Complex Plane Transformations for Manipulation and Visualization of Panoramas

Leonardo Sacht and Luiz Velho
IMPA - Instituto Nacional de Matematica Pura e Aplicada, Rio de Janeiro, Brazil

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Abstract: We present a method for manipulation and visualization of wide-angle images using transformations defined on the complex plane $\mathbb{C}$. We map the unit sphere $\mathbb{S}^2$ to $\mathbb{C}$ using the stereographic projection, multiply the complex plane by a given complex number, and map the result back to the sphere using the inverse of the stereographic projection. Since all these transformations preserve angle, we obtain a result containing only distortions due to the latitude/longitude representation of the sphere, which were already present in the input image. We then explore the possibility given by our technique of mapping wide fields of view to narrower ones. This makes possible to apply perspective projection to wider fields of view, leading to a natural generalization of the perspective projection in the context of panoramic images. Our results are generated in real-time and compare competitively with state-of-the-art methods used to project the viewing sphere to the image plane.

1 INTRODUCTION

Wide-angle images have become increasingly popular during the last years due to the development of computational photography techniques and equipment to capture this kind of images. Although they represent much better the information of a scene, they usually present artifacts such as bending of straight lines.

A format commonly used to represent images of the full sphere around a viewpoint is the equi-rectangular format (Figure 1 (a), bottom). Each point on the the viewing sphere is represented by its longitude/latitude coordinates on the equi-rectangular image. Thousands of images in this format are available on the Internet, for example in (Flickr, 2012).

Our method allows the possibility of applying conformal transformations to these images, i.e., transformations that preserve angles on the sphere. Due to this angle-preserving property, the resulting modified equi-rectangular images (Figure 1 (d)) could be projected on a spherical surface such as a dome and the perceived result would not present unpleasant distortions. We see this possibility as a promising future application of our technique.

Other possibility than projecting the modified spherical image on a dome, is to apply some planar projection and map it to a new image. It is well known that the only projection that preserves all possible straight lines in the scene is the perspective projection, but it distorts objects for fields of view wider than 90 degrees. Our technique allows mapping a wide field of view on the sphere to a narrower one, and then applying the perspective projection to this narrow field of view (Figure 1 (e)). This process leads to high-quality results, naturally extends the perspective projection and is very simple to implement in real-time, making it possible to be incorporated to panorama viewers such as Google Street View (Google Street View, 2012) and fieldOfView (fieldOfView, 2012). Incorporating our technique to these viewers would improve the navigation offered by them, since their visualization is limited to a narrow filed of view to avoid the distortions of the standard perspective projection.

Figure 1 describes the pipeline of our method: it receives as input an equi-rectangular image representing the full viewing sphere around a given viewpoint (Figure 1 (a)). We map this sphere to the complex plane using the stereographic projection (Figure 1 (b)), which is known to be conformal and surjective. We then transform the complex plane by multiplying each point by some given real number (Figure 1 (c)). The transformation that we explore most is uniform scaling of the complex plane, but our technique can handle full spatially-varying Mobius transformations as well. This transformed complex plane is mapped back to the unit sphere using the inverse stereographic projection, leading to a transformed equi-rectangular

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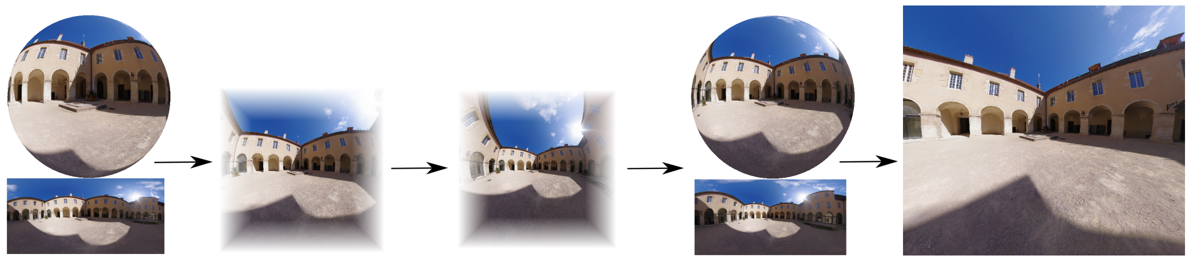

Figure 1: The input image of our method is a viewing spehre at a viewpoint, which is represented by an equi-rectangular image (a). We map the sphere to the complex plane using the stereographic projection (figure (b) shows only a limited part of the complex plane), apply a transformation to the complex plane (c) and map the result back to the sphere (d). Finally, perspective projection is applied to the modified viewing sphere, allowing for visualization of wide fields of view with a good balance between straight line and conformality (figure (e) shows a 160 degree field of view).

image (Figure 1 (d)). We then apply a perspective projection to the resulting sphere (Figure 1 (e)).

All these transformations are independent for each sphere vertex, allowing us to implement them in parallel and obtain real-time interaction. As a limitation, our results may present straight line bending for some scenes and especially for fields of view that are too wide (wider than 180 degrees).

To summarize, the main contributions of our work are the following:

- Use of complex transformations to map wide fields of view on the sphere to narrower ones in a conformal way;
- Generalization of the perspective projection to visualize wide-angle images real-time.

The rest of the paper is organized as follows: in section 2 we review the panoramic image literature and relate previous works to ours; In section 3 we describe in detail our technique for manipulation of equi-rectangular images and perspective reprojection; in section 4 we present implementation details; in section 5 we describe our results and compare with previous works; Finally, in section 6 we describe limitations and possibilities for future work.

2 RELATED WORK

Due to their increasing popularity and interest, panoramic images have become a theme of intense discussion in the Computer Graphics and Image Processing communities in the last twenty years.

The impossibility of obtaining a global projection from the sphere to the plane that preserves all possible straight lines and object shapes was shown in the seminal work by Zorin and Barr (Zorin and Barr, 1995).

The alternative to this problem proposed in (Zelnik-Manor et al., 2005) was to use different perspective projections in the same scene. In that work the user specified different projection planes and view directions to define the different projections. The discontinuities caused by using different projections for different regions of the panorama were hidden (if possible) by choosing the projection planes in a way that fit well orientation discontinuities that were already present in the scene.

The work in (German et al., 2007) explored conformal mappings to preserve the shape of the objects in a panoramic image. They investigated the stereographic projection and scaling of the complex plane but only for artistic and exploratory purposes. Since their focus was on shape preservation, their results present bent lines. In our work we go beyond and improve these ideas to map wide fields of view to narrower ones and add a perspective re-projection step, which allows for better quality results because the straight lines are less bent.

Other methods relied on both user interaction and energy-minimization formulations. (Carroll et al., 2009) used the important lines in the scene provided by the user and detected faces to control straight line preservation and conformality in these regions. (Kopf et al., 2009) used regions specified by the user where the projection should be nearly planar to formulate their optimization framework. In these methods, user interaction was usually laborious and the optimization formulations made them impossible to be implemented in real-time.

Another important technique was proposed in (Kopf et al., 2007). Their viewer changes the projection depending on the filed of view, what can be achieved by our viewer by applying different scales to the complex plane as the field of view of the perspective projection changes. But in our work, we let the user specify both parameters, what gives her or him more control. Also, our visualization simulates better camera movements since it is a natural generalization

Figure 2: Left: perspective projection applied to a 160 degree field of view. Center: stereographic projection. Right: Our result using the value $s = 0.8$ to scale the complex plane.

of the perspective projection.

Another advantage of our method compared to previous ones is that we do not rely on heavy user interaction. The user is only asked to vary the field of view of the perspective projection and the scale to be applied to the complex plane in our viewer, what makes our panorama viewer a pleasant experience, instead of laborious. Also, our formulation is simple and does not rely on heavy optimizations, which makes our method possible to be implemented in real-time. We elaborate more on comparisons with previous works in Section 5.

A work that is not related to panoramic images but has some connection with ours is (Crane et al., 2011). In this work, the authors look for conformal transformations of surfaces by associating $\mathbb{R}^3$ with the imaginary part of the Quaternions $Im(\mathbb{H})$. We work with the complex plane $\mathbb{C}$ instead of $Im(\mathbb{H})$, since our surface of interest is the unit sphere $\mathbb{S}^2$ and $\mathbb{S}^2 \backslash \{(0,0,-1)\}$ is confomally equivalent to the complex plane. Applying their work to our context would produce conformal results, but straight lines in the scene would appear bent.

3 METHOD

In this section we explain details of our method to manipulate equi-rectangular images and visualize them with less distortions. We first give an overview and motivation of our technique and then we present the Mathematics involved in each transformation of our pipeline.

3.1 Overview

The importance of conformal mappings is well known in geometry applications. Intuitively, a mapping from a surface to another is conformal if it is locally a composition of a scale and a rotation. This property prevents the map to shear the surface, an important property for mesh quality and texture mapping of the final surface (for more details, see the work (Crane et al., 2011)). Regarding panoramic images, conformality is also essential (Carroll et al., 2009).

In this work, we apply a sequence of conformal transformations to a given viewing sphere. The only transformation that is not conformal is the final perspective projection. Although this seems to be a problem, we apply this final mapping to a narrow field of view of the (conformally) transformed sphere, and perspective projection does not deviate too much from conformality for narrow fields of view. The sequence of transformations we use is described in Figure 1 and detailed in the next sub-section.

The choices we have made in our method are justified in Figure 2. We first show the results of applying the perspective (left) and stereographic (center) projections to a 160 degree field of view. The first result preserves straight lines but presents excessive shearing especially in the periphery of the image. On the other hand, the stereographic projection is conformal, but lines are clearly bent. We propose to combine the good properties of both methods. We first observe in the detail of Figure 2 (left) that the perspective projection is almost conformal for narrow fields of view. By applying the sequence of conformal transformations described in Figure 1 using the value $s = 0.8$ to scale the complex plane, we are able to map 160 degree FOV to a narrower FOV and then apply perspective transformation to this narrow field of view. Figure 2 (right) shows the final result of our method.

We leave to the user the specification of the field of view of the original sphere to visualized and the scale of the complex plane to be applied. Of course, we recommend that these parameters are such that the final perspective projection is applied to a narrow field of view.

3.2 Transformations

We first map the unit sphere to the complex plane using the stereographic projection, which is illustrated in Figure 1 (b). This projection consists of mapping each point on $\mathbb{S}^2\backslash\{(0,0,-1)\}$ to the $z=1$ plane through lines emanating from the pole opposite to the point of tangency $(0,0,-1)$, for which the projection is not well-defined. The expression for this projection is given by:

$$\begin{array}{rccc} \mathbf{S}: & \mathbb{S}^2\backslash\{(0,0,-1)\} & \rightarrow & \mathbb{C} \\ & (x,y,z) & \mapsto & (u,v)=\left(\frac{2x}{z+1},\frac{2y}{z+1}\right) \end{array} \quad (1)$$

It is well known that the stereographic projection is a conformal mapping from the sphere to the complex plane (Snyder, 1987).

The next step of our method is to multiply the complex plane by some given complex number. We first write the points in their polar form

$$\begin{array}{rcl} u+iv=(u,v) & \mapsto & (\sqrt{u^2+v^2},\arctan 2(u,v))= \\ & & =(r,\theta)=re^{i\theta} \end{array} \quad (2)$$

In this form, multiplication by a given complex number $se^{i\alpha}$ is given by the simple expression

$$(r,\theta)\mapsto(\tilde{r},\tilde{\theta})=(rs,\theta+\alpha) \quad (3)$$

From complex analysis (Conway, 1978), it is known that multiplication by a complex number is a conformal mapping from the complex plane onto itself. We show in Fig. 1 (c) the effect of multiplying the plane by 0.6, i.e., a uniform scale of 0.6 and no rotation.

Finally, we rewrite the complex plane in Cartesian coordinates

$$(\tilde{r},\tilde{\theta})\mapsto(\tilde{u},\tilde{v})=(\tilde{r}\cos(\tilde{\theta}),\tilde{r}\sin(\tilde{\theta})) \quad (4)$$

and map the transformed points back to the unit sphere using the inverse stereographic projection, which is also conformal:

$$\begin{array}{rccc} \mathbf{S}^{-1}: & \mathbb{C} & \rightarrow & \mathbb{S}^2\backslash\{(0,0,-1)\} \\ & (\tilde{u},\tilde{v}) & \mapsto & \left(\frac{4\tilde{u}}{\tilde{u}^2+\tilde{v}^2+4},\frac{4\tilde{v}}{\tilde{u}^2+\tilde{v}^2+4},\frac{\tilde{u}^2+\tilde{v}^2-4}{\tilde{u}^2+\tilde{v}^2+4}\right) \end{array} \quad (5)$$

The result of this transformation is presented in Figure 1 (d).

At this point we have a modified viewing sphere containing the information of a wide FOV represented in a narrower one. Since all the manipulations performed until now were conformal, one could project this modified sphere in a spherical surface such as a dome and see it without angle distortions, allowing for acceptable visualizations of wide fields of view on a dome.

In this work we apply a perspective transformation to the modified sphere, in order to see it in a flat surface such as a computer screen:

$$\begin{array}{rccc} \mathbf{P}: & \mathbb{S}^2\backslash\{z<0\} & \rightarrow & \mathbb{C} \\ & (x,y,z) & \mapsto & \left(\frac{x}{z},\frac{y}{z}\right) \end{array} . \quad (6)$$

The result of this projection is shown in Figure 1 (e).

4 IMPLEMENTATION

To implement all the transformations we have just described, we represent the unit sphere as a triangle mesh. Since all operations from (1) to (6) can be performed independently for each vertex on the mesh, we implemented them as a GLSL vertex shader.

The shader is loaded by a Qt Viewer Application that implements our technique. This application consists of an interface where the user specifies the view direction, the field of view of the input viewing sphere to be visualized and the scale to be applied to the complex plane. The interface also has an option to calculate a good scale value depending on the specified field of view, so the user specifies only one of the two parameters.

To illustrate the real-time performance of our viewer, we show some timing numbers in Table 1. All these numbers were generated with a screen resolution of 1024×768 pixels in a PC with an Intel Xeon Quad Core 2.13GHz and 12 GB of RAM and a GeForce GTX 470 GPU. We emphasize that mesh and image resolutions shown in Table 1 generate visualizations without discretization artifacts.

Table 1: Frame rates generated by our technique while the user interacts, for different mesh resolutions (vaying on the rows) and different equi-rectangular panorama image resolutions (varying on the columns).

Vertices \ Pixels	4000×2000	8000×4000
200×200	93 fps	89 fps
400×400	85 fps	84 fps
800×800	35 fps	33 fps

5 RESULTS AND DISCUSSION

In the accompanying video of this paper we show three results of our technique. The first one shows the effect of applying different scales to the viewing sphere, without applying a perspective transformation. The proportions of the objects in the equirectangular image change due to the scaling parameter, but the angles are well preserved. We also observe that the transformations are bijective, i.e., the whole content of the sphere appear in all transformed results.

The next two results in the accompanying video show the result of applying a scale to the sphere, followed by a perspective transformation. Our interface allows the user to look around the sphere and control the field of view and scaling factor. In both results, we zoom out until the perspective projection (our result with scale $s=1$) becomes too stretched. To correct

(a) Equi-rectangular (b) Perspective (c) Mercator (d) Ours, $s = 0.75$

Figure 3: Comparison with standard projections for a 160 degree field of view. Our result is the only one with a good balance between straight line and object shape preservation.

this problem, we decrease the scale s which makes possible to see a much wider field of view with less distortions. We observe that the FOV shown in the video is the one used to define the perspective projection on the *modified* viewing sphere, which is not the same as the original viewing sphere. For instance, when we have a FOV of 130 degrees and $s = 0.5$ we are seeing a FOV wider than 180 degrees in the original sphere.

We also compare our method with standard projections to map the sphere to the image plane in Figure 3. We observe that for the field of view of 160 degrees the perspective projection (b) distorts objects too much but preserves all straight lines. On the other hand, Mercator (c) projection preserves object shapes because it is conformal, but bends lines. The result obtained with our method (d) using a scale parameter $s = 0.75$ is the only one that presents a good balance between these two properties.

Finally, we compare our method with recent works on the same topic in Figure 4. The result obtained by the technique proposed in (Zelnik-Manor et al., 2005) (a) shows discontinuities on the floor produced by using different projections for different areas of the image. This strategy works successfully for the building in the image, since these discontinuities are hidden by natural discontinuities in the scene, but it fails to fit the geometry of the floor. In Fig. 4 (b) we show a result produced by our implementation of the technique in (Carroll et al., 2009). All straight lines specified by the user (please see their work for more details) are well-preserved, but the lines on the floor appear bent, since they are too many to be marked by the user. Although their energy minimization formulation guarantees conformality and smoothness of the final result, it has the problem of taking some seconds to be performed. Our result ((c), for which we apply a scale $s = 0.8$ to the complex plane) does not rely on heavy user interaction nor on any optimization and is not restricted to scenes with any particular geometry.

6 LIMITATIONS AND FUTURE WORK

A limitation of our method appears when one uses our interface to visualize too wide fields of view (Figure 5). Applying standard perspective projection ($s = 1$) is prohibitive in this case, and applying a small scale to the complex plane makes the final result of our method deviate too much from the standard perspective projection and present bent lines. One possibility to overcome this limitation would be to apply content-dependent scale to the complex plane, i.e., regions with straight lines would be forced to have a scaling parameter close to 1. However, the specification of important lines in the scene could be laborious, as already happened in previous works.

We also intend to use our technique to map wide fields of view to narrower ones and project the resulting viewing sphere on a spherical dome, instead of applying a planar perspective projection as we described in Section 3. This visualization would not have the distortions caused by the planar projections, and would benefit from our conformal pipeline.

Another interesting direction for future work is to extend our technique to panoramic videos, i.e., temporally varying viewing spheres. Distortions as the ones observed in (Sacht et al., 2011) would have to be considered and a time-varying warping could have to be formulated.

Since the technique we have presented in this paper is simple and can be implemented in real-time, we think it can be easily incorporated to current panorama viewers such as Google Street View.

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(a) (Zelnik-Manor et al., 2005) (b) (Carroll et al., 2009) (c) Ours, $s = 0.8$

Figure 4: Comparison with other methods to project the viewing sphere to the image plane for a field of view of 150 degrees. In the result obtained by the method in (Zelnik-Manor et al., 2005) (a), the different perspective projections used for different areas of the image appear clear and unpleasant on the floor of the scene. The method by (Carroll et al., 2009) (b) preserves all straight lines marked by the user, but fails to preserve the ones on the floor (which are too many to be marked by the user). Our result (c) has all straight lines in the scene with very little bending.

Figure 5: Top-left: input equi-rectangular image. Top-right: result of applying a scale $s = 0.05$ on the viewing sphere. Bottom: result of perspective projection applied to the transformed viewing sphere (showing only the area inside the red rectangle of the top-right figure). Although almost the entire viewing sphere is being shown, unpleasant straight line distortions appear.

for making available their images under the Creative Commons license: Janne., gadl and HamburgerJung.

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