Geometric Image of Neurodynamics

Germano Resconi[1] and Robert Kozma[2]
[1]Dept. of Mathematics and Physics, Catholic University Brescia, Brescia, I-25121, Italy
[2]Dept. of Mathematical Science, Computational Neurodynamic Laboratory, University of Memphis, Memphis, TN 38152, U.S.A.

Keywords: Conceptual Intention, Material Intention, Electrical Circuit, Memristor, Neuromorphic Computing, Geodesic Conductance Matrix, Impedance Matrix, Software, Hardware, Digital Computer, Multidimensional Space of Currents, Charges and Voltages.

Abstract: We know that the brain is composed of simple neural units given by dendrites, soma, and axons. Every neural unit can be modelled by electrical circuits with capacitors and adaptive resistors. To study the neural dynamic we use special Ordinary Differential Equations (ODE) whose solutions give us the behaviour or trajectory of the neural states in time. The problem with ODE is in the definition of the parameters and in the complexity of the solutions that in many cases cannot be found. The key elements that we use are the multidimensional vector spaces of the electrical charges, currents and voltages. So currents and voltages are geometric references for states in the central neural system (CNS). Any neuro –biological architecture can be modelled by an adaptive electrical circuit or neuromorphic network that relates voltage with current by conductance matrix or on the contrary by impedance matrix. Given a straight line with a change of reference we reshape the straight line in a geodetic and in a new form for the distance. The change of the reference transforms a set of variables into another so this transformation is similar to a statement in the digital computer that we associate to the software. Every change of variables can be reproduced by a similar change of voltages (currents) into currents (voltages) by conductance (impedance) matrix. We use the CNS as a material support or hardware in the digital computer to realise the wanted transformation. In conclusion geometry fuses the digital computer structure with neuromorphic computing to give efficient computation where conceptual intention is the change of the reference space , while material intention is given by the neurodynamical processes modelled by the change of the electrical charge space where we define the metric geometry and distance.

1 INTRODUCTION

This work studies a possible mathematical formulation of intentional brain dynamics following Freeman's half century-long dynamic systems approach (Freeman, 1975; 2007); (Kozma, 2008) We consider the electrical behaviour of the brain. In 1980 an artificial neural network was built that works but has high precision components, slow unstable learning, it is non adaptive and needs an external control. Now we want low precision components, fast stable learning, adapt to environment and autonomous. How can we get this? We can make dynamical components, add feedback (positive & negative) and close the loop with the outside world. The ordinary differential equations or ODEs to control the neural dynamic are a stiff and nonlinear system. Why not just program this on a computer? We know that stiff and nonlinear dynamical systems are inefficient on a digital computer. An example is the IBM Blue Gene project with 4096 CPUs and 1000 Terabytes RAM, which, to simulate the Mouse cortex uses 8 10^6 neurons, 2 10^{10} synapses 109 Hz, 40 Kilowatts and digital. The brain uses 10^{10} neurons, 10^{14} synapses 10 Hz and 20 watts analog system which is more efficient than digital by many orders of magnitude.

(Snider, 2008) suggests to use analog electrical circuit denoted CrossNet or neuromorphic computing with memristor to solve the problem of the neural computation. Let's recall that for Turing the physical devise is not computable by a Turing machine, which is the theoretical version of the digital computer. (Carved, 1990) suggests that the physics or analog computer is more efficient to solve the neural network problem. In fact, for analog system we do not have

Resconi G. and Kozma R..
Geometric Image of Neurodynamics.
DOI: 10.5220/0004110204570465
In *Proceedings of the 4th International Joint Conference on Computational Intelligence* (NCTA-2012), pages 457-465
ISBN: 978-989-8565-33-4

algorithms to program the neurons. Rather, the digital program is substituted by the dynamics in non Euclidean space. We can program the CrossNet (Takashi Kohno, 2008), (Rinzel, 1998) electrical system as it was used by Snider to compute the parameters useful to generate the desired trajectories to solve problems. Geometric and physical description of the intentionality (Freeman, 1975) is beyond any algorithmic or digital computation. To clarify better the new computation paradigm, we can refer the following principle: "Animals and humans use their finite brains to comprehend and adapt to infinitely complex environment." (Kozma, 2008) We show that this adaptive system has a geometric interpretation that gives us the possibility to implement the required parameters in ODE to achieve the desired behaviours. The geometric interpretation uses three main spaces. One is the current multidimensional space, the other is the electrical charge multidimensional space and the last is the voltage space (Resconi, 2007; 2009). In (Mandzel, 1999) we can found geometric method to study human motor control.

2 GEOMETRY AND ELECTRICAL CIRCUITS

Because the brain is a complex electrical circuit with capacity and resistors, a network of neurons or an electronic network is a general transformation or MIMO from many voltages in inputs to many currents in output

$$\begin{cases} i_1 = f_1(v_1, v_2, \ldots, v_p) \\ i_2 = f_2(v_1, v_2, \ldots, v_p) \\ \ldots \\ i_n = f_n(v_1, v_2, \ldots, v_p) \end{cases} \tag{1}$$

where the currents are vectors in a n-dimensional space of the currents and the voltages are vectors in a p-dimensional space. In figure 1 we show as example the three dimensional current space.

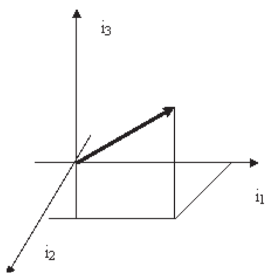

Figure 1: Vector of current in the current space.

For one dimension the (1) is written in this form $i = f(v)$ that in electronics is denoted characteristic function. In Figures 2 and 3 we show two different cases for (1) in one dimension.

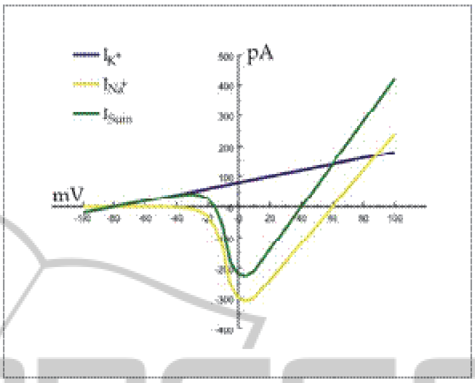

Figure 2: An approximation of the potassium and sodium ion components of a so-called "whole cell" I–V curve of a neuron.

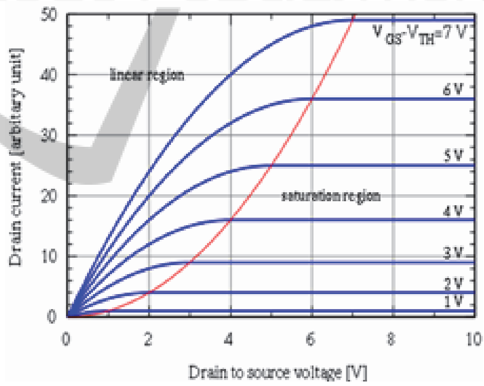

Figure 3: MOSFET drain current vs. drain-to-source voltage for several values of the *overdrive voltage*, V_{GS} - V_{th}; the boundary between **linear** (**Ohmic**) and **saturation** (**active**) modes is indicated by the upward curving parabola.

The instrument to match intentionality with the electrical circuit is the metric geometry of the brain state space or electrical charge space. The metric geometry in the state space can be obtained by the instantaneous electrical power p in the current space or in voltage space as we show in equation (2). For the linear form of the (1) we have the expression of the power.

$$(\frac{ds}{dt})^2 = power = i_1 v_1 + i_2 v_2 +,.....,+ i_n v_n$$

for

$$\begin{bmatrix} i_1 \\ i_2 \\ \dots \\ i_n \end{bmatrix} = \begin{bmatrix} C_{1,1} & C_{1,2} & \dots & C_{1,n} \\ C_{2,1} & C_{2,2} & \dots & C_{2,n} \\ \dots & \dots & \dots & .. \\ C_{n,1} & C_{n,2} & \dots & C_{n,n} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ \dots \\ v_n \end{bmatrix}$$

we have

$$power = C_{1,1} v_1^2 +,..,+2\ C_{1,2} v_1 v_2 + 2C_{1,3} v_1 v_3 + ..$$
$$= \sum_{\alpha,\beta} C_{\alpha,\beta}\, v_\alpha v_\beta = \sum_{\alpha,\beta} Z_{\alpha,\beta}\, i_\alpha i_\beta$$

$$(\frac{ds}{dt})^2 = \sum_{\alpha,\beta} Z_{\alpha,\beta} \frac{dq_\alpha}{dt} \frac{dq_\beta}{dt} \tag{2}$$

and

$$ds^2 = \sum_{\alpha,\beta} Z_{\alpha,\beta}\, dq_\alpha dq_\beta \tag{3}$$

Where $C_{\alpha,\beta}$ and $Z_{\alpha,\beta}$ are the conductance matrix and the impedance matrix, v are the voltages , i are the currents and q are the charges. For example, given the electrical circuit.

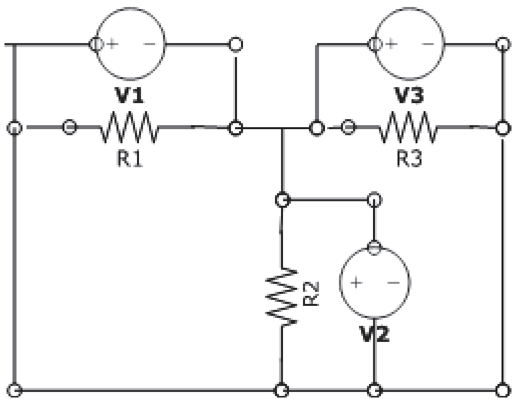

Figure 4: Simple electrical circuit with three generators V1, V2, V3.

For the Kirchhoff current and voltage laws and for Ohm's law we have for the electrical circuit in Figure 4 the following system of equations.

$$V_1 + V_2 = E_1$$
$$V_3 + V_2 = E_2$$
$$i_2 = i_1 + i_3$$
$$E_1 = R_1 i_1 + R_2 i_2$$
$$E_2 = R_3 i_3 + R_2 i_2$$

whose solution is

$$\begin{cases} i_1 = \dfrac{(R_2 + R_3)V_1 + R_3 V_2 - R_2 V_3}{R_1 R_2 + R_1 R_3 + R_2 R_3} \\ i_2 = \dfrac{(R_1 + R_3)V_2 + R_3 V_1 + R_1 V_3}{R_1 R_2 + R_1 R_3 + R_2 R_3} \\ i_3 = \dfrac{(R_1 + R_2)V_3 + R_1 V_2 - R_2 V_1}{R_1 R_2 + R_1 R_3 + R_2 R_3} \end{cases} \tag{4}$$

For the vector space of currents and voltages the Kirchhoff current and voltage laws and Ohm's laws can be represented in this vector form

$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_3 \end{bmatrix} = \begin{bmatrix} i_1 \\ i_1 + i_3 \\ i_3 \end{bmatrix}$$

$$\begin{bmatrix} E_1 \\ E_2 \end{bmatrix} = \begin{bmatrix} V_1 + V_2 \\ V_3 + V_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}^T \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} R_1 & 0 & 0 \\ 0 & R_2 & 0 \\ 0 & 0 & R_3 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix}$$

where the solutions can be written in an operational way in this form

$$\begin{bmatrix} E_1 \\ E_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}^T \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} =$$

$$\begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}^T \begin{bmatrix} R_1 & 0 & 0 \\ 0 & R_2 & 0 \\ 0 & 0 & R_3 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_3 \end{bmatrix}$$

$$i = \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_3 \end{bmatrix}$$

$$i = (\begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} (\begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}^T \begin{bmatrix} R_1 & 0 & 0 \\ 0 & R_2 & 0 \\ 0 & 0 & R_3 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix})^{-1} \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}^T) v$$

$$v = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

So for $i = Cv$

$$C = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} (\begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}^T \begin{bmatrix} R_1 & 0 & 0 \\ 0 & R_2 & 0 \\ 0 & 0 & R_3 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix})^{-1} \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}^T$$

$$C = \begin{bmatrix} R_2 + R_3 & R_3 & -R_2 \\ R_3 & R_1 + R_3 & R_1 \\ -R_2 & R_1 & R_1 + R_2 \end{bmatrix} (R_1 R_2 + R_1 R_3 + R_3 R_2)^{-1}$$

where C is the conductance matrix for which

$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} C_{1,1} & C_{1,2} & C_{1,3} \\ C_{2,1} & C_{2,2} & C_{2,3} \\ C_{3,1} & C_{3,2} & C_{3,3} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

or

$$\begin{cases} i_1 = C_{1,1} v_1 + C_{1,2} v_2 + C_{1,2} v_3 \\ i_2 = C_{2,1} v_1 + C_{2,2} v_2 + C_{2,3} v_3 \\ i_3 = C_{3,1} v_1 + C_{3,2} v_2 + C_{3,3} v_3 \end{cases}$$

For relation (1) we can compute the dynamical conductance

$$C = \begin{bmatrix} C_{1,1} & C_{1,1} & \dots & C_{1,1} \\ C_{1,1} & C_{1,1} & \dots & C_{1,1} \\ \dots & \dots & \dots & \dots \\ C_{1,1} & C_{1,1} & \dots & C_{1,1} \end{bmatrix} = \begin{bmatrix} \frac{\partial i_1}{\partial v_1} & \frac{\partial i_1}{\partial v_2} & \dots & \frac{\partial i_1}{\partial v_p} \\ \frac{\partial i_2}{\partial v_1} & \frac{\partial i_2}{\partial v_2} & \dots & \frac{\partial i_2}{\partial v_p} \\ \dots & \dots & \dots & \dots \\ \frac{\partial i_n}{\partial v_1} & \frac{\partial i_n}{\partial v_2} & \dots & \frac{\partial i_n}{\partial v_p} \end{bmatrix} \quad (5)$$

C is the dynamical conductance which is the function of the voltages

$$C = C(v_1, v_2, \dots, v_p)$$

The instantaneous power is

$$power = \sum_j i_j v_j = \sum_{j.k} C_{j,k} v_k v_j \text{ where}$$

$$i = Cv, v = C^{-1} i = Zi, Z = C^{-1}$$

and

$$\sum_{j,k} C_{j,k} v_k v_j = v^T C v = (C^{-1} i)^T C (C^{-1} i)$$

$$= i^T C^{-1} i = i^T Z i = \sum_{j,k} Z_{j,k} i_k i_j \quad (6)$$

In the next chapter we show how it is possible by the geodesic in the charge space to generate the ordinary differential equation (ODE) that controls the dynamics of the neural network and of the electrical circuit.

3 SIMPLE ELECTRICAL CIRCUIT AND GEODESIC

Given the trivial electrical circuit

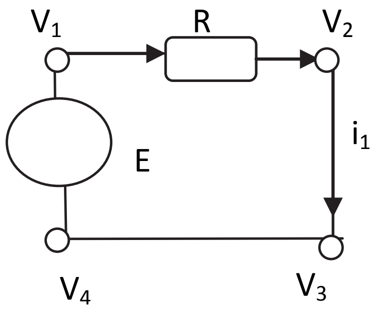

Figure 5: Simple electrical circuit with one generator E and one resistor R.

We compute the power p that is dissipated by the resistance R. We define the infinitesimal distance ds in this way:

$$\frac{ds}{dt} = \sqrt{power} = \sqrt{Ri^2} = \sqrt{R(\frac{dq}{dt})^2}$$

We know that in the electrical circuit the currents flow in the circuit in such a way to dissipate the minimum power. The geodesic line in the one dimension current space i is the trajectory in time. For the minimum dissipation of the power or cost C, we have

$$\delta C = \delta \int ds = \delta \int \sqrt{W}\, dt = \delta \int \sqrt{R\left(\frac{dq}{dt}\right)^2}\, dt = 0$$

We can compute the behavior of the charges for which we have the geodesic condition of the minimum cost. We know that this problem can be solved by the Euler Lagrange (Izrail, 1963) differential equations or ODE (ordinary differential equation)

$$\frac{d}{dt}\frac{\partial R\left(\frac{dq}{dt}\right)^2}{\partial \frac{dq}{dt}} - \frac{\partial R\left(\frac{dq}{dt}\right)^2}{\partial q} = 0$$

When R is independent of the charges then R has no memory , so the previous equation can be written as follows

$$\frac{d\left(\frac{dq}{dt}\right)}{dt} = \frac{d^2 q}{dt^2} = 0 \;,\; q(t) = at + b \;,\; i = \frac{dq}{dt} = a = \frac{E}{R}$$

The geodesic is a straight line in the space of the charge. In Figure 6 we show the behaviour of the geodesic in the charge space and the current space as derivatives of the electrical charges

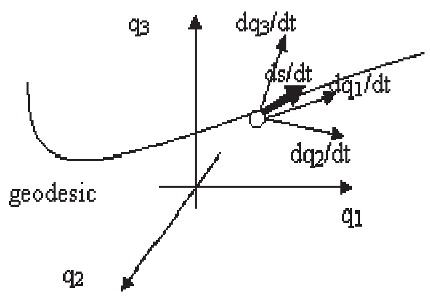

Figure 6: Charge space and derivatives in time of charges or currents.

4 DIGITAL AND NEURAL COMPUTING

To stress the difference between digital computer and geometric map of the brain we refer the interesting discussion of (Carver, 1990) where are present all the main ideas that we use and improve in this paper. Biological solutions in formation processing systems operate on completely different principles from those with which most engineers are familiar. For many problems, particularly those in which the input data are ill-conditioned and the computation can be specified in a relative manner, biological solutions are many orders of magnitude more effective than those we have been able to implement using digital methods. This advantage can be attributed principally to the use of elementary physical phenomena as computational primitives, and to the representation of information by the relative values of analog signals, rather than by the absolute values of digital signals. A typical microprocessor does about 10 million operations and uses about 1 W. In round numbers, it costs about 10^{-7} J to do one operation, the way we do it today, on a single chip. If we go off the chip to the box level, a whole computer uses about 10^{-5} J /operation. A whole computer is thus about two orders of magnitude less efficient than is a single chip. Back in the late 1960's we analyzed what would limit the electronic device technology as we know it; those calculations have held up quite well to the present. The standard integrated circuit fabrication processes available today allow us to build transistors that have minimum dimensions of about 1 μ (10^{-6} m). By ten years from now, we will have reduced these dimensions by another factor of 10, and we will be getting close to the fundamental physical limits: if we make the devices any smaller, they will stop working. It is conceivable that a whole new class of devices will be invented that are not subject to the same limitations. But certainly the ones we have thought of up to now-including the superconducting ones-will not make our circuits more than about two orders of magnitude more dense than those we have today. The factor of 100 in density translates rather directly into a similar factor in computation efficiency. So the ultimate silicon technology that we can envision today will dissipate on the order of 10^{-9} J of energy for each operation at the single chip level, and will consume a factor of 100-1000 more energy at the box level. We can compare these numbers to the energy requirements of computing in the brain. There are about 10^{16} synapases in the brain. A nerve pulse arrives at each synapse about ten times, on average. So in rough numbers, the brain accomplishes 10^{16} complex operations. The power dissipation of the brain is a few watts, so each operation costs only 10^{-16} J. The brain is a factor of 1 billion more efficient than our present digital technology, and a factor of 10 million more efficient than the best digital technology that we can imagine.

From the first integrated circuit in 1959 until today, the cost of computation has improved by a factor about 1 million. We can count on an additional factor of 100 before fundamental limitations are encountered. At that point, a state-of-the-art digital system will still require 10MW to process information at the rate that it is processed by a single human brain. The unavoidable conclusion, which (Carver, 1990) reached about ten years ago, is that we have something fundamental to learn from the brain about a new and much more effective form of computation. Even the simplest brains of the simplest animals are awesome computational instruments. They do computations we do not know how to do, in ways we do not understand. We might think that this big disparity in the effectiveness of computation has to do with the fact that, down at the device level, the nerve membrane is actually working with single molecules. Perhaps manipulating single molecules is fundamentally more efficient than is using the continuum physics with which we build transistors. If that conjecture were true, we would have no hope that our silicon technology would ever compete with the nervous system. In fact, however, the conjecture is false. Nerve membranes use populations of channels, rather than individual channels, to change their conductances, in much the same way that transistors use populations of electrons rather than single electrons. It is certainly true that a single channel can exhibit much more complex behaviors than can a single electron in the active region of a transistor, but these channels are used in large populations, not in isolation (Carver, 1990). We can compare the two technologies by asking how much energy is dissipated in charging up the gate of a transistor from a 0 to a 1. We might imagine that a transistor would compute a function that is loosely comparable to synaptic operation. In today's technology, it takes about 10^{-13}j to charge up the gate of a single minimum-size transistor. In ten years, the number will be about 10^{-15} j within shooting range of the kind of efficiency realized by nervous systems. So the disparity between the efficiency of computation in the nervous system and that in a computer is primarily attributable not to the individual device requirements,/operation. A whole computer is thus about two orders of magnitude less efficient than is a single chip. The disparity between the efficiency of computation in the nervous system and that in a computer is primarily attributable not to the individual device requirements,

but rather to the way the devices are used in the system.

Where did all the energy go? There is a factor of 1 million unaccounted for between what it costs to make a transistor work and what is required to do an operation the way we do it in a digital computer. There are two primary causes of energy waste in the digital systems we build today.

1) We lose a factor of about 100 because, the way we build digital hardware, the capacitance of the gate is only a very small fraction of capacitance of the node. The node is mostly wire, so we spend most of our energy charging up the wires and not the gate.

2) We use far more than one transistor to do an operation; in a typical implementation, we switch about 10 000 transistors to do one operation. So altogether it costs 1 million times as much energy to make what we call an operation in a digital machine as it costs to operate a single transistor. (Carver, 1990) does not believe that there is any magic in the nervous system, that there is a mysterious fluid in there that is not defined, some phenomenon that is orders of magnitude more effective than anything we can ever imagine.

There is nothing that is done in the nervous system that we cannot emulate with electronics if we understand the principles of neural information processing by suitable conceptual or software transformations in general reference (geometry).

We can starts by letting the device physics define elementary operations. These functions provide a rich set of computational primitives, each a direct result of fundamental physical principles. They are not the operations out of which we are accustomed to building computers, but in many ways, they are much more interesting. They are more interesting than AND and OR. They are more interesting than multiplication and addition. But they are very different. (Carver,1990) tries to fight them, to turn them into something with which we are familiar, he thinks to end up making a mess. We show in this paper that this is not true. In fact (Carver,1990) forgot that the new operations must be oriented to a specific goal or intension. Now we are in agreement with and his neuromorphic network but we add a new dimension to the electrical system by the geometry in multidimensional space of charges to mimic the wanted transformation in the multidimensional space of the states.

So the real trick is to invent a vector representation of the electrical charges that takes advantage of the inherent capabilities of the medium, such as the abilities to mimic the wanted transformation. These are powerful primitives. In conclusion we use the nervous *system as an instrument to simulate system-design strategy oriented to the wanted goal or intentionality.*

5 GEOMETRY AND CONCEPTUAL PART IN NEURAL NETWORK

Now the electrical power gives us the *material* aspect of intentionality. The other part of intentionality is the *conceptual one* which is given by the wanted transformation

$$\begin{cases} y_1 = y_1(x_1, x_2, \ldots x_p) \\ y_2 = y_2(x_1, x_2, \ldots x_p) \\ \ldots \\ y_q = y_q(x_1, x_2, \ldots x_p) \end{cases} \tag{7}$$

where $(x_1, x_2, \ldots, x_p)$ are the initial variables and $(y_1, y_2, \ldots, y_n)$ are the wanted final variables. Now with the transformation (7) we can write the local linear equation

$$\begin{cases} dy_1 = \frac{\partial y_1}{\partial x_1} dx_1 + \frac{\partial y_1}{\partial x_2} dx_2 + \ldots + \frac{\partial y_1}{\partial x_p} dx_p \\ dy_2 = \frac{\partial y_2}{\partial x_1} dx_1 + \frac{\partial y_2}{\partial x_2} dx_2 + \ldots + \frac{\partial y_2}{\partial x_p} dx_p \\ \ldots \\ dy_q = \frac{\partial y_q}{\partial x_1} dx_1 + \frac{\partial y_q}{\partial x_2} dx_2 + \ldots + \frac{\partial y_q}{\partial x_p} dx_p \end{cases}$$

So we have

$$ds^2 = dy_1^2 + dy_2^2 + \ldots + dy_q^2 = \left(\frac{\partial y_1}{\partial x_1} dx_1 + \ldots + \frac{\partial y_1}{\partial x_p} dx_p\right)^2$$

$$+\left(\frac{\partial y_2}{\partial x_1} dx_1 + \ldots + \frac{\partial y_2}{\partial x_p} dx_p\right)^2 + \ldots + \left(\frac{\partial y_q}{\partial x_1} dx_1 + \ldots + \frac{\partial y_q}{\partial x_p} dx_p\right)^2 = \sum_{j,k} G_{j,k} dx_j dx_k$$

$$G = G_{j,k} = \begin{bmatrix} \frac{\partial y_1}{\partial x_1} & \frac{\partial y_2}{\partial x_1} & \ldots & \frac{\partial y_q}{\partial x_1} \\ \frac{\partial y_1}{\partial x_2} & \frac{\partial y_2}{\partial x_2} & \ldots & \frac{\partial y_q}{\partial x_2} \\ \ldots & \ldots & \ldots & \ldots \\ \frac{\partial y_1}{\partial x_p} & \frac{\partial y_2}{\partial x_p} & \ldots & \frac{\partial y_q}{\partial x_p} \end{bmatrix}^T \begin{bmatrix} \frac{\partial y_1}{\partial x_1} & \frac{\partial y_2}{\partial x_1} & \ldots & \frac{\partial y_q}{\partial x_1} \\ \frac{\partial y_1}{\partial x_2} & \frac{\partial y_2}{\partial x_2} & \ldots & \frac{\partial y_q}{\partial x_2} \\ \ldots & \ldots & \ldots & \ldots \\ \frac{\partial y_1}{\partial x_p} & \frac{\partial y_2}{\partial x_p} & \ldots & \frac{\partial y_q}{\partial x_p} \end{bmatrix} = J^T J \tag{8a}$$

The identity between the geometric metric G with the electrical circuit metric Z is the fundamental equation that connects conceptual transformation (7) with physical transformation (1).

$G_{i,j} = Z_{i,j}$ where the distance is in the charge space

With the fundamental equation we can compute the parameters of the distance as the square of the power in the electrical circuit. The square of the power is a non Euclidean distance in the state space of the electrical circuit that simulates the non Euclidean space of the classical geometry.

Example:

Let's begin with an example. When the *conceptual intention* moves on a sphere given by simple equation

$$y_1^2 + y_2^2 + y_3^2 = r^2$$

Figure 7: sphere where the green, red and blue lineas are geodetic.

we have the transformations (*conceptual intention*)

$$\begin{cases} x_1 = r\sin(\alpha)\cos(\beta) \\ x_2 = r\sin(\alpha)\sin(\beta) \\ x_3 = r\cos(\alpha) \end{cases}$$

Let's compute the geodesic in the space (x_1 , x_2 , x_3)
So we have

$$ds^2(\alpha, \beta) = \left(\frac{dx_1}{dt}\right)^2 + \left(\frac{dx_2}{dt}\right)^2 + \left(\frac{dx_3}{dt}\right)^2]$$
$$= \left(\frac{dx_1}{d\alpha}\frac{d\alpha}{dt} + \frac{dx_1}{d\alpha}\frac{d\beta}{dt}\right)^2 + \left(\frac{dx_2}{d\alpha}\frac{d\alpha}{dt} + \frac{dx_2}{d\alpha}\frac{d\beta}{dt}\right)^2 \tag{8b}$$
$$+\left(\frac{dx_3}{d\alpha}\frac{d\alpha}{dt} + \frac{dx_3}{d\alpha}\frac{d\beta}{dt}\right)^2 = r^2\left(\frac{d\alpha}{dt}\right)^2 + r^2\sin^2(\alpha)\left(\frac{d\beta}{dt}\right)^2$$

for the ***fundamental equation $G_{i,j}= Z_{i,j}$ we have***

$$Z = \begin{bmatrix} r^2 & 0 \\ 0 & r^2\sin^2(\alpha) \end{bmatrix}$$

The current is

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} \frac{d\alpha}{dt} \\ \frac{d\beta}{dt} \end{bmatrix}, \; power = r^2 i_1^2 + r^2 \sin^2(q_1) i_2^2 \tag{9}$$

6 NEURAL SYSTEM AS A COMPLEX ELECTRICAL CIRCUIT (FIGURE 12)

In opposition to actual digital sequential computers where computations are carried out by a single complex processor there are Cellular Neural/Non-linear Networks (CNN) (Torralba, 1999) which are analog parallel machines with a high number of simple processors, which are disposed in a regular array, and each processor is connected to the other processors in a reduced neighborhood. One of these analog processors is represented by the electrical activity of the synapse given by the electrical circuit

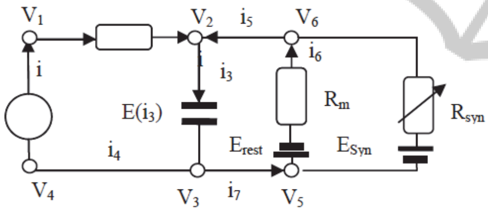

Figure 8: Electrical circuit of the synapse.

The impedance matrix is

$$Z = \begin{bmatrix} R_{ins} + 3 & 1 & 0 \\ 1 & R_m + 3 & R_m \\ 0 & R_m & R_{syn} + R_m + 2 \end{bmatrix}$$

The geodesic trajectory of the synapse activity is controlled by the relation

power = $i^T Z i$

where Z is the impedance matrix in the currents space. In an extensive form we have

$$power = \left(\frac{ds}{dt}\right)^2 = (R_{ins} + 3) i_2^2 + (R_m + 3) i_5^2 +$$

$$+ (R_m + R_{sys} + 2) i_8^2 + 2 i_2 i_5 + 2 R_m i_5 i_8$$

$$= (R_{ins} + 3)\left(\frac{dq_2}{dt}\right)^2 + (R_m + 3)\left(\frac{dq_5}{dt}\right)^2 +$$

$$+ (R_m + R_{sys} + 2)\left(\frac{dq_8}{dt}\right)^2 + 2\left(\frac{dq_2}{dt}\right)\left(\frac{dq_5}{dt}\right)$$

$$+ 2 R_m \left(\frac{dq_5}{dt}\right)\left(\frac{dq_8}{dt}\right) \tag{10}$$

We will show examples of simulation of a neuronal network by an equivalent electrical circuit.

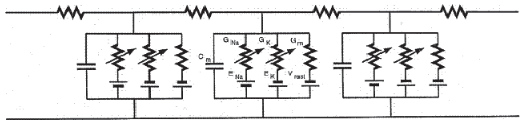

Figure 9: Example of axon and electrical circuit.

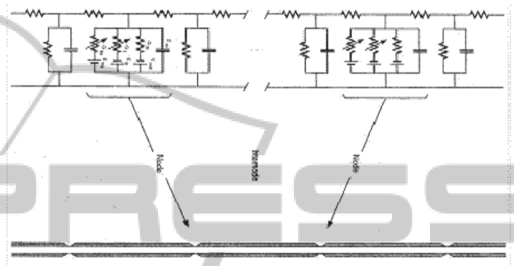

Figure 10: Axon with myelin and equivalent electrical circuit.

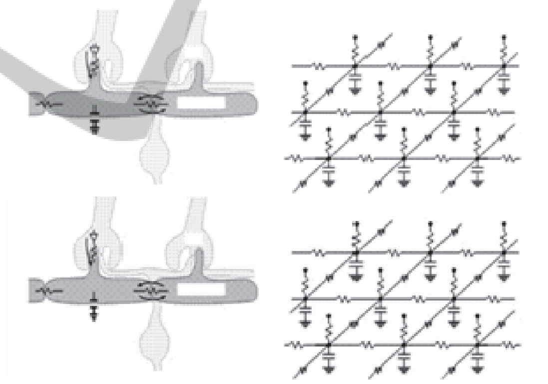

Figure 11: On the left there are the cones, the orizontal neurons and bipolar neurons. On the right there are the neuromorphic diagram or equivalent electrical circuit.

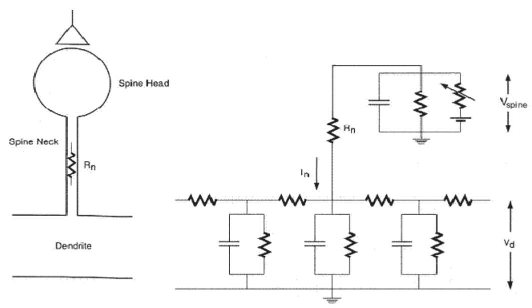

Figure 12: Complex electrical circuit of neural network system.

For more complex neural networks, we can derive the corresponding geodesics in a similar fashion. For example, we could consider have the electrical representation of a neural network as shown in Figure 12.

7 CONCLUSIONS

With the neural network we can simulate the geodesic movement for any transformation of reference. For a given transformation of reference, we can build the associate geodesic, which allows to implement the transformation of reference in the neural network. The neural network as analog computer gives the solution of the ODE of the geodesic inside the wanted reference. The Freeman K set (Freeman, 1975) is the ODE of the geodesic that is the best trajectory in the space of the electrical charges. (*Freeman, 1975) introduced the concept of intentionality, which can be recognized and studied in its manifestations of goal-directed behavior. Intention is interpreted as, respectively, an attribute of mental representations, the expression of motivations and biological driver. The mental representation is the conceptual part (software) of the intention, the biological driver is the material part (hardware) of the intention.*

In this paper we showed that any part of the brain can be represented by a complex electrical circuit. Intention has two different parts: the one is the conceptual part given by wanted transformation of the brain states. In the new reference the deformed straight line, geodesic, is the minimum distance between two points in the state space as in the classical straight line. The other part is the material part of the intention. In fact because any part of the brain can be modelled by an electrical circuit, and because the transformations between the voltages and currents give us the change of the reference ,the real transformation in the brain states is the material part of the intention. The conceptual parameters and the material parameters G (conductance C or impedance Z) must be equal. When the two parts are equal we have defined the central nervous system CNS dynamics in agreement with the wanted transformation in the conceptual space. The CNS realizes in the material world the wanted transformation. We define a task in the conceptual domain and we can implement the task in the material neural network parametric structures. In comparison with the traditional digital computer the conceptual part of intention is the software and the material part of intention is the hardware. The difference in the geometry of intention theory and digital computer is in the representation of the software and hardware. In the digital computer we have logic statements for the software and logic gates for the hardware. In the geometry of the intention we have geometric changes of the references in the multidimensional space as software and neural network as hardware.

REFERENCES

Walter J. Freeman 1975 Mass Action in The Nervous System. *Academic Press,* New York San Francisco London.

R. Kozma W. J. Freeman, (2008) “Intermittent spatial – temporal desynchronization and sequenced synchrony in *ECoG signal“ Interdisciplinary J. Chaos* 18.037131.

Freeman, W. J. A 2007 pseudo-equilibrium thermodynamic model of information processing in nonlinear brain dynamics. *Neural Networks* (2008), doi:10.1016/ j.neunet..12.011.

Takashi Kohno and Kazuyuki Aihara, 2008) A Design Method for Analog and Digital Silicon Neurons Mathematical-Model-Based Method-, *AIP Conference Proceedings*, Vol. 1028, pp. 113–128.

J. Rinzel and B. Ermentrout, 1998 “Analysis of Neural Excitability and Oscillations,” in “*Methods in Neural Modeling*”, ed. C. Koch and I. Segev, pp. 251–291, MIT Press, 1998.

Germano Resconi 2007, Modelling Fuzzy Cognitive Map by Electrical and Chemical Equivalent Circuits *Joint Conference Information Science* July8-24 Salt lake City Center USA.

Germano Resconi, Vason P.Srini 2009 Electrical Circuit As A Morphogenetic System, *GEST International Transactions on Computer Science and Engineering volume* 53, Number 1 pag.47-92.

Amir A. Mandzel, Tomar Flach 1999 Geometric Methods in the study of human motor control. *Cognitive Studies* 6 (3) p.309 – 321.

Greg S. Snider 2008 Hewlett packard Laboratory, *Berkeley conference on memristors*.

Carver Mead 1990, Neuromoprhic Electronic Systems, *Proceeding of the IEEE* vol.78 No.10

Antonio B. Torralba 1999, *Analogue Architectures for Vision Cellular Neural Networks and Neuromorphic Circuits*, Doctorat thesis, Institute national Polytechnique Grenoble, Laboratory of Images and Signals.

Izrail Moiseevich Gelfand 1963. *Calculus of Variations.* Dover.