

# Two Modes of Scheduling in a Simple Economic Agent-Based Model

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Abstract: Agent-based models (ABMs), and with them simulation, are gaining importance in economics. As they allow to study coordination problems in a dynamic setting, they can be helpful tools for identifying win-win strategies for climate policy. This paper argues that strongly simplified models can support a better understanding of economic ABMs. We present work in progress on an example case: while in economic systems in the real world many actions and interactions by various agents take place in parallel, often ABMs use sequential computation. With a simple economic agent-based model of firms that trade and produce goods, we explore and discuss two alternative modes of scheduling: the *timetable model*, where all agents complete one step after the other, and the *heliotropic model*, where one agent after the other completes steps. We find that the timetable model is better suited for working with data from national statistics, while the heliotropic model dispenses with random shuffling that is often introduced to guarantee symmetric expectations for agents. The latter can be used in a completely deterministic fashion, providing a baseline case for studying the system's dynamics.

## 1 INTRODUCTION

Simulation plays an ever more important role in economics as agent-based models (ABMs) are used more frequently. These represent the economy as a complex system in which macro-features emerge from the interaction of many heterogeneous agents. Implementing a system of agents on a computer and observing simulation runs to study the system's behaviour poses the question whether some of the observations owe to computational features of the implementation rather than being characteristic of the system under study. For example, in real-world economic systems, many actions take place in parallel, while on the computer, parallel actions are often represented by sequential steps. Some observations might occur due to the sequencing chosen by the modeller<sup>1</sup>.

Various platforms for agent-based modelling (such as Swarm, Repast, MASON, Netlogo, etc.) provide tools for representing time and scheduling ac-

tions by different agents. ABMs may implement a simple sequence of agents all conducting the same action one after the other, or complex message passing systems between agents that trigger actions in event-driven simulation. In some cases, randomness needs to be introduced to guarantee symmetric expectations for agents. For example, in a representation of trade, the first firms buying goods might find full inventories of all others, while the last ones may find inventories rather empty. To avoid such a bias, which would be an artefact of computing sequentially, the order in which agents act is often determined by random shuffling. This means that randomness is introduced for computational reasons<sup>2</sup>.

Most works related to ABMs provide little detail on how simulations are executed and rather focus on describing agents and their environment, as stated by Mathieu and Secq (2012), who find that the representation of time and scheduling in the simulator used, as well as sequential or parallel execution of actions can have crucial impacts on simulation results.

The present paper focuses on the case of sequen-

<sup>1</sup>Parallel computation may provide ways to avoid this problem, but parallelisation is beyond the scope of this short paper: interdependence between the agents makes the model used difficult to parallelise. Also, we aim at a simple model, while parallel computation raises complexity.

<sup>2</sup>Other sources of randomness, such as random mutations to represent innovation, may be essential to the model, but are not of interest here.

tial computations with a fixed schedule<sup>3</sup>. We use a simplistic economic ABM of firms that trade and produce goods in order to explore and discuss two alternative modes of scheduling: the *timetable model*, where all agents complete one step after the other, and the *heliotropic model*, where one agent after the other completes steps. In particular, our work in progress focuses on the questions how much randomness is necessary, whether and how the model behaviour becomes mathematically tractable, and how model input and output relates to economic data from national statistics. We argue that a better understanding of these questions, and of economic ABMs in general, is relevant to climate policy making. Therefore, the climate policy background is sketched in Section 2 before Section 3 briefly describes the simplistic model. First results are presented in Section 4 and discussed in Section 5, before Section 6 concludes.

## 2 CLIMATE POLICY AND ABMs

The motivation for our work on economic ABMs stems from the climate change context. Climate policy analysis and recommendations are generally based on standard economic modelling; the most frequently used models are computable general equilibrium models (Capros et al., 1999, for example) or optimal growth models (Nordhaus, 2008; Stern, 2007).

The general set-up is to concentrate on a business as usual (BAU) growth path as the single stable equilibrium path of the system, that is optimal in the short run, and compare this with a situation where measures for the mitigation of greenhouse gas emissions are taken. Mitigation is framed as a welfare trade-off between present and future generations: greenhouse gas emissions are an external effect of the present upon future generations, so that for these the path is not optimal. The current generation uses the atmosphere (as also a few previous generations did) as a “waste dump” for emissions without considering the future negative effects this will due to climate change resulting from the emissions. Introducing a price on emissions, this externality can be internalized, that is, eliminated. However, on the BAU growth path this implies costs in the short run, usually expressed in terms of a reduction in GDP, because the current generation will have to pay for using the atmosphere, that before it simply used “for free”.

This widely accepted welfare trade-off argument has coined a narrative of mitigation as a problem of

<sup>3</sup>Here fixed is meant as in opposition to event-driven. The schedule may still involve randomness in the order in which agents act.

burden sharing (Jaeger et al., 2012). While the mitigation costs are legitimated by the benefits of avoided climate change and its impacts, these benefits lie in a rather far away future, so that on shorter planning horizons, such as the election periods of politicians, the costs seem much more relevant. In this setting, international negotiations have made little progress towards significant world-wide reductions of emissions.

There is, however, in this argument a fundamental assumption that is problematic: the existence of a unique stable equilibrium growth path of the system is warranted neither by economic theory nor, much more importantly, by real-world observations as summarized in empirical data. For example, Ormerod et al. (2009) find that the US, the UK, and the German economic system from time to time switch from a steady to a weak pattern. Also, the comparison of countries that at a certain point in time were in similar situations but now differ in economic growth, such as Poland and Hungary at the moment, deserves the consideration of different growth paths.

Likewise for economic theory: the Arrow-Debreu framework of general equilibrium shows the existence of equilibria in an abstract setting – a growth path is determined by prices that, for each time-step under consideration, balance supply and demand. However, equilibria need not be unique nor stable. Assumptions made to guarantee uniqueness and stability include that of a single representative agent, discussed for example by Kirman (1992).

Hence, the narrative of climate change mitigation as a problem of burden sharing is not the only story to be told. Jaeger et al. (2012) suggest to “reframe the problem of climate change, from zero-sum game to win-win solutions”, i.e., mitigation measures which are beneficial for the economy. Win-win strategies for climate policy can be identified when widening the focus – from concentrating on a single equilibrium to considering several possible equilibria (Jaeger et al., 2010; Shi and Zhang, 2011). A low carbon economy needs a good deal of restructuring as compared with the current economic situation, for example from fossil to renewable energy. Such a structural change, from the perspective of economic theory, involves a shift to another equilibrium growth path. Win-win opportunities arise when the new path is in some sense “better” than the current one. In this case, micro-costs which occur due to mitigation measures, such as increased energy prices due to emission trading, can be more than compensated by macro-benefits, such as higher growth and less unemployment, that the new growth path entails (Jaeger et al., 2010).

Equilibrium selection is a coordination problem (Jaeger, 2012): as for the case of conventions (such

as driving on the left or driving on the right) there are several viable alternatives – which one is in place depends on interactions of many agents and on institutions evolving between them. In order to shift an economic system to a different equilibrium, agents need to re-coordinate to another alternative. Agent-based models are a promising tool for studying such problems as they allow to consider coordination problems between many agents in a dynamic setting. However, few ABMs have been applied in the climate change context, and even less to study climate economics (see Balbi and Giupponi, 2009, for a review).

Macro-economic ABMs, of particular interest in the climate policy context, are often rather complex models with a huge amount of state variables (Gintis, 2007; Dawid et al., 2011; Mandel et al., 2009). Studying equilibrium selection as a coordination problem between many agents requires much more complex models than the standard representative agent models. However, while the simulations of a complex ABM allow to observe the modelled system’s dynamics, this does not necessarily mean that one understands these dynamics. In a simulation, iterations compute one new state after the other from a given initial state of all agents and their environment. Hence, an ABM could be described as a dynamical system given by a state space together with a transition function. However, models are too complex to write these down explicitly, and it is not even clear whether the dynamical system should be deterministic (Epstein, 2006) or a Markov process, that is, probabilistic (Tesfatsion, 2006; Gintis, 2007). For the model’s implementation, randomness can be discussed away with the argument that the computer generates *pseudo* random numbers. Many ABMs have random elements that are essential to the modelled system, but others might be introduced for implementational/computational reasons.

Macro-economic agent-based modelling for (climate) policy does not always start from simple models that are then further elaborated. Therefore, as a second track besides complex economic ABMs, we advocate strongly simplified economic ABMs to gain a better understanding of their properties as dynamical systems and related methodological issues. In the following, we present a preliminary example in this spirit. While this work is further away from real-world applications such as climate policy analysis, it aims at small improvements of our understanding of economic ABMs – which in turn may be very helpful in that context. In particular, the interdependence between agents, that makes scheduling an issue here, is an essential aspect also in more closely climate change related problems.

### 3 THE SIMPLE ABM

The model used here is based on the Lagom model family of economic ABMs (Mandel et al., 2009; Wolf et al., 2012) but it radically simplifies the economic system considered: 100 firms, grouped into 5 sectors, “trade”<sup>4</sup> and produce the good of their sector. Production requires intermediary inputs, the production structure is based on an input output table.

Firms’ parameters are a desired production  $d \in \mathbb{R}_{\geq 0}$ , input coefficients for circulating capital  $\gamma \in [0, 1]^5$ , and an inventory depreciation rate<sup>5</sup>  $r \in [0, 1]$ . It suffices to consider their inventory  $I \in \mathbb{R}_{\geq 0}$  as state variables in this simplistic model. Economic input data (total production, an input-output table, and the inventory depreciation rate) is used to initialize the model.

#### 3.1 Activities: Trade and Production

For each sector  $s = 1, \dots, 5$ , firms try to “buy” the inputs for their desired production  $d$ . Being myopic, they only see a few firms; the number of firms observed, and whether these are fixed or randomly drawn firms, are model input parameters. In the case of fixed suppliers, a firm “sees” the firms listed just before itself in the model’s firm list. The set of observed firms from sector  $s$  is denoted  $O_s$ . Firms first find out the available supply of inputs among the firms they observe for each sector. This is  $a_s = \sum_{i \in O_s} I_i$  for sector  $s$ . The quantity to be produced is determined as  $q = \min\left(d, \min_{s \in S} \frac{a_s}{\gamma_s}\right)$  and the required amounts of inputs are then given by  $q \cdot \gamma_s$ . The firm then buys  $q \cdot \gamma_s$  for each sector  $s$ , and the sellers subtract the respective amounts from their inventories.<sup>6</sup>

Production corresponds to a simple change of state variables for a firm: the produced quantity  $q$  is added to the inventory. In particular, production changes state variables for the active firm only, while

<sup>4</sup>The model uses a caricature of trade, without any payments being made. Rather than of “buying goods”, one might speak of “obtaining presents”.

<sup>5</sup>This rate can be seen as a work-around for the missing consumption in the simplistic model: inventory is decreased, which it would also be if the good was sold to consumers. As money does not play a role here, this approximation is good enough for our purposes.

<sup>6</sup>This is a typical case of interaction in an economic ABM that is easy to describe in words and rather easy to implement, but not so easy to describe as a mathematical dynamical system. State variables of “passive” seller firms change, and the amount  $q$  that is bought may depend on the inventory of some firm in  $O_s$  that again may depend on how much other firms have previously bought from this firm.

trade makes changes to the state variables of other firms as well.

### 3.2 Two Modes of Scheduling

Time evolves discretely. In each period, each firm carries out each activity once. Within periods, we consider two modes of scheduling.

In the *timetable model*, two steps constitute a period: trade and production. In each step, all firms carry out the corresponding activity. The real-world time interval that a period represents may be chosen by the model user by calibrating the input data accordingly. In each period, goods are produced using only goods from previous periods as inputs. This means that the aggregate production of all firms in one period corresponds to the production amount in the time span represented by this period. Therefore, data from national statistics can easily be used in this model, both to calibrate parameters in such a way as to represent a given real-world economic system at a given moment in time, and to validate the model output by comparison with national statistics data. The trade step, however, has a “first come first serve”-element, therefore, the order in which firms trade is important.

In the *heliotropic model*, firms enter activity phases one after the other. Within an activity phase, a firm first trades and then produces. Here, all agents are “in the same situation”: each firm finds some other firms that have just produced goods with a full inventory, while others may have sold most of their inventory since they last produced. That is, in this model no bias arises from the order in which agents act, or in other words, firms have symmetric expectations even though they always act in the same order. However, production is not simultaneous here. A period cannot easily be mapped to a time interval in the real world because the production from one period is not clearly separated from that of another one: firms can use goods that others have already produced in the same period as inputs for production. This complicates the link with national statistics data.

## 4 SIMULATIONS

Input parameters have been calibrated in order to identify the range of parameter values where the model is susceptible to changes in scheduling. This means considering an economic system in which the production inputs are scarce, so that trading first or last in the timetable has an effect. The inputs used are toy data, for example, all entries in the input-output table are equal to 1 for simplicity. With a production

value of 5.1 for all sectors, input coefficients are high compared with the desired production and initial inventories. Also, the inventory depreciation rate is set rather high, so that inventories stay low compared to the demand of production inputs in later periods.

### 4.1 Shuffling in the Timetable Model

Figure 1 shows that shuffling of firms at the beginning of the trade step is indeed necessary to avoid a bias that favours firms who trade first: reduced productions occur only for the last firms in the list when no shuffling takes place, while, when firms are shuffled at the beginning of each trade step, the reduced production amounts are scattered over all firms.

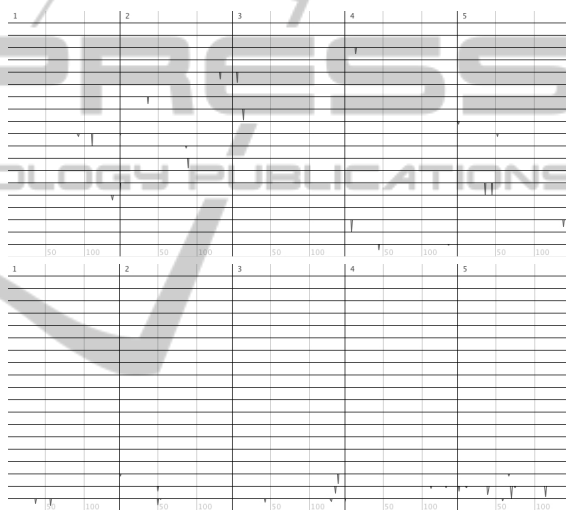


Figure 1: Timetable model with (above) and without (below) shuffling before trade. The model output shown is the produced quantity of firms, columns are sectors, cells show firms. All parameters (including random seed) are the same in the two example runs. The desired production is at the upper border of the firms’ cells, the downward spikes show when a firm produces less than desired.

Without scarcity, the order in which firms trade would not have mattered here. However, it might not always be as obvious that shuffling is required to provide symmetric expectations for agents. In more complex models, higher prices or less skilled workers might be examples of how agents acting last are disadvantaged. Such biases between agents are artificially introduced by sequential operations on the computer that represent parallel actions in the real world.

### 4.2 Scarcity in the Heliotropic Model

The heliotropic model can – without a need to shuffle firms – produce output that resembles the one seen in



Fig. 1 *with* shuffling. However, with the same input data, one sees less scarcity effects, and the inventory depreciation rate needs to be raised to obtain a similar picture as above. This was to be expected because firms trading late in a period may buy goods from firms that have already produced in that period, but it underlines the difficulty of mapping output to national statistics data. Produced goods may enter back into the production scheme in the same period when they were produced, meaning the separation between production inputs and produced goods is lost.

With fixed suppliers, the heliotropic model can be used to construct a completely deterministic baseline case in which randomness is eliminated. Considering this model as a dynamical system, it is a truly deterministic system. Writing down state space and transition function is still a lengthy undertaking, and therefore not done in this short paper. However, it becomes clearer how the system functions: depending on the choice of data for the initialisation one can create an economy in equilibrium, where stocks remain the same or grow, or an economy in decline that will in the end have to crash. For simplicity, the inventory depreciation rate can be “switched off”, that is, set to 0. With an input of 1 unit from each sector, the economy stagnates when the production is set to 5. For production values below this, the economy declines, and production goes down to 0 (except for round-off errors), and this the more quickly, the lower the initial production value is set, of course. In fact, it suffices to have one sector with a production that is not sustainable to create an economy in decline when all goods are needed as inputs. With an initial production value greater than 5, firms always succeed in producing their desired production, and inventories simply increase. The heliotropic model can thus be a starting point for adding bits and pieces of complexity (e.g., an evolving, instead of a fixed desired production), for observing, and hopefully understanding, its effects.

## 5 DISCUSSION

The results presented are of course no deep modelling results. Nevertheless, this simple model highlights some interesting points concerning economic ABMs that use sequential computation. In particular, both modes of scheduling come with advantages and disadvantages.

In the timetable model, shuffling introduces randomness that is not actually essential to the modelled system, but required in order to provide symmetric expectations for agents, owing to the sequential implementation of the model. Randomness in

ABMs implies that the interpretation of model output necessitates many model runs and statistical evaluations of these to learn about the distributions of states. Looking at single trajectories, one might see some effects of improbable events having taken place in just this run. While in ergodic systems the probability distributions over states converge regardless of the initial state chosen, one does not necessarily know when the system arrives “close enough” to the limit and while distributions converge, this does not mean one only sees “average” trajectories. The heliotropic model eliminates this randomness, allowing for a simpler starting point for analysing the ABM. In the efforts that are being made to create economic ABMs which are mathematically tractable, this is a step forward. In fact, the deterministic dynamical system that arises when using fixed matching rules for trade in this model shows behaviour that depends on the input data used in a clearly understandable way.

At the same time, efforts are being made to create economic ABMs which are empirically satisfactory (Fagiolo et al., 2006; Boero and Squazzoni, 2005, for example). Here, the timetable model shows an advantage. The “separation” of periods, meaning that all goods produced in a given period can be used as production inputs at the earliest in the next period, allows to consider the aggregate production of all firms in a period as corresponding to the real-world production of the time span that this period represents. Greater ease of mapping data from national statistics to input data, and vice versa mapping model output to data from national statistics facilitates the empirical validation of the model. This is an important advantage also in the climate policy context, where empirically relevant models are badly needed to “compete” with single equilibrium models. As most data are in a format fit for standard modeling approaches, ABMs that can be fed and validated with these data formats have the edge over those that cannot.

## 6 CONCLUSIONS AND OUTLOOK

This paper presented work in progress on the comparison of two scheduling modes for a simple economic ABM. We find that the output is similar. However, the timetable model, where all firms complete one step after the other, is closer to data from national statistics, facilitating their use as input and the interpretation of aggregates in the model output. The heliotropic model, where one firm after the other completes all steps, can do without shuffling that in the other model is necessary to avoid favouring agents who act first.

Thus, the heliotropic model eliminates randomness – artificial, because required due to sequential computations – producing a simpler model.

For the task of helping to generate theoretical insights, it can be helpful to start from simple economic ABMs and add complexity in a step-by-step manner. In fact, this is the aim of the models used here: too simple to study real-world economic systems, they may allow a better understanding of issues that can be studied already at this simple stage. Indirectly, a better understanding of economic ABMs as dynamical systems then may contribute to policy analysis of real-world problems – as for example the questions of multiple equilibria and win-win strategies for climate policy sketched above.

Further work on simple economic ABMs, in particular on a completely deterministic version of the heliotropic model as a benchmark and on these models as dynamical systems in the mathematical sense, seems worthwhile.

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