Use of Multi-touch Gestures for Capturing Solution Steps in Arithmetic Word Problems

Adewale Adesina[1], Roger Stone[1], Firat Batmaz[1] and Ian Jones[2]

[1]Department of Computer Science, Loughborough University, Loughborough, U.K.
[2]Mathematics Education Centre, Loughborough University, Loughborough, U.K.

Keywords: Mathematics, Problem Solving, Multi-touch Interfaces, e-Assessments, e-Learning Tools.

Abstract: Multi-touch interfaces are becoming popular with tablet PCs and other multi-touch surfaces increasingly used in classrooms. Several studies have focused on the development of learning and collaboration potentials of these tools. However, assessment and feedback processes are yet to leverage on the new technologies to capture problem solving steps and strategies. This paper describes a computer aided assessment prototype tool that uses an innovative approach of multi-touch gestures to capture solution steps and strategies. It presents a preliminary effort to investigate the capturing of solution steps involving a two-step arithmetic word problem using the approach. The results suggest that it is possible to perform two step arithmetic work with multi-touch gestures and simultaneously capture solution processes. The steps captured provided detailed information on the students' work which was used to study possible strategies adopted in solving the problems. This research suggests some practical implications for development of automated feedback and assessment systems and could serve as a base for future studies on effective strategies in arithmetic problem solving.

1 INTRODUCTION

Assessment is central to the learning experience (JISC, 2010). In recent years, there has been an increasing interest in using technology to enhance assessment and feedback processes. New technologies are revolutionizing work, play and study. The technologies suggest new opportunities to include touch and physical movement, which can benefit learning, in contrast to the less direct, somewhat passive mode of interaction suggested by a mouse and keyboard. Current research reveals that the ownerships of technologies such as tablets and hand held devices among learners are likely to be widespread (Heinrich, 2011). Other studies have shown that hand held tablet devices and smart phones have significant and very positive impact on learning and motivation of students; leading to increased capacities to research, communication and collaboration(Banister, 2010; Gasparini, 2012; Heinrich, 2011).

Despite the digitally enhanced landscape in which learning now takes place, assessment and feedback practices are yet to fully leverage on the technology to provide innovative solutions to identified problems. A criticism of some implementations of computer aided assessment is that the design sometimes limits creative problem solving. The most common question type used in such systems tends to be based on convergent, selected responses. Some practitioners have argued that the practice has little pedagogic value beyond testing surface learning (Hommel et al. 2011).In solving a two-step arithmetic word problem for instance, selecting a single best answer among other options for grading presents some difficulties. First, only the final answer is compared against the correct answer, making it difficult to obtain intermediate results or award partial marks as possible with paper based assessments. Second, solution paths or strategies are not explicit. The limited information on the steps and strategies makes it difficult to give detailed and personalized feedback on a student's work.

This paper discusses a new and innovative approach to computer-aided assessment that uses multi-touch gestures to capture solution steps and strategies. A small pilot study was conducted using the prototype tool to obtain and examine solution steps of a two-step arithmetic word problem.

2 APPROACH

Effectively capturing solution steps and strategies requires a tool that is educationally justified. It must follow sound pedagogic principles and contribute to learning, and it should provide an environment that freely allows creative problem solving without increasing cognitive load. It should be possible to capture solution steps without disturbing the user.

Multi-touch interaction is a new technique that allows users to interact naturally with digital objects in a physical way, and could help to address the requirements. The pedagogic advantages of using gestures have been studied (Drews & Hansen, 2007; Goldin-Meadow & Beilock, 2010; Segal, 2011) .The studies show that multi-touch technologies can benefit cognition and learning (Barsalou, Niedenthal, Barbey, & Ruppert, 2003), augment working memory (Goldin-Meadow, 2009) . Also, the mode of interaction allows for bimanual input which increase the parallelism of manipulations and reduce the time of task switching(Jiao, Deng, & Wang, 2010).

Consider a two-stage arithmetic word problem that involves three numbers, say 2+5+8. Students are typically taught to solve the problems in two separate stages i.e. by adding numbers in pairs. Fischbein et al. (1985) argued that intuitive models associate addition with *putting together*. The first stage adds 2 and 5; using bimanual multi-touch interaction makes it possible to simultaneously work on the two numbers. Although it is possible to use single touch to interact with the numbers one at a time, it is rather cumbersome, less intuitive and requires too many steps. The first step produces an intermediate result which is used in the next stage. It is interesting to note the first step has six possible combinations (2+5, 2+8, 5+2, 5+8, 8+2, and 8+5) and the second step similarly has six possible correct combinations of the number pairs (7+8, 10+5, 7+8, 13+2, 5+10, and 2+13). The diversity of solution paths increases if the other arithmetic operators (-, x, ÷) are required to solve the problems. Capturing the particular number choices made by the student during the interactions should provide detailed feedback on the steps the student has taken to solve the problem. This feedback provides an opportunity to examine the strategies adopted in tackling the problem.

To capture the solution steps without increasing the cognitive load,(Chandler & Sweller, 1991) the tool needs to implement a smooth user interface which allows students to enter the solutions freely and easily. The interface should present the question and the solution work areas. For this study, the problem text and the solution workspaces are placed together on the same page. This aids the student memory of the problem context and requirements. This arrangement is known to have pedagogical value and has been used in different studies (Suraweera & Mitrovic 2002; Stone et al. 2009; Batmaz et al. 2009) .Also, it allows the student to focus fully and continuously on the task at hand without having to flip back and forth between pages. Another advantage is that it facilitates user interactions between the workspaces with minimal disruption. The solution space will not provide any toolbox, options or hints and should allow free form entry design space.

The method of capturing steps and strategies is comparable in complexity to that used for design rationale capture – an area widely studied. Design rationale has been defined as the reasoning and argument that leads to the final decision of how the design intent is achieved (Sims, 1997). A variety of methods have been used to capture the rationale, each has its advantage and disadvantages. A method known as reconstruction method captures the rationale after the design. This approach does not interrupt the flow of the design effort but does not provide accurate or complete rationale capture, because people usually do not accurately explain how or why they do things. Another method referred to in literature as apprentice system Sims 1997), requires asking the designer questions as the design action is carried out. This method is time consuming and frequently interrupts the design effort. A third approach captures the rationale implicitly. This approach is used for this work as it does not obstruct the process and has minimal time overheads.

2.1 The Multi-touch Arithmetic Tool

The prototype tool developed on the iPad is called the multi-touch arithmetic tool (MAT). The tool supports questions of different complexities including all arithmetic operations and provides and captures solution steps. Figure 1 presents a description of the tool. It has word problem pane on which questions are presented to the student and the solution pane.

The word problem text section presents problems with numeric values that can be dragged to the solution area by using simple touch and drag gestures with one or both hands. The numbers dragged on the solution pane are referenced to the numbers on the problem text using techniques developed by Batmaz and Hinde (2006). The bottom
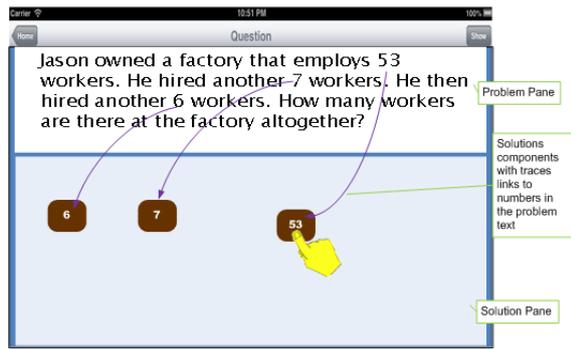
Figure 1: The Multi-touch Arithmetic tool.

pane is the working pane where the student solves the problem From the problem pane, the student chooses the numeric values and drags them to the solution as illustrated in Figure 1. Two or more numbers can be moved this way. When this is done the student simultaneously selects any pair of numbers by touch holding them for about 3 seconds (so called long press gesture), this action brings up a pad containing arithmetic operators from which the user selects an appropriate operator to solve the problem (shown in Figure 2).
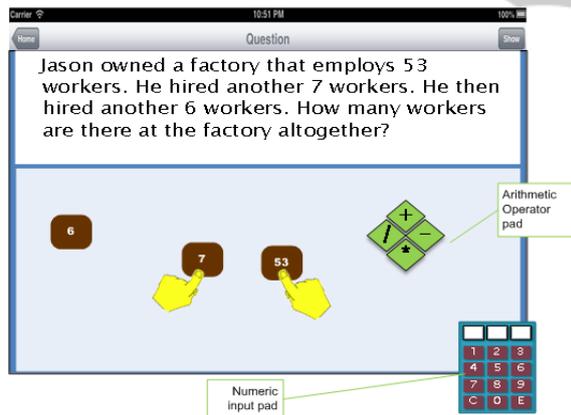

Figure 2: Performing arithmetic operation with two hands.

For example Figure 2 shows a question involving the addition of employees, the problem has three numbers in it which can be carried to the solution pane. The user selects the two numbers to apply an arithmetic operator by dragging the numbers together. Note users can only apply an arithmetic operator on the number pair they choose. This gives the opportunity to capture the two numbers the student is working on. A successful selection of an arithmetic operator results in a display of a numeric key pad, through which a calculated result is inputed.

The result of this intermediate step is fed to the next stage of the solution process by the same drag or pan gesture, whiles the touch and hold gesture is used as above for the arithmetic operation (Figure 3).
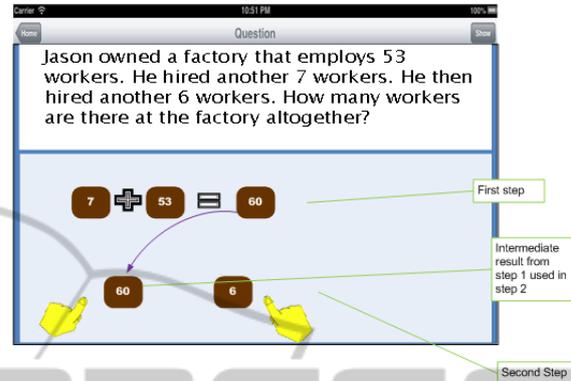

Figure 3: Using intermediate results as inputs to other steps.

The same process is repeated for all numbers and intermediate results in the problem text until a final solution is arrived at. Figure 4 shows the feedback of the solution process.
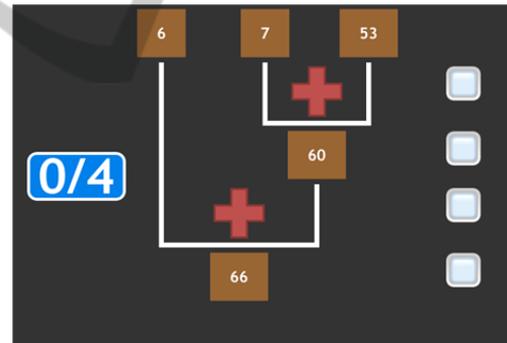

Figure 4: Feedback on solution steps.

The figure indicates that the first step used for solving the problem is $7 + 53$. The result of this step is 60 – an intermediate result which was used in the second step. The second step used the result with the third number i.e. $60 + 6$ to obtain a final result of 66. The individual steps and intermediate results can be assigned marks and graded.

3 PILOT STUDY

The study described in this section was set out to determine if students can successfully solve the arithmetic problems using the prototype tool.

Participants were Loughborough University students. It is assumed that they will not have difficulty with arithmetic tasks but are unfamiliar with the multi-touch approach. Although the participants are university students, we believe the findings may be relevant to younger learners as well. Seventeen students were enrolled for the study. An introduction session was given to each participant on sample question to intimate them on how to use the tool to solve problems. After this, they were asked to solve three word problems using the techniques demonstrated. The word problems used are shown in Table 1.

Table 1: Two Step Arithmetic Word Problems.

Problem	Strategy
Q1.William had 7 bottles of wine. His father gave him 41 more bottles of wine. His mother gave him 9 more bottles of wine. How many bottles of wine did William have altogether?	start with 41 then add 9 then add 7
Q2. Sara has 8 sugar donuts. She also has 5 plain donuts and 32 jam donuts. How many donuts does Sara have altogether?	start with 32 then add 8 then add 5
Q3. Jason owned a factory that employs 53 workers. He hired another 7 workers. He then hired another 6 workers. How many workers are there at the factory altogether?	start with 53 then add 7 then add 6

It was hypothesized that two main strategies would be detected from the output results, namely the *place-value strategy* in which the student starts by selecting two numbers that sum to a multiple of 10 in order to reduce computational burden (e.g. 41 + 9 would be the first number bond in the example given above), and the '*as presented' strategy* – where the order numbers appear in the question is followed from left to right. We also anticipated that some students would select numbers arbitrarily.

The numbers were chosen to support the use of the place-value strategy by students such that in each problem there is a large (two-digit) number, and a corresponding small (single-digit) number that sum to a multiple of 10. In each problem the two-digit number is presented in a different position: 2nd in question 1; 3rd in question 2; 1st in question 3. The particular values were selected so that adding the single digit numbers was not too easy, i.e. every single digit addition requires a carry over. The large numbers were selected such that each question is

most easily answered by starting with the large number, and then one of the smaller numbers (i.e. the place-value strategy). Question 3 presents the numbers in strategic order. This is a control question to help us work out if any participants consistently either (i) just chose numbers from left to right or (ii) just choose numbers arbitrarily.

It can be hypothesized that those with a conceptual grasp of addition will consistently use the place-value strategy described above. Those who do not have a conceptual grasp of addition will go left to right, select numbers arbitrarily or only make partial use of the place-value strategy.

4 RESULTS AND DISCUSSION

The present study was designed to determine the suitability of the multi-touch approach in solving arithmetic word problems without constraining problem solvers. The solution steps were captured for feedback and assessment purposes. The results obtained from the students showed each step to have five to six different solution paths.

To assess the usability of the multi-touch approach, the participants on completing the tasks were asked to respond on their being able to solve the problems. The overall response to this question was very positive, all the participants expressed that they were able to successfully carry out the tasks. Analysis of the detailed results generated on the tool showed that 98% of the participants had correct answers. Only one participant approached a question using subtraction rather than addition, and this may be due to his lack of proper understanding of the question. To assess how comfortable the participants were with the solution process, they were asked to response to a Likert-type question on a six point scale on how easy it was to use the tool. Over half (53%), responded that the found the tool moderately easy to use, 35% found it very easy while the others (15%) reported using it was sort of easy. While the study did not set out to test arithmetic ability, the results suggest that the tool did not prevent the students from solving the questions and inputting answers thought to be correct.

Turning now to the question on strategies, the steps and order in which the participants answered the problems were all captured. An analysis of the responses showed different patterns or strategies to solving the problems can be detected. It was hypothesized that two major strategies can be implied from the order the numbers were paired, (e.g. here we do not discriminate between

participants paired 41 + 7 from those who paired 7 + 41). The strategies output from the tool are summarized in Table 2. Across all participants just over half of the questions were solved by starting with a place-value addition that resulted in a round number (e.g. 53 + 7). However the use of strategies varied across the three questions. Fewer than half of the participants used the place-value strategy for questions 1 and 2 whereas most participants appeared to use it for question 3. These between-question differences were significant, $\chi^2(2, N = 17) = 6.75$, $p = .034$, suggesting participants were more disposed to using the place-value strategy for question 3 than they were for questions 1 and 2.

Table 2: Strategies used by the participants to solve the questions.

STRATEGY	Q1	Q2	Q3	TOTALS
place-value	6	7	13	26
other	11	10	4	25

This result is consistent with our hypothesis that some participants would use the place-value strategy, and others would use the 'as presented' strategy. For questions 1 and 2 those using the place-value strategy could be discriminated from those using the 'as presented' strategy. However question 3 was deliberately designed such that the place-value and 'as presented' strategies were the same (53 then 7 then 6). Therefore the reason most participants appeared to use the same strategy in question 3 is that the place-value and 'as presented' strategies were counted together.

The results suggest that while some participants were disposed to using the place-value strategy, overall most participants used the 'as presented' strategy on most questions. In light of this finding we scrutinized the data for evidence of our hypothesized distinct groupings of participants. The small sample size (17) and small number of questions (3) meant this was merely a descriptive exercise and no generalizable conclusions can be drawn. Nevertheless, we anticipated a 'larger' group who consistently answered all three questions by adding the numbers as presented, and a 'smaller' group who consistently used the place-value strategy. To a limited extent this is what we found: 3 of the 17 participants consistently added the numbers as presented, whereas only 1 consistently used the place-value strategy. Although these numbers are small they are encouraging given the size of the data set, and demonstrate how in

principle the tool might enable the detection of distinct arithmetic strategies.

5 CONCLUSIONS

This paper has investigated the approach of using multi-touch gestures to solve two step arithmetic questions. The pilot study set out to capture solution steps as the problems were solved and obtain feedback from the participants on usefulness of the approach. The results showed that students were able to freely solve arithmetic problems without being constrained to limited options or solution paths. The tool demonstrates that detailed information on solution steps can be captured without obstructing a creative problem solving process. Analysis of captured data suggests that solution strategies can be detected.

However, the findings are subject to at least two limitations. First, the study used a convenience sample size – which was sufficient for descriptive purposes, but may not suffice to reach generalizable conclusions on the strategies. Second, university students were the participants used to acquire feedback on the approach and to generate multiple solution paths. While the findings are useful and applicable to students, they may not be transferable to children.

Nevertheless, the study suggests several courses of action: Further experimental investigations on a larger population involving primary school children are required to determine if a relationship exists between strategies and successful problem solving. The diversity of solution paths is likely to increase the marking and feedback workloads of teachers if done manually, a next step will be the study and development of automated or semi-automated marking techniques.

REFERENCES

Banister, S., 2010. Integrating the iPod Touch in K–12 Education: Visions and Vices. *Computers in the Schools*, 27(2), pp.121–131.

Barsalou, L.W. et al., 2003. Social Embodiment. In *Psychology of Learning and Motivation*. Academic Press, pp. 43–92.

Batmaz, F. & Hinde, Chris J., 2006. A diagram drawing tool for semi–automatic assessment of conceptual database diagrams. In *10th CAA International Computer Assisted Assessment Conference*. Lougborough University, pp. 71–84.

Batmaz, F., Stone, R. & Hinde, C. J., 2009. Personal Feedback with Semi-Automated Assessment Tool for Conceptual Database Model. In *10th Annual Conference of HEA-ICS*. University of Kent, Canterbury UK, pp. 115–119.

Chandler, P., & Sweller, J. (1991).). Cognitive Load Theory and the Format of Instruction *Cognition and Instruction*, *8*(4),pp.293–332.

Drews, D. & Hansen, A., 2007. *Using Resources to Support Mathematical Thinking: Primary and Early Years* 1st ed., Learning Matters.

Fischbein, E. et al., 1985. The Role of Implicit Models in Solving Verbal Problems in Multiplication and Division. Journal for Research in Mathematics Education, 16(1), pp.3–17.

Gasparini, A.A., 2012. Touch, learn, play - what children do with an iPad in the classroom. Available at: https://www.duo.uio.no/bitstream/handle/123456789/9015/Gasparini.pdf?sequence=2 [Accessed January 17, 2013].

Goldin-Meadow, S., 2009. How Gesture Promotes Learning Throughout Childhood. Child Development Perspectives, 3(2), pp.106–111.

Goldin-Meadow, S. & Beilock, S.L., 2010. Action's Influence on Thought: The Case of Gesture. Perspectives on Psychological Science, 5(6), pp.664–674.

Heinrich, P., 2011. The ipad As A Tool For Education A study of the introduction of iPads at Longfield Academy, Kent. Available at: http://www.naace.co.uk/publications/longfieldipadresearch [Accessed December 20, 2012].

Hommel, B. et al., 2011. Bilingualism and Creativity: Benefits in Convergent Thinking Come with Losses in Divergent Thinking. Frontiers in Psychology, 2.

Jiao, X., Deng, H. & Wang, F., 2010. An Investigation of Two-Handed Manipulation and Related Techniques in Multi-touch Interaction. In 2010 International Conference on Machine Vision and Human-Machine Interface (MVHI). 2010 International Conference on Machine Vision and Human-Machine Interface (MVHI). pp. 565 –568.

JISC, 2010. Effective Assessment in a Digital Age: A guide to technology-enhanced assessment and feedback.

Kabbash, P., Buxton, W. & Sellen, A., 1994. Two-handed input in a compound task. In Proceedings of the SIGCHI Conference on Human Factors in Computing Systems. CHI '94. New York, NY, USA: ACM, pp. 417–423.

Segal, A., 2011. Do Gestural Interfaces Promote Thinking? Embodied Interaction: Congruent Gestures and Direct Touch Promote Performance in Math. ProQuest LLC.

Sims, R.C.H., 1997. Interactive learning as an" emerging" technology: A reassessment of interactive and instructional design strategies. Australian Journal of Educational Technology, 13, pp.68–84.

Stone, R.G., Batmaz, F. & Hinde, Chris J., 2009. Drawing and marking graph diagrams. ITALICS, 8(2).

Suraweera, P. & Mitrovic, A., 2002. KERMIT: A constraint-based tutor for database modeling. *Intelligent Tutoring Systems*.