed that velocity of considered mobile manipulator is small, not exceeding 10 $[\frac{\text{km}}{\text{h}}]$.

All wheels of mobile platform are identical, therefore constraints related to absence of longitudinal slippage can be expressed as follows

$$\dot{\phi}_{1r} = v_x + c\omega \quad \text{right side of the platform,} \tag{2}$$

$$\dot{\phi}_{2r} = v_x - c\omega \quad \text{left side of the platform,} \tag{3}$$

where r is a radius of wheel and c is a half of platform width. Equations (2)-(3) describing nonholonomic constraints can be expressed in so-called Pfaffian form in the following way

$$A(q_m)\dot{q}_m = \begin{bmatrix} \cos\theta & \sin\theta & -c & -r & 0 \\ \cos\theta & \sin\theta & c & 0 & -r \end{bmatrix} \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \dot{\phi}_1 \\ \dot{\phi}_2 \end{pmatrix} = 0. \tag{4}$$

3.1 Model in Generalized Coordinates

A complex object, which is the mobile manipulator, can include holonomic or nonholonomic subsystems. If any nonholonomic subsystem is present then whole robotic object must be treated as nonholonomic system. Dynamics of such systems can be obtained from d'Alembert principle as

$$M(q)\ddot{q}+C(q,\dot{q})\dot{q}+D(q)+F(q,\dot{q})= = B(q)u+A^T(q_m)\lambda. \qquad (5)$$

Dynamics (5) can be presented more in detail as

$$\begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}\begin{pmatrix} \ddot{q}_m \\ \ddot{q}_r \end{pmatrix}+\begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}\begin{pmatrix} \dot{q}_m \\ \dot{q}_r \end{pmatrix}+\begin{pmatrix} 0 \\ D \end{pmatrix} +\begin{pmatrix} F \\ 0 \end{pmatrix}=\begin{bmatrix} B & 0 \\ 0 & I \end{bmatrix}\begin{pmatrix} u_m \\ u_r \end{pmatrix}+\begin{pmatrix} A^T\lambda \\ 0 \end{pmatrix}.$$

Symbol $M(q)$ denotes inertia matrix of whole mobile manipulator, matrix $C(q,\dot{q})$ describes Coriolis and centripetal forces, vector $D(q)$ is a vector of gravity (only for manipulator because platform moves on horizontal equipotential surface), while vector $F(q,\dot{q})$ is a vector of non-potential forces, mostly reaction forces of terrain and friction forces. Torques $B(q)u$ represent control inputs (actuators) whereas $A^T\lambda$ are forces coming from nonholonomic constraints. Input matrix $B(q)$ describes relationship between input signals u and control torques

$$B\cdot u_m=\begin{bmatrix} 0 & 0 & -\sin\theta \\ 0 & 0 & \cos\theta \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}\begin{pmatrix} u_1 \\ u_2 \\ u_v \end{pmatrix}.$$

Symbols u_1 and u_2 denote control signal associated with coupled wheels on the right and left side of the skid-steering mobile platform. Third column of matrix $B(q)$ is responsible for hypothetical virtual force u_v, influenced by platform orientation.

3.2 Model in Auxiliary Velocities

Mobile manipulator with skid-steering mobile platform is a control system with nonholonomic constraints. These constraints can be transformed to auxiliary velocities η

$$\begin{aligned}\dot{q}_m &= G(q)\eta \\ &= \begin{bmatrix} \cos\theta & \cos\theta & -\sin\theta \\ \sin\theta & \sin\theta & \cos\theta \\ \frac{1}{c} & -\frac{1}{c} & 0 \\ 0 & \frac{2}{r} & 0 \\ \frac{2}{r} & 0 & 0 \end{bmatrix}\begin{pmatrix} \eta_1 \\ \eta_2 \\ \eta_{3v} \end{pmatrix}\end{aligned} \qquad (6)$$

where η_1, η_2 are scaled wheels velocities on right and left side of the mobile platform and η_3 is hypothetical velocity of orientation changes.

In turn, dynamics expressed in auxiliary velocities for nonholonomic skid-steering platform have a form

$$M^*\begin{pmatrix} \dot{\eta} \\ \ddot{q}_r \end{pmatrix}+C^*\begin{pmatrix} \eta \\ \dot{q}_r \end{pmatrix}+\begin{pmatrix} 0 \\ D \end{pmatrix}+\begin{pmatrix} F^* \\ 0 \end{pmatrix}=B^*\begin{pmatrix} u_m \\ u_r \end{pmatrix}, \qquad (7)$$

where:

$$\begin{aligned} M^* &= \begin{bmatrix} G^TM_{11}G & G^TM_{12} \\ M_{21}G & M_{22} \end{bmatrix}, \\ C^* &= \begin{bmatrix} G^T(C_{11}G+M_{11}\dot{G}) & G^TC_{12} \\ M_{21}\dot{G}+C_{21}G & C_{22} \end{bmatrix}, \\ F^* &= G^TF, \\ B^* &= \begin{bmatrix} G^TB & 0 \\ 0 & I \end{bmatrix}. \end{aligned}$$

Equations (6) and (7) constitute complete model of nonholonomic mobile manipulator with skid-steering platform, expressed in auxiliary coordinates. This model is a point of departure to design a control algorithm based on artificial force approach.

It is worth to mention that a mobile manipulator with a wheeled platform has a special property, which is not valid for its subsystems, (Duleba, 2000).

Property 1. *For a mobile manipulator with a wheeled nonholonomic mobile platform a skew-symmetry between inertia matrix M^* and the matrix of Coriolis and centripetal forces C^* does not hold anymore. To regain the skew-symmetry, a special non-trivial correction matrix C_K has to be added*

$$\frac{d}{dt}M^*=(C^*+C_K)+(C^*+C_K)^T. \qquad (8)$$

Any matrix, for which the relation (8) holds, can play role of correction matrix. The following expression describing a form of C_K matrix, e.g.

$$C_K=C_K^T=\frac{1}{2}\left\{M^*-C^*-(C^*)^T\right\}$$

should be calculated before starting the regulation process.

4 CONTROL PROBLEM STATEMENT

In this paper mobile manipulator with skid-steering mobile platform is considered. Such robotic object should move along desired trajectory without longitudinal slippage of its wheels. Desired trajectory is defined separately for each subsystem, i.e. platform should move along $q_{md}(t)$ and rigid onboard manipulator have to track vector of desired joint positions $q_{rd}(t)$, defined relative to local frame of platform.

Our goal is to address the following control problem:

1. Determine control law u such that mobile manipulator with skid-steering platform with known dynamics follows the desired trajectory even if terrain parameters are unknown or some measurement disturbances occur.
2. All virtual forces have to be equal to zero, because they do not exist in reality.

In order to design trajectory tracking controller for considered object, it is necessary to consider a complete mathematical model of the nonholonomic system (6)-(7) expressed in auxiliary variables as a cascade composed of two groups of equations: kinematics (nonholonomic constraints) and dynamics, see Figure 2:

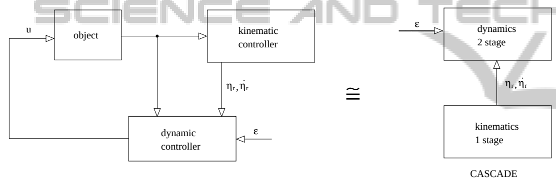

Figure 2: Structure of the control algorithm (backstepping): cascade with two stages.

For this reason the structure of the controller is divided into two parts working simultaneously (backstepping control approach, see (Krstić et al., 1995)):

- kinematic controller η_r – represents a vector of embedded control inputs, which ensure realization of the task for the kinematics (nonholonomic constraints) if the dynamics were not present. Such a controller generates 'velocity profile' which has to be executed in practice to realize the trajectory tracking for nonholonomic subsystem. Kinematic controller is called 'motion planner'.
- dynamic controller – as a consequence of cascaded structure of the system model, the system's velocities cannot be commanded directly, as it is assumed in the design of kinematic control signals, and instead they must be realized as the output of the dynamics driven by u. Dynamics are calculated for whole system, not for each subsystem separately.

5 ROBUST CONTROL ALGORITHM

As we have mentioned in previous section, control algorithm consists of two parts, i.e. kinematic controller and dynamic controller. Both control algorithms, working simultaneously, are necessary to solve the control problem of nonholonomic mobile manipulator.

5.1 Kinematic Control Algorithm for Trajectory Tracking

Considering nonholonomic constraints (6), for real case of two active controls, they are equivalent to the unicycle model. On that basis, the kinematic controller is suggested in the form given by Samson and Ait-Abderrahim (Samson and Ait-Abderrahim, 1991). This algorithm allows trajectory tracking for a simple unicycle vehicle. Unicycle velocities appropriate for tracking of desired trajectory q_{md} are described by the following equation (first and second column of matrix $G(q)$ in equation (6))

$$\dot{q}_{md} = G(q_d)\eta_d = \begin{bmatrix} \cos\theta_d & \cos\theta_d \\ \sin\theta_d & \sin\theta_d \\ \frac{1}{c} & -\frac{1}{c} \\ 0 & \frac{2}{r} \\ \frac{2}{r} & 0 \end{bmatrix} \begin{pmatrix} \eta_{1d} \\ \eta_{2d} \end{pmatrix}, \tag{9}$$

where η_{1d} and η_{2d} are desired scaled velocities for platform wheels located on the right and left side.

Desired linear and angular velocities of skid-steering mobile platform are

$$v_d = \eta_{1d} + \eta_{2d}, \qquad \omega_d = \frac{1}{c}(\eta_{1d} - \eta_{2d}).$$

The kinematic algorithm for model described by (6) and desired velocities (9) requires

$$\begin{pmatrix} v_r \\ \omega_r \end{pmatrix} = \begin{pmatrix} v_d e_\theta - e_x \\ \omega_d - k_1 e_\theta - k_2 \frac{\sin e_\theta}{e_\theta} v_d e_y \end{pmatrix}, \quad k_1, k_2 > 0, \tag{10}$$

where

- v_r, ω_r are reference linear and angular velocities for robot vehicle (signals coming from kinematic controller),
- v_d, ω_d are desired linear and angular velocities,
- k_1 and k_2 are control parameters,
- $e_\xi = (e_x, e_y, e_\theta)^T$ are reference trajectory tracking errors.

The reference trajectory tracking errors are defined as follows:

$$e_\xi = \begin{pmatrix} e_x \\ e_y \\ e_\theta \end{pmatrix} = Rot(z, -\theta) \begin{pmatrix} x - x_d \\ y - y_d \\ \theta - \theta_d \end{pmatrix}.$$

The asymptotic convergence of tracking errors e_ξ to zero implies asymptotic trajectory tracking. Reference velocities η_{1r} and η_{2r} could be obtained from relationship

$$\eta_{1r} = \frac{v_r + c\omega_r}{2}, \quad \eta_{2r} = \frac{v_r - c\omega_r}{2}.$$

The third component η_{3r} is responsible for maintain the apparent force u_v at 0. It can be obtained by solving the equation $u_v = 0$.

5.2 Dynamics Controller

Let's consider model of mobile manipulator with skid-steering platform (7), in which some disturbances $\varepsilon(t)$ in dynamics coming from platform can occur

$$M^* \begin{pmatrix} \dot\eta \\ \ddot q_r \end{pmatrix} + C^* \begin{pmatrix} \eta \\ \dot q_r \end{pmatrix} + \begin{pmatrix} 0 \\ D \end{pmatrix} + \begin{pmatrix} F^* \\ 0 \end{pmatrix} + \begin{pmatrix} \varepsilon(t) \\ 0 \end{pmatrix} = = B^* \begin{pmatrix} u_m \\ u_r \end{pmatrix}. \quad (11)$$

We assume that such disturbances are uniformly bounded i.e.

$$\forall \ t \geq 0 \qquad \| \varepsilon(t) \| \leq E_M.$$

Lets choose the dynamic control algorithm based on modification of passivity-based sliding mode control (Slotine and Li, 1988) given by Slotine & Li for robotic manipulators. We assume that dynamics of the mobile manipulator are known although parameters of reaction forces (friction coefficients) coming from ground are unknown but bounded. Therefore we use control law which tries to estimate these parameters as follows

$$\begin{pmatrix} u_m \\ u_r \end{pmatrix} = (B^*)^{-1} \Big\{ M^* \begin{pmatrix} \dot\eta_r \\ \ddot q_{ref} \end{pmatrix} + C^* \begin{pmatrix} \eta_r \\ \dot q_{ref} \end{pmatrix} + \begin{pmatrix} 0 \\ D \end{pmatrix} + \begin{pmatrix} Y_F \hat a \\ 0 \end{pmatrix} - C_K \begin{pmatrix} e_\eta \\ s \end{pmatrix} - K_d \begin{pmatrix} e_\eta \\ s \end{pmatrix} - K\mathrm{sgn} \begin{pmatrix} e_\eta \\ s \end{pmatrix} \Big\}, \quad (12)$$

where $K_d = k_d I$ and $K = kI$ are positive definite diagonal matrices of regulation parameters and C_K is correction matrix necessary to have skew-symmetry (8).

Matrix Y_F is so-called regression matrix which makes possible linear parametrization of reaction forces

$$F^*(\dot q_m) = Y_F(\dot q_m) a, \quad (13)$$

with $a \in R^p$ – a vector of real friction coefficients of terrain. If parameters a are unknown then we can use some constant estimates instead real values of these parameters.

In turn, tracking errors are defined in the following way

$$\begin{pmatrix} e_\eta \\ s \end{pmatrix} = \begin{pmatrix} \eta - \eta_r \\ \dot q_r - \dot q_{ref} \end{pmatrix} = \begin{pmatrix} \eta - \eta_r \\ \dot e_q + \Lambda e_q \end{pmatrix}, \quad (14)$$

where

$$e_q = q_r - q_{rd}$$

is joint position error and $\Lambda > 0$ is regulation matrix.

Closed-loop system (7) with feedback control (12) is given by

$$M^* \begin{pmatrix} \dot e_\eta \\ \dot s \end{pmatrix} + (C^* + C_K) \begin{pmatrix} e_\eta \\ s \end{pmatrix} + \begin{pmatrix} \varepsilon \\ 0 \end{pmatrix} + + K_d \begin{pmatrix} e_\eta \\ s \end{pmatrix} + K\mathrm{sgn} \begin{pmatrix} e_\eta \\ s \end{pmatrix} = \begin{pmatrix} Y_F(\dot q_m) \tilde a \\ 0 \end{pmatrix}, \quad (15)$$

where

$$\tilde a = \hat a - a$$

is constant parameter estimation error.

5.3 Proof of the Convergence

For the system (15) we propose the Lyapunov-like function

$$V(e_\eta, s) = \frac{1}{2} \begin{pmatrix} e_\eta & s \end{pmatrix} M^*(q) \begin{pmatrix} e_\eta \\ s \end{pmatrix} \geq 0, \quad (16)$$

which is non-negative definite.

We compute time derivative of V along solutions of the closed-loop system (15)

$$\dot V = \frac{1}{2} \begin{pmatrix} e_\eta & s \end{pmatrix} \dot M^*(q) \begin{pmatrix} e_\eta \\ s \end{pmatrix} + \begin{pmatrix} e_\eta & s \end{pmatrix} M^*(q) \begin{pmatrix} \dot e_\eta \\ \dot s \end{pmatrix}$$

$$= - \begin{pmatrix} e_\eta & s \end{pmatrix} \left[K\mathrm{sgn} \begin{pmatrix} e_\eta \\ s \end{pmatrix} + \begin{pmatrix} \varepsilon(t) \\ 0 \end{pmatrix} - \begin{pmatrix} Y_F(\dot q_m) \tilde a \\ 0 \end{pmatrix} \right] - \begin{pmatrix} e_\eta & s \end{pmatrix} K_d \begin{pmatrix} e_\eta \\ s \end{pmatrix}$$

First term in the time derivative of V is a sum of following components

$$-e_{\eta i} [k\mathrm{sgn}(e_{\eta i}) + \varepsilon_i - \sum_j Y_{F,ij} \tilde a_j] - s_i \, k \, \mathrm{sgn}(s_i) =$$

$$- \mid e_{\eta i} \mid \left[k + \varepsilon_i \mathrm{sgn}(e_{\eta i}) - \sum_j Y_{F,ij} \tilde a_j \mathrm{sgn}(e_{\eta i}) \right] - \mid s_i \mid k$$

We define parameter k as follows

$$k > \sum_j Z_{ij}\tilde{a}_{max} + E_M + \beta,$$

where $\mid Y_{F,ij} \mid \leq Z_{ij}$, $\tilde{a}_{max}$ is maximal error between real and estimated friction coefficients (known from practical measurements) and β is some positive constant. Then we obtain evaluation of $\dot{V}$ as below

$$\begin{aligned}\dot{V} &\leq -k_d(\| s \|^2 + \| e_\eta \|^2) - \sum_i \beta_i \mid e_{\eta i} \mid -k \sum_i \mid s_i \mid \\ &\leq 0. \end{aligned} \quad (17)$$

From La Salle & Yoshizawa theorem, see (Krstić et al., 1995) for details, it could be concluded that the errors e_η and s converge asymptotically to zero. Using definition of s given by (14) and positive definiteness of parameter Λ we get that position tracking error e_q for manipulator joints goes asymptotically to zero. It ends the proof.

On the other hand the convergence of e_η to zero means that the the velocity profile generated by kinematic controller is successfully followed, and therefore one can conclude that the nonholonomic system (skid-steering platform) is tracking a desired trajectory q_{md}.

5.4 Artificial Force

The third reference velocity η_{3r} can be calculated from the assumption that third component of control (12), i.e. u_v equals to zero. Solving implicit function relative to η_{3r}, we obtain

$$\dot{\eta}_{3r} = \frac{1}{m_t}[F_y + K_d(\eta_3 - \eta_{3r}) - m_t\dot{\theta}(\eta_{1r} + \eta_{2r})], \quad (18)$$

for any initial condition.

6 SIMULATION STUDY

The simulations were run with MATLAB package and SIMULINK toolbox. As an object of simulations we have taken skid-steering mobile platform equipped with two axes of fixed wheels with 2R rigid manipulator. The parameters of the platform were: mass of the platform m_p, mass of the wheel m_k, platform moment of inertia I_p relative Z_p axis, wheel moment of inertia I_k relative Z_p axis, half of platform width c, distances a and b from mass center to front and back axis of wheels.

In this section we want to show a behavior of skid-steering mobile platform tracking different trajectories – admissible and inadmissible (obtained as joining of different admissible trajectories). Simulations should show if proposed control strategy works properly for mobile manipulator with skid-steering platform. In practice the measurements of state variables are disturbed by noise, therefore we want to check influence of possible perturbations on simulations.

Table 1: Simulation parameters.

Param.	Value	Unit	Param.	Value	Unit
m_p	60	kg	c	0.75	m
m_k	3	kg	I_p	10	Nm
R_w	0.15	m	I_k	0.034	Nm
l_1	0.5	m	m_1	20	kg
l_2	1	m	m_2	15	kg
a	0.6	m	b	0.6	m

Disturbances are modeled as a white noise linked to measurement of wheels' velocities and as integration noise ε occurring in third reference velocity responsible for making artificial force equal to zero, see Figure 2 for details.

Desired trajectory has been defined as follows:

- for the platform:
  - for $t \in [0, 30]$: $(x_d, y_d)(t) = (-10 + t, -10)$,
  - for $t \in [30, 61.5]$: $(x_d, y_d)(t) = (20 + 10\sin\frac{t}{10}, -20 + 10\cos\frac{t}{10})$,
  - for $t \in [61.5, 100]$: $(x_d, y_d)(t) = (20, 10 - 1 \cdot (t - 61.5))$
- for the manipulator:

$$q_{rd}^T(t) = (2, 1).$$

It can be observed that after the time 30 s desired trajectory changes from straight line to part of circle. Next, in time 61.5 s mobile platform should change trajectory from circle to other straight line. It means that in time t=30 s and t=61.5 s trajectory has to change kinematic control from one solution (obtained for straight line task) to the another (obtained for the circle). In other words, in these time moments some impulse in tracking errors should occur.

Simulation has been made for changing parameters of ground. First, real friction coefficients has been selected as $a_1 = 0.9$ and $a_2 = 0.1$ for time interval [0,40] s. Next, after 10 s parameters changed rapidly on values $a_1 = 0.6$ and $a_2 = 0.4$. But for control law (12) these coefficients have been chosen as $\hat{a}_1 = 0.1$ and $\hat{a}_2 = 0.9$.

Regulation parameters have been chosen as follows: for dynamic controller $K_d = 500$ and $K = 10$, for kinematic controller $K_1 = 10$ and $K_2 = 10$.

6.1 Tracking without Disturbances

Real trajectory realized by the platform can be observed in Figure 3. Tracking errors for the platform and for the manipulator have been presented in Figure 4.

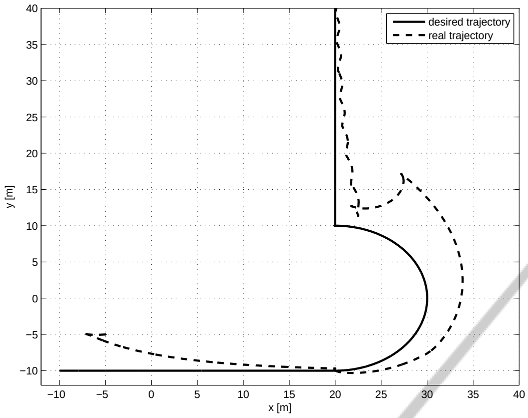

Figure 3: Trajectory realized by skid-steering platform without disturbances.

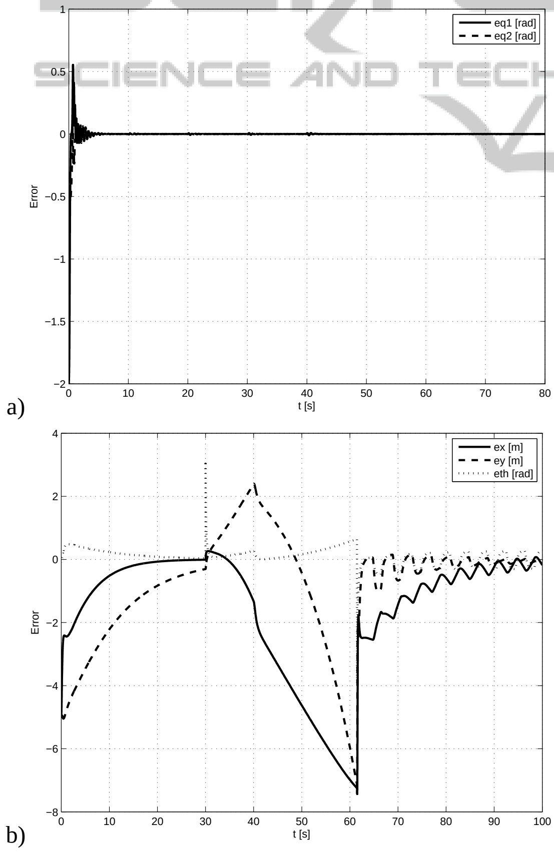

Figure 4: Errors obtained by trajectory tracking without disturbances: a) joint errors in manipulator, b) platform errors.

6.2 Tracking with Disturbances

Simulations have been made once again for system disturbed by white noise with gain 0.1 added to the signals coming from encoders and for integration of third reference velocity η_{3r}.

Real trajectory performed by the platform can be seen in Figure 5. Tracking errors for the platform and for the manipulator have been presented in Figure 6.

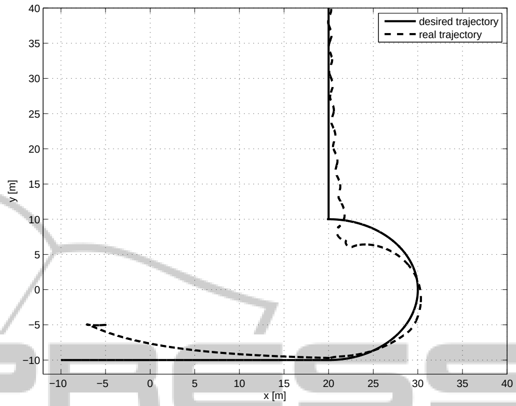

Figure 5: Trajectory realized by skid-steering platform with disturbances.

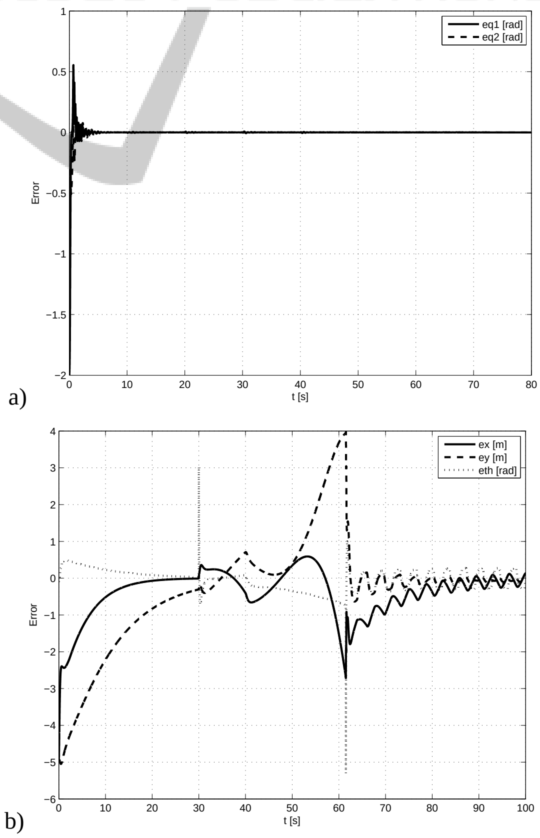

Figure 6: Errors obtained by trajectory tracking with disturbances: a) joint errors in manipulator, b) platform errors.

7 CONCLUSIONS

This paper presents a solution to the trajectory tracking problem for mobile manipulators including skid-steering platform (platform with tracks). Lack of any longitudinal slippage for the platform with nonholonomic constraint was considered as valid. Due to cascaded structure of mathematical model of the nonholonomic system, control algorithm consists of the two stages: kinematic and dynamic level. As a kinematic controller the Samson & Ait-Abderrahim algorithm dedicated to unicycle was used. This algorithm generates piecewise admissible velocity profiles. Dynamic controller is similar to sliding mode control. From Figures 4 and **??** it is easy to observe that, unawares, realized trajectory is better for the case with disturbances than for this undisturbed case. It can be explained by too small values of friction coefficients taken for control law (12). In such a case disturbance can be treated as additional excitation in control law, making feedforward part in control algorithm nearer real value of this part. Presented control law is robust on parametric and structural uncertainty in dynamics; it can work without any adaptation of unknown parameters, even if the terrain, which platform is moving in, is unknown and changes during motion.

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