Mathematics, Technology Interventions and Pedagogy

Seeing the Wood from the Trees

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Abstract: This research explores recent technological interventions in mathematics education and examines to what extent these make use of the educational opportunities offered by the technology and the appropriate pedagogical approaches to facilitate learning. In an attempt to address this, a systematic literature review has been carried out, and a classification is presented that categorises the types of technology as well as the pedagogical foundations of the interventions in which those technologies are used. The potential of technology to fundamentally alter how mathematics is experienced is further investigated through the lens of the SAMR hierarchy, which identifies four levels of technology adoption: substitution, augmentation, modification and redefinition. Classification of the interventions in this paper thus ranges from enhancing traditional practice, to transforming teaching and learning through redefinition of how tasks and activities are planned and carried out. The results of the research will be beneficial for guiding teaching, increasing our understanding of learning in a technology rich environment, and improving mathematics education.

1 INTRODUCTION

When mathematics teachers first consider the integration of technology into their daily class activities, they can be faced with an overwhelming array of devices, software, and instructional approaches, with no clear guide as to best practice. Hoyles and Noss (2003) highlight that the focus of research on digital technologies for mathematics education tends to concentrate on identification of the potential of a particular technology and the pitfalls or obstacles to its integration, and then discusses its mediation through activities, and the role of the teacher. The aim of this research is to gain some clarity regarding pedagogical approaches to technology interventions in post-primary mathematics education, as documented in recent literature, with the goal of generating an overview of the properties of interventions that are deemed to be successful. The main objectives are to increase the understanding of the kinds of teaching and learning of mathematics that technology has the potential to enhance and the generation of a set of guidelines for the implementation of such activities. A long-term goal is to create, and test, a pragmatic and comprehensive 21st Century model of classroom practice for mathematics education.

In order to address these issues, this paper first conducts a review of research that discusses issues and approaches to technology interventions in mathematics education. Following from this, the need for a system of classification is investigated, along with some existing models. A systematic literature review on a selection of twenty five papers that discuss specific interventions is carried out in accordance with the methodology discussed in section 4. The resulting data are analysed through the lenses provided by the emergent classification. A set of guiding principles for the appropriate use of technology in mathematics education is presented in the final section of the paper.

2 BACKGROUND

2.1 Pedagogic Approach

Many of the empirical studies examined for this paper are limited in that they concentrate on implementations of specific technology and do not focus on the more pragmatic issues around technology interventions that teachers require.

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However, there is a clear trend towards a socially constructivist approach, indicating that technology interventions in mathematics education may be suited to an active and collaborative environment. This pedagogical theory has its foundations in the work of Kolb, Vygotsky and Bruner, and its positive effects have been borne out through the results of the longitudinal SPRinG study in the UK (Blatchford et al., 2003).

The emphasis on sense-making and problem solving, in particular in a social context, that becomes evident through the classification of the literature, informs the development of some of the guiding principles presented in this paper.

2.2 Further Areas of Consideration

Prior to the initiation of any intervention, it is of utmost importance to look at the circumstances under which learning can be enhanced by technology (Means, 2010). Oldknow (2009) suggests that the transformative potential of ICT is not restricted to new, or purpose built technology, but also lies in the innovative uses of everyday equipment.

Oates (2011) and Geiger, V., Farragher, R., and Goos, M. (2010) provide evidence that the outsourcing of computation through the use of technologies such as Computer Algebra Systems (CAS) has the potential to do more than just improve speed and accuracy. It can also provide increased opportunity for the development of investigative skills and problem solving.

Means (2010) and Oates (2011) highlight that if technology is to be truly integrated into teaching and learning then the assessment potential that it offers needs to be utilised where possible. Assessment can be administered through computer based testing, intelligent tutoring systems, use of collaborative documents or knowledge fora (Lazakidou and Retalis, 2010), or student devices networked to the teacher console (Noss et al., 2012).

It is also noted (Means, 2010) that teachers who actively facilitate and scaffold their students interactions with the technology are in a position to use their insights to refine the activities and inform instruction. In essence, the students' interactions with the technology can contribute to their formative assessment.

Innovation with regard to the working environment and class routine are seen as necessary in order to fully exploit the potential of technology in the teaching and learning of mathematics. Means (2010) points out that, contrary to popular belief, higher learning gains are evident when there is not a one-to-one relationship between the student and the technology, thereby encouraging collaboration and team-work.

Issues such as the professional development of teachers also emerge as essential for the successful integration of technology in educational settings, but these concerns are beyond the scope of this research

3 CLASSIFICATIONS

Classifications are not definitive descriptions, but should reflect a theory about the current situation; they should be dynamic and able to keep pace with the changes to the status quo. They should permit generalisation, and provide a basis for explanation of the emerging argument. In this case, the classification system is being developed to shed light on the current trends in technology usage in mathematics education, with a view to informing a set of guidelines for future interventions in the field.

Prior to the development of the classification of the literature presented in this paper, some existing systems were identified and considered. Four areas emerged as being of interest: technology, levels of adoption, learning theory and instructional approach. These will now be discussed in more depth.

3.1 Existing Classifications of Technology

The classification systems of Clarebout and Elen (2006) and Passey (2012) were considered, but were deemed unsuitable due to issues around relevance to mathematics and levels of complexity. Two classifications of technology for mathematics education by Hoyles and Noss however, are influential in this research. They are specific to mathematics education and, while being concise, provide an appropriate level of detail.

The first, (Hoyles and Noss, 2003) distinguishes between *programming* tools and *expressive* tools. Programming tools, such as microworlds, are defined as lending themselves to individual expression and collaboration. Expressive tools on the other hand, provide easy access to the results of algorithms and procedures, without the user being required to understand the intricacies of their calculation. The category of expressive tools is further broken down into pedagogic tools, designed specifically for the exploration of a mathematical domain, and calculational instruments, which are frequently adapted to, rather than designed for, pedagogic purposes. Dynamic Graphical

Environments, such as GeoGebra, are examples of pedagogic tools, and spreadsheet programs would fall into the category of calculational instruments.

In their later research, Hoyles and Noss (2009) classify tools according to how their usage shapes mathematical meanings. They refine and extend their previous framework differentiating between *dynamic and graphical* tools such as Cabri and Geometers Sketchpad; tools that *outsource the processing power*, of which computer algebra systems are an example; new *semiotic* tools, which may have the potential to influence how mathematics is represented; and tools that increase *connectivity*, such knowledge fora.

3.2 Emerging Classification of Technology

In this study, the classifications by Hoyles and Noss are further refined and amalgamated to provide the foundation for the technological component of the emerging classification. There is no evidence in the papers reviewed of semiotic tools that change the representational infrastructure of mathematics, and it has thus been removed from the presentation of the findings. Through the review of the papers it emerged however, that an extension of the Hoyles and Noss classification was required. The category of *toolkit* is therefore added as a distinct class. Integral to the definition of this new category is the design of technologies in accordance with a specific pedagogical approach, along with the provision of support for the student and the teacher through tasks and lesson plans, and feedback for assessment, all founded in the relevant didactic theory. Examples include Noss et al. (2009) and Tangney et al. (2010).

The technologies in the literature reviewed in this paper are thereby classified as follows:

- Outsourcing of Processing power
- Dynamic Graphical Environments (DGE)
- Purposefully Collaborative
- Simulations/Programming
- Toolkit

3.3 Classifications of Technology Adoption

Two perspectives on the adoption of technology in mathematics interventions were considered: the FUIRE model (Hooper and Rieber, 1995) and the SAMR hierarchy (Puentedura, 2006). While the FUIRE model provides information on an individual's use of the technology and their level of adoption of it in the classroom, the SAMR model is better fitted to describing the level of adoption present in a given intervention and as such, is the model selected for this classification of the papers.

The SAMR hierarchy (Figure 1) is broken down into the two broad categories of Enhancement and Transformation, each of which has two further subsections. The lowest level of Enhancement is classed as Substitution. This describes situations in which the technology is used as a direct substitute for the traditional method, without functional change as exemplified by the reading of classic texts online. The second level is that of Augmentation, in which the technology is used as a substitute for an existing tool, but with some functional improvement, e.g. if the text being read contains links to online study guides.

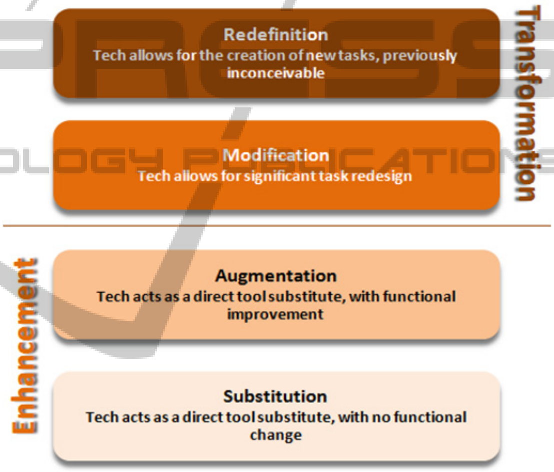


Figure 1: The SAMR Hierarchy (Puentedura, 2006).

The Transformation space on the SAMR hierarchy describes interventions that either significantly redesign the tasks provided through the use of the technology (modification), or that have used the affordances of the technology to design new tasks that would previously have been inconceivable (redefinition).

3.4 Additional Dimensions of the Classification

In addition to grouping the papers reviewed in this study by technology and level of adoption, this research also classifies them according to learning theory and instructional approach. These additional dimensions will now be expanded.

3.4.1 Learning Theories

The learning theories considered fall into two main camps: *Behaviourism* (Skinner, 1938) and *Cognitivism* (Bruner, 1977). Some cognitive learning activities can be further classified as *Constructivist* (Kolb, 1984), and within this, as *Constructionist* (Papert, 1980) or *Social Constructivist* (Vygotsky, 1978).

3.4.2 Instructional Approaches

The Instructional Approaches taken into account are:

- Drill and practice.
- Task Based.
- Individual work.
- Contextual.
- Inquiry.
- Plenary or whole class discussion.
- Realistic.
- Sense making.
- Active learning.
- Problem solving.
- Collaborative.

Most of the interventions examined adopted more than one instructional approach, with up to five distinguishable in some cases.

4 METHODOLOGY

The electronic databases searched in the review of recent literature were chosen for their relevance to education, information technology and mathematics: ERIC (Education Resources Information Center), Science Direct, and Academic Search Complete. The general search terms used were:

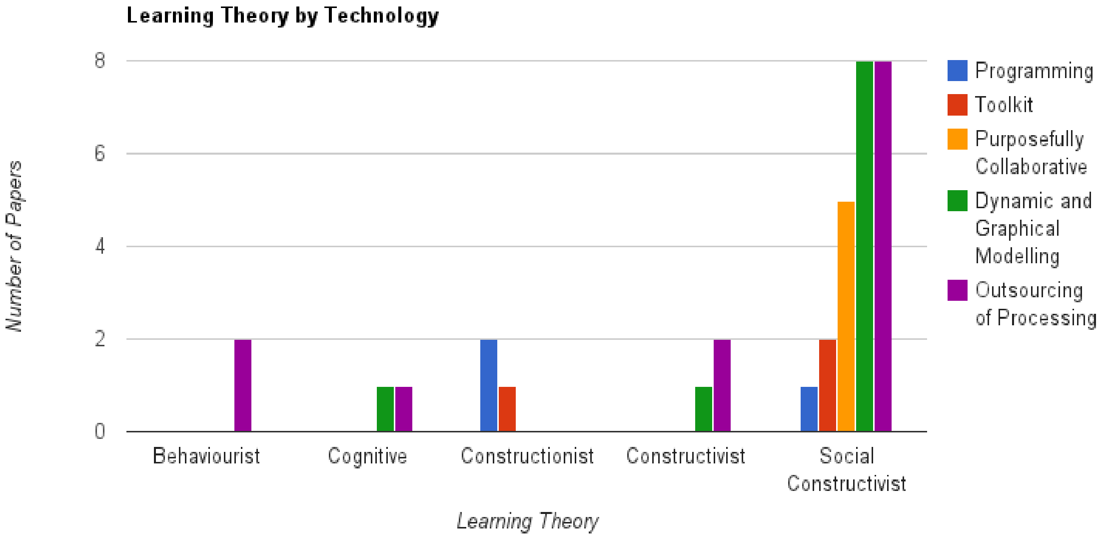
math* AND (technolog* OR tool*) AND education

These were used in an initial pass over the databases, and the results were then refined by limiters such as "secondary education". In order to further restrict results, only peer-reviewed, journal articles, issued between 2009 and 2012, were considered. A preliminary set of thirty four papers were selected for initial analysis, and of these, twenty five make up the final data set. The remaining nine papers were not included as they did not discuss specific interventions. However, a number of them did compare interventions in general and were useful in informing the set of guidelines that aim to describe a method of successful integration of technology in mathematics education.

5 ANALYSIS OF THE DATA

The data emerging from the literature review were coded and stored in a spreadsheet pivot table. This allowed the information to be arranged, related and visualised in diverse and meaningful ways. A summary of the classified papers is presented in scss.tcd.ie/~braya/csedu/The%20Papers.pdf.

Through this process, a number of interesting patterns became available. Figure 2 illustrates the clear socially collaborative trend in the literature, as well as the concentration on Outsourcing of Processing, and Dynamic Graphical Modelling Environments as the technologies of choice.



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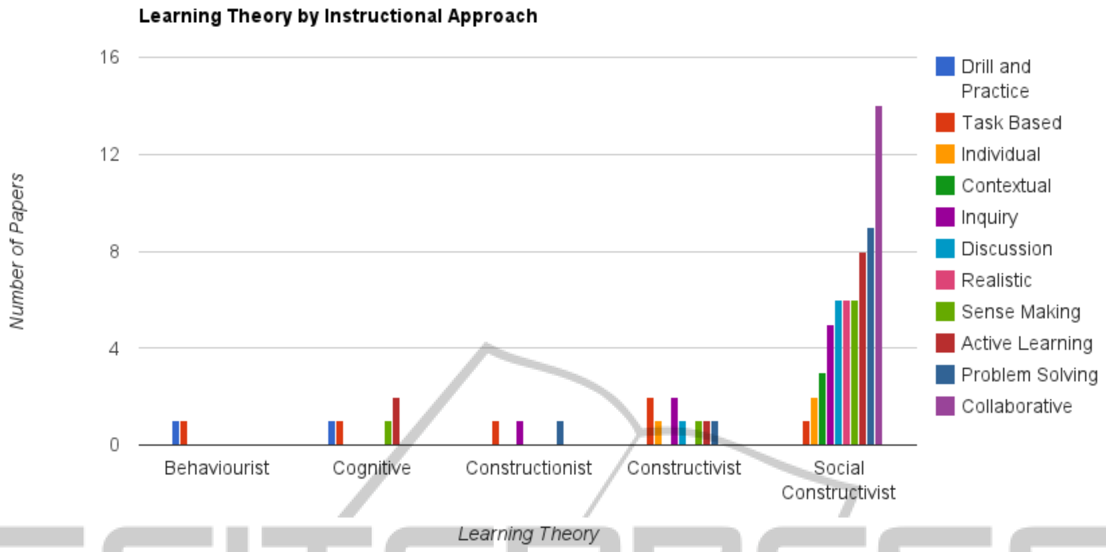

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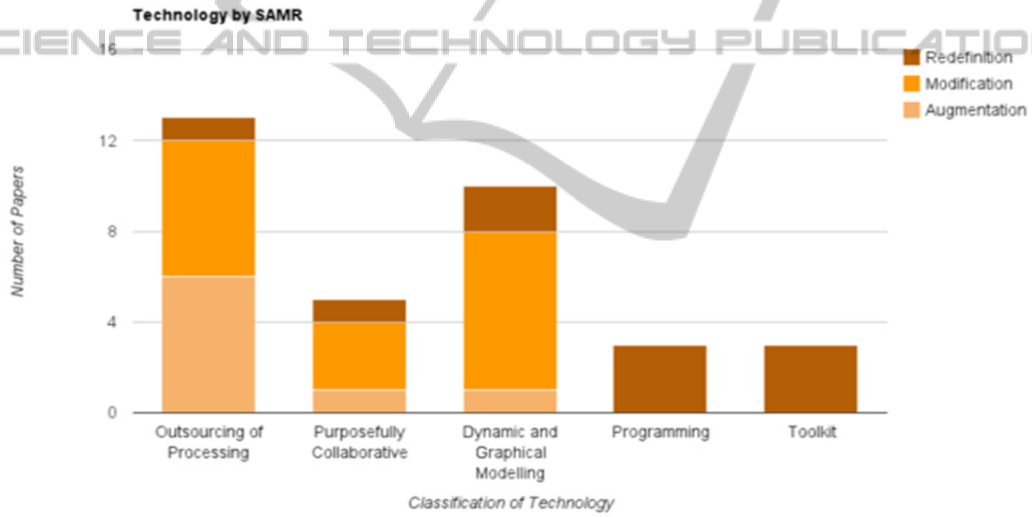

Figure 4: Technology according to SAMR hierarchy.

A correlation between instructional approach and learning theory also emerged from the data, as illustrated by Figure 3.

The interventions were also coded according to where they fell on the SAMR scale. It became clear that none of the papers considered dealt with technology as a direct substitute for traditional methods, without functional change (Figure 4). Roughly 30% fell into the sphere of *augmentation*, an example of which is the paper by Kay and Kletskin (2012), who evaluate the use of video podcasts in mathematics education.

Over 40% of the papers came under the heading of *Modification*, these include Ruthven, Deaney et al., (2009) analysis of the use of graphing software to teach about algebraic forms, and Lazakidou and Retalis' (2010) work on the use of technology supported collaborative learning strategies for problem solving in mathematics education.

The technology in each of the interventions classified in this way has facilitated significant redesign of the tasks and the learning experience.

Articles within the category of *Redefinition* make up the remaining 30% of the papers. These describe tasks and activities that would not have been possible without the use of the technology in question (e.g. Noss et al., 2012; Tangney et al., 2010). All of the studies that fell into the technology classifications of *programming* (e.g., Noss et al., 2012) or *toolkit* (e.g., Noss et al., 2009; Tangney et

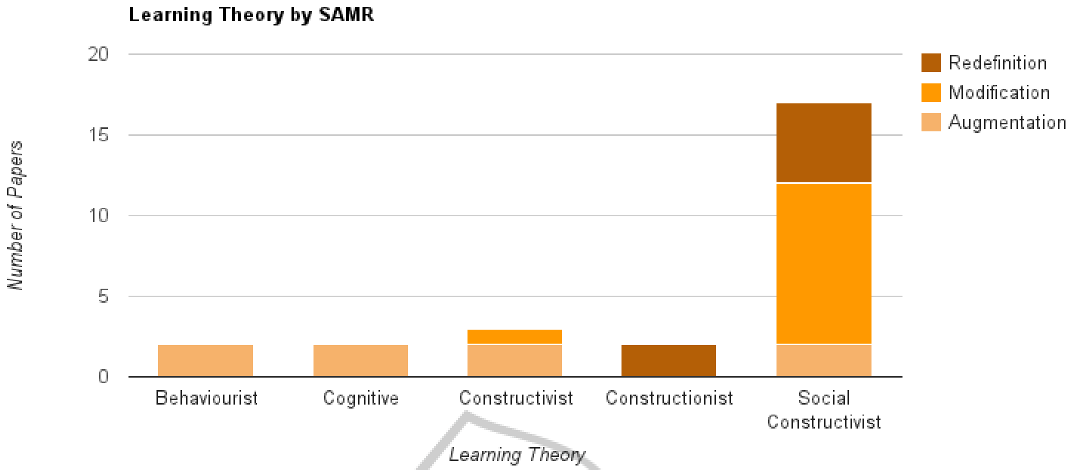

Figure 5: Learning Theory according to SAMR hierarchy.

al., 2010), are also classified under the remit of *redefinition*. Figure 4 illustrates the breakdown of the technologies according to the SAMR hierarchy, and Figure 5 indicates how the learning theories observed in the interventions are grouped in terms of the hierarchy.

6 DISCUSSION

Having looked at the general literature on uses of technology in mathematics education, this study took a structured approach, facilitated by a classification system, to analyse specific interventions.

It is interesting to note that all of the *constructionist* interventions use programming tools or toolkits (Figure 2), and are classed in the sphere of redefinition (Figure 5). The only other incidences of redefinition are seen within *socially constructivist* settings. Another interesting result is the lack of papers in the literature review that are classified at the level of Substitution on the SAMR hierarchy. There are a variety of possible reasons for this: one may be that while it is occurring in everyday usage of technology, it simply may not be reported in the literature. Alternatively, it is possible that the selection process of the papers under review was too narrow, an issue that will be addressed in future work.

Based on the general literature review and the classification of selected papers, a set of guidelines is now proposed outlining an approach to the design of learning experiences that fully employ the educational potential of technology and appropriate pedagogical approaches to facilitate learning. Interventions designed in accordance with these guidelines should significantly modify or redefine the learning experience through the affordances of the technology. That is, interventions of this type are likely to be classified within the Transformation space on the SAMR hierarchy.

6.1 Guidelines

An appropriate and innovative technology intervention in mathematics education should thus:

1. Be collaborative and team-based in accordance with a socially constructivist approach to learning.
2. Exploit the transformative as well as the computational capabilities of the technology.
3. Involve problem solving, investigation and sense-making, moving from concrete to abstract concepts.
4. Make the learning experience interesting and immersive/real wherever possible, adapting the environment and class routine as appropriate.
5. Use a variety of technologies (digital and traditional) suited to the task, in particular, non-specialist technology that students have to hand such as mobile phones and digital cameras.
6. Utilise the formative and/or summative assessment potential of the technology intervention.

7 CONCLUSIONS AND FUTURE WORK

Through the literature review and the development of a classification system, pedagogic approaches have been identified which are appropriate for use in technology enhanced teaching and learning. These are based in socially constructivist/constructionist learning theory and emphasise problem solving, investigative, and realistic instructional approaches. Use of assessment potential provided by the technology and flexibility with regard environment and class routine also emerged as important aspects of a successful intervention. From this data, a set of guiding principles were extracted that have the potential to form the basis of a 21st Century model for the integration of technology into mathematics education.

In order to gauge the efficacy of the guidelines, initial exploratory interventions are being developed. As part of this, a number of pilot activities have already been implemented, with very encouraging results.

Further studies which implement the guidelines will be used to build up a strong evidence base for the potential of such activities to enhance mathematics education. Such activities will require execution in traditional school settings as well as purpose designed environments, in order to investigate their potential to scale.

The literature review will continue to be expanded in order to confirm the results and keep the system of classification up to date. This will be an iterative process and will, along with the results of the studies, continue to inform and refine the guidelines.

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