Adaptive Gravitational Search Algorithm for PI-fuzzy Controller Tuning

Radu-Codruţ David, Radu-Emil Precup
Dept. of Automation and Appl. Inf., "Politehnica" University of Timisoara, Bd. V. Parvan 2, 300223, Timisoara, Romania

Emil M. Petriu
School of Electrical Eng. and Computer Science, University of Ottawa, 800 King Edward, ON, K1N 6N5, Ottawa, Canada

Mircea-Bogdan Rădac, Constantin Purcaru, Claudia-Adina Dragoş, Stefan Preitl
Dept. of Automation and Appl. Inf., "Politehnica" University of Timisoara, Bd. V. Parvan 2, 300223, Timisoara, Romania

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Abstract: This paper proposes an adaptive Gravitational Search Algorithm (aGSA) focused on tuning of Takagi-Sugeno PI-fuzzy controllers (T-S PI-FCs). The algorithm adapts two depreciation laws of the gravitational constant to the iteration index, a parameter in the weighted sum of all forces exerted from the other agents to the iteration index, and the reset at each stage of agents' worst fitnesses and positions to their best values. Two fuzzy logic blocks carry out the adaptation of the ratios of exploration runs and explanation runs using the ratio between the minimum and maximum Popov sums as an input variable. A tuning method for T-S PI-FCs dedicated to a class of nonlinear servo systems with an integral component and is offered, and T-S PI-FCs with reduced process gain sensitivity are tuned. A case study and digital simulation results illustrate the optimal tuning of a T-S PI-FC for the position control of a laboratory servo system.

1 INTRODUCTION

Fuzzy control systems are successful in many applications as relatively easily understandable nonlinear control approaches (Blažič et al., 2003; Sala et al., 2005). Evolutionary algorithms are employed in the optimal tuning of fuzzy control systems; the current approaches include simulated annealing (Haber et al., 2009; Precup et al., 2011a), elite-guided continuous ant colony optimization (Juang and Chang, 2011), Particle Swarm Optimization (PSO) eventually combined with genetic algorithms (Ling et al., 2008; Precup et al., 2011b; Valdez et al., 2011), and iterative genetic optimization (Onieva et al., 2011).

This paper proposes a new aGSA dedicated to the optimal tuning of Takagi-Sugeno PI-fuzzy controllers (T-S PI-FCs). Our aGSA is developed around the popular GSA (Rashedi et al., 2009), and it is characterized by the several new properties: two Single Input-Single Output (SISO) fuzzy logic blocks ensure the adaptation of the ratios of exploration runs and of explanation runs using the ratio between the minimum and maximum Popov sums as an input variable; the inclusion of Popov sums guarantees the convergence; the adaptation of two depreciation laws of the gravitational constant to the iteration index; the adaptation of a parameter in the weighted sum of all forces exerted from the other agents to the iteration index.

These properties are advantageous compared to the state-of-the-art because of the improved search process offered by our new algorithm. Therefore our approach, which is different to that proposed by Askari and Zahiri (2011), exhibits an additional improvement with respect to GSAs (Precup et al., 2011b).

This paper offers twofold new contributions. First, aGSA is applied to optimal tuning of T-S PI-FCs is proposed. Second, a class of T-S PI-FCs

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which ensures a reduced process gain sensitivity of the fuzzy control systems is suggested. A tuning method is given to minimize objective functions which depend on the control error and on the squared sensitivity function defined with respect to process gain variations from the state sensitivity models of fuzzy control systems.

This paper treats the following topics: the new aGSA is presented in the next section. The tuning method for optimal T-S PI-FCs is described in Section 3. Section 4 discusses the case study of T-S PI-FCs optimally tuned for the angular position control of a laboratory DC nonlinear servo system. The conclusions are pointed out in Section 5.

2 ADAPTIVE GSA

The standard GSA uses agents (particles), and two equations are usually used as depreciation laws of the gravitational constant versus GSA's iterations:

$$g(k) = g_0(1-\psi k / k_{\max}), \quad (1)$$

$$g(k) = g_0 \exp(-\zeta\, k / k_{\max}), \quad (2)$$

where $g(k)$ is the gravitational constant at current iteration index k, g_0 is the initial $g(k)$, $0<\psi<1$ and $\zeta>0$ are parameters which affect GSA's convergence and search accuracy, and $k_{\max}$ is the maximum number of iterations.

Considering N agents and a q-dimensional search space, the position of i^{th} agent is defined in terms of the vector $\mathbf{X}_i$

$$\mathbf{X}_i = [x_i^1 \quad \ldots \quad x_i^d \quad \ldots \quad x_i^q]^T,\ i=1\ldots N, \quad (3)$$

where: x_i^d – the position of i^{th} agent in d^{th} dimension, $d=1\ldots q$, T – matrix transposition. The acceleration $a_i^d(k)$ of i^{th} agent in d^{th} dimension is

$$\begin{aligned} a_i^d(k) &= [1/m_{Ii}(k)] \sum_{j=1,\, j\neq i}^{N} \sigma_j \{ g(k) \\ & m_{Pi}(k) m_{Aj}(k) [x_j^d(k) - x_i^d(k)] / (r_{ij}(k)+\varepsilon) \}, \end{aligned} \quad (4)$$

where: $0 \le \sigma_j \le 1$ – a random generated number, $m_{Pi}(k)$ and $m_{Aj}(k)$ – the active and passive gravitational mass related to i^{th} and j^{th} agent, $\varepsilon>0$ – a relatively small constant, $m_{Ii}(t)$ – the inertia mass related to i^{th} agent, and $r_{ij}(k)$ – the Euclidian distance between i^{th} and j^{th} agents:

$$r_{ij}(k) = \| \mathbf{X}_i(k) - \mathbf{X}_j(k) \|. \quad (5)$$

The next velocity of an agent, $v_i^d(k+1)$, and the next position of an agent, $x_i^d(k+1)$, result from the state-space equations (Rashedi et al., 2009)

$$\begin{aligned} v_i^d(k+1) &= \rho_i v_i^d(k) + a_i^d(k), \\ x_i^d(k+1) &= x_i^d(k) + v_i^d(k+1), \end{aligned} \quad (6)$$

with $0 \le \rho_i \le 1$ – a uniform random variable.

The active gravitational mass and the inertial mass are (Rashedi et al., 2009)

$$\begin{aligned} & m_i(k) = n_i(k) / [\sum_{j=1}^{N} n_j(k)],\ m_{Ai} = m_{Ii} = m_i, \\ & n_i(k) = [f_i(k) - w(k)]/[b(k) - w(k)], \\ & b(k) = \min_{j=1\ldots N} f_j(k),\ w(k) = \max_{j=1\ldots N} f_j(k), \end{aligned} \quad (7)$$

where $f_i(k)$ is the fitness value of i^{th} agent at iteration index k, f is the fitness function, the term $b(k)$ corresponds to the best agent, and the term $w(k)$ corresponds to the worst agent.

The convergence of the aGSA is guaranteed by hyperstability analysis results derived from (Landau, 1979; Precup et al., 2003). A sufficient condition for GSA's convergence is

$$\begin{aligned} & \nu_i(k_0,k_1) = \sum_{k=k_0}^{k_1} (\mathbf{w}_i^{\text{LTI}}(k))^T \mathbf{v}_i^{\text{LTI}}(k) \ge -\mu_0^2, \\ & \forall\, k_1 \ge k_0 \ge 0, \mu_0 = \text{const}, \mu_0 \ne 0, \mathbf{w}_i^{\text{LTI}}(k) = \\ & -[a_i^1(k) \quad \ldots \quad a_i^q(k) \quad a_i^1(k) \quad \ldots \quad a_i^q(k)]^T, \\ & \mathbf{v}_i^{\text{LTI}} = [v_i^1 \quad \ldots \quad v_i^q \quad x_i^1 \quad \ldots \quad x_i^q]^T, \end{aligned} \quad (8)$$

where $\nu_i(k_0,k_1)$, $i=1\ldots N$, is the Popov sum, and the superscript LTI points out a discrete-time linear time-invariant block resulted after the organization of equations (4) to (7) as a dynamical feedback system structure.

aGSA is formulated in terms of the flowchart presented in Figure 1. Stage II allows the algorithm to discover the extent of the search space. This stage is characterized by a linear decrease of $g(k)$ according to (1) during the first $r_{e1} k_{\max}$ runs of the search process, where r_{e1} is the ratio of exploration runs $0<r_{e1}<1$. The input variable iv is

$$\begin{aligned} & iv = \min_{i=1\ldots N} \nu_i(k_0,k_1) / \max_{i=1\ldots N} \nu_i(k_0,k_1), \\ & \forall\, k_1 \ge k_0 \ge 0, \end{aligned} \quad (9)$$

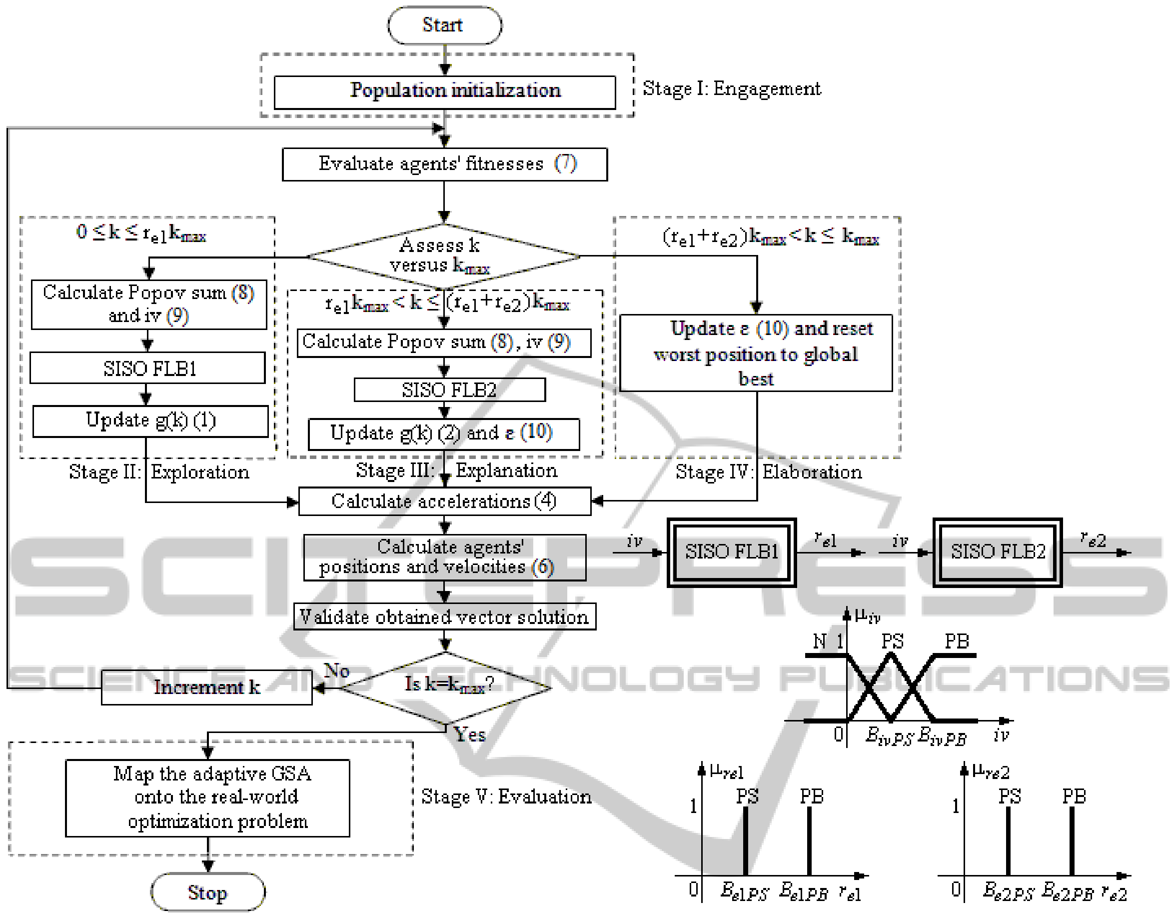

Figure 1: Flowchart of aGSA, and structures of SISO FLB1 and SISO FLB2.

and it is applied to the Mamdani fuzzy block SISO FLB1 (with the structure and membership functions presented in Figure 1) to calculate r_{e1}.

aGSA restricts agents' movements in stage III by the introduction of a more aggressive depreciation schedule of $g(k)$ in terms of (2) and by the linear depreciation of ε

$$\varepsilon = \varepsilon_0[1-(k-r_{e1}k_{\max})/(k_{\max}-r_{e1}k_{\max})]. \qquad (10)$$

The input variable iv is calculated and applied to the second SISO fuzzy logic block (SISO FLB2) which calculates the ratio of explanation runs $0 < r_{e2} < 1$. The next $r_{e2}k_{\max}$ runs in the search process are conducted using (10).

The rule base of SISO FLB1 is

$$\begin{aligned} &R^1: \text{IF } iv \text{ IS N THEN } r_{e1} \text{ IS PS},\\ &R^2: \text{IF } iv \text{ IS PS THEN } r_{e1} \text{ IS PB}, \qquad (11)\\ &R^3: \text{IF } iv \text{ IS PB THEN } r_{e1} \text{ IS PS}.\end{aligned}$$

Rule R^1 points out that some runs do not fulfil (8); therefore a small number of runs must be conducted. Rule R^2 outlines that agents' positions are oscillating far away from the solution, so a high number of runs is needed. Rule R^3 indicates that aGSA is close to the solution. We suggest the following parameter settings of SISO FLB1 to ensure a trade-off to convergence speed and number objective function evaluations: $B_{ivPS} = 0.3$, $B_{ivPB} = 0.8$, $B_{e1PS} = 0.15$, and $B_{e1PB} = 0.2$.

Mamdani's MAX-MIN compositional rule of inference is used in the inference engines of SISO FLB1 and SISO FLB2. The defuzzification for both SISO FLB1 and SISO FLB2 is carried out by the center of gravity method for singletons.

The rule base of SISO FLB2 is similar to (11), and the parameters of output membership functions of SISO FLB2 are set as follows (similar settings to to SISO FLB1): $B_{e2PS} = 0.45$ and $B_{e2PB} = 0.5$.

Stage IV sets the general position for the optimal value of fitness function and dedicates the remaining time to the refinement of obtained results. The value of $g(k)$ stops decaying and only ε continues the depreciation process. Worst agents' positions are reset to the best values obtained so far after each run.

Stage V evaluates the real-world optimization problem's performance for the location of the best position vector obtained during the search process. The obtained solution is mapped onto the real-world optimization problem and tested at this stage. The optimization problem which leads to a new class of T-S PI-FCs with a reduced process gain sensitivity is

$$\boldsymbol{\rho}^* = \arg\min_{\boldsymbol{\rho}\in D_{\boldsymbol{\rho}}} I_{IAE}^{k_P}(\boldsymbol{\rho}),$$
$$I_{IAE}^{k_P}(\boldsymbol{\rho}) = \sum_{t=0}^{\infty}[|\,e(t)\,| + (\gamma^{k_P})^2[\sigma^{k_P}(t)]^2], \quad (12)$$

where: $\boldsymbol{\rho}$ – the parameter vector of the controller, $\boldsymbol{\rho}^*$ – the optimal parameter vector, $D_{\boldsymbol{\rho}}$ – the feasible domain of $\boldsymbol{\rho}$, $I_{IAE}^{k_P}(\boldsymbol{\rho})$ is the objective function, $t \in \mathbf{N}$ – the independent discrete time argument, $e(t)$ – the control error, $\sigma^{k_P}(t)$ – the output sensitivity function (Precup et al., 2011b), γ^{k_P} – the weighting parameter, IAE – the Integral of Absolute Error, and all variables depend on $\boldsymbol{\rho}$.

3 OPTIMAL TAKAGI-SUGENO PI-FUZZY CONTROLLERS

Many processes in servo systems can be described by the continuous-time nonlinear time-invariant SISO state-space models saturation and dead zone static nonlinearity (Precup et al., 2011b). A simplified process model (with variable parameters) used in controller tuning is the transfer function

$$P(s) = k_P/[s(1+T_\Sigma s)], \quad (13)$$

where k_P is the process gain and T_Σ is the small time constant.

PI controllers are recommended for processes of type (13) as shown in (Åström and Hägglund, 1995; Visioli, 2004). PI controllers' transfer functions are

$$C(s) = k_c(1+sT_i)/s = k_C[1+1/(sT_i)], \quad (14)$$

where k_c is the controller gain, T_i is the integral time constant and $k_C = k_c T_i$. Very good control system performance indices and a trade-off to these indices can be achieved if the PI controllers are tuned by the Extended Symmetrical Optimum (ESO) method (Preitl and Precup, 1999) which uses a single design parameter β and the tuning conditions

$$k_c = 1/(\beta\sqrt{\beta}\,T_\Sigma^2\,k_P),\ T_i = \beta T_\Sigma,\ 1 < \beta < 20\,. \quad (15)$$

T-S PI-FCs are tuned in order to ensure the performance improvement. The Two Inputs-Single Output fuzzy controller (TISO-FC) block (Figure 2) uses the SUM and PROD operators in the inference engine and the weighted average defuzzification method. The rule base of T-S PI-FC is

$$\begin{aligned}
&R^1: \text{IF } e(t) \text{ IS N AND } \Delta e(t) \text{ IS N THEN}\\
&\quad \Delta u(t) = \eta K_P[\Delta e(t) + \mu e(t)],\\
&R^2: \text{IF } e(t) \text{ IS N AND } \Delta e(t) \text{ IS ZE THEN}\\
&\quad \Delta u(t) = K_P[\Delta e(k) + \mu e(t)],\\
&\ldots\\
&R^8: \text{IF } e(t) \text{ IS P AND } \Delta e(t) \text{ IS ZE THEN}\\
&\quad \Delta u(t) = K_P[\Delta e(t) + \mu e(t)],\\
&R^9: \text{IF } e(t) \text{ IS P AND } \Delta e(t) \text{ IS P THEN}\\
&\quad \Delta u(t) = \eta K_P[\Delta e(t) + \mu e(t)],\\
&K_P = k_c\,(T_i - T_s/2),\ \mu = 2T_s/(2T_i - T_s),
\end{aligned} \quad (16)$$

where $\Delta e(t) = e(t) - e(t-1)$ is the increment of e, $\Delta u(t) = u(t) - u(t-1)$ is the increment of u, T_s is the sampling period, and the parameter $0 < \eta \le 1$ reduces the overshoot. The tuning condition resulted from the modal equivalence principle is

$$B_{\Delta e} = \mu\,B_e = 2T_s B_e/(2\,\beta\,T_\Sigma - T_s)\,. \quad (17)$$

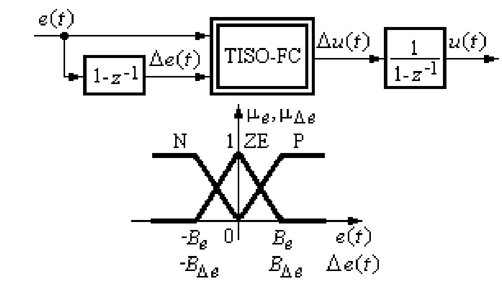

Figure 2: Structure of T-S PI-FC.

The ESO method and the modal equivalence principle lead to the reduction of the number of T-S PI-FC parameters and to the simplification of the number of variables of the objective function. The parameter vector $\boldsymbol{\rho} \in R^3$ of T-S PI-FCs is $\boldsymbol{\rho} = [\rho_1 \quad \rho_2 \quad \rho_3]^T$, with $\rho_1 = \beta,\ \rho_2 = B_e,\ \rho_3 = \eta$. Our aGSA is mapped onto the optimization problem

(12) which ensures the optimal tuning of T-S PI-FC parameters by means of

$$f_j(k) = I_{IAE}^{k_P}(\boldsymbol{\rho}),\ a = 1...m,\ j = 1...N,$$
$$\mathbf{X}_i = \boldsymbol{\rho},\ i = 1...N. \quad (18)$$

The tuning method consists of the following steps which have to be proceeded to get the optimal parameter vector $\boldsymbol{\rho}^*$:

- Step A. Apply (15), set T_s, apply (16), derive the sensitivity model with respect to k_P.
- Step B. Set γ^{kP} to meet the performance specifications, validation condition, and feasible domain of $\boldsymbol{\rho}$ in (12), $D_{\boldsymbol{\rho}} \in R^{q=3}$, to include all constraints.
- Step C. Apply aGSA to get optimal parameter vector $\boldsymbol{\rho}^*$, apply (17) to compute $B_{\Delta e}^*$.

4 CASE STUDY AND SIMULATION RESULTS

The case study applies the new tuning method to a T-S-PI-FC for the angular speed control of a laboratory servo system built around the INTECO DC servo system laboratory equipment experimental setup. The parameters in (13) are (Precup et al., 2011b) $k_P = 139.88$ and $T_\Sigma = 0.9198$ s.

Steps A – C are applied and a set of results is presented as follows in terms of setting. T_s =0.01s and γ^{kP} =100. A good trade-off to convergence speed and number of evaluations of $I_{IAE}^{k_P}(\boldsymbol{\rho})$ is ensured by aGSA parameters set to N=20, k_{max}=100, ζ=30, ε_0=0.01 and g_0=100. The optimal values of T-S PI-FC parameters are $B_e^* = 22.2572$, $B_{\Delta e}^* = 0.0806183$, $\eta^* = 0.981566$, and the reduced objective function of $I_{IAE\,\min}^{k_P} = 488237$. The comparison was done for the same parameter values for GSA; the optimal parameters of T-S PI-FC are $B_e^* = 20$, $B_{\Delta e}^* = 0.072393$, $\eta^* = 1$, and the reduced objective function is $I_{IAE\,\min}^{k_P} = 488523$. The convergence speed is defined as the number of evaluations of objective function until reaching the minimum. The average values of best five runs of both algorithms show the convergence speed of 532.4 for GSA and of 1622.8 for aGSA.

The comparison of these results shows that the aGSA leads to improved optimal values of $I_{IAE}^{k_P}(\boldsymbol{\rho})$ compared to GSA for the same value of γ^{kP}.

A sample of simulation results is presented in Figure 3. The simulations were carried out for the step-type angular position reference input of r = 20 rad and $\beta = 3.6$ for control systems with T-S-PI-FC tuned by the new aGSA and for the fuzzy control system with T-S-PI-FC tuned by GSA.

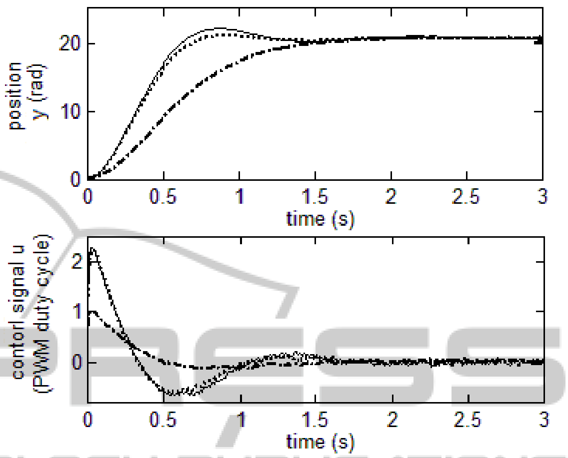

Figure 3: Time responses of fuzzy control systems with initial T-S PI-FC (line-dotted), GSA tuned T-S PI-FC (solid), aGSA tuned T-S PI-FC (dotted).

Based on these simulation results, our solutions can be accepted as very close to the optimal ones. However different conclusions can be drawn for other objective functions eventually controlling other processes (Baranyi et al., 1997; Ferreira and Ruano, 2009; Hladek et al., 2009; Johanyák, 2010; Leva and Maggio, 2011).

5 CONCLUSIONS

This paper has introduced an aGSA which employs the adaptation of two parameters of a classical GSA to the iteration index and on the fuzzy logic-based adaptation of the number of algorithm's runs in two stages. Popov's hyperstability results guarantee the convergence of our aGSA.

The simple and effective implementation of aGSA in the optimal tuning of parameters of T-S PI-FCs is obtained by the application of the ESO method and of the modal equivalence principle. In addition, aGSA offers a better usage of the algorithms resources by extending the search process to the entire search duration.

Future research will concern the extension of our aGSA such that to be applied to other optimization problems which will offer robustness properties of the fuzzy control systems. Nonlinear MIMO processes will be targeted.

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REFERENCES

Askari, H., Zahiri, S.-H., 2011. Decision function estimation using intelligent gravitational search algorithm. *International Journal of Machine Learning and Cybernetics*. DOI 10.1007/s13042-011-0052-x.

Åström, K. J., Hägglund, T., 1995. *PID Controllers Theory: Design and Tuning*. Research Triangle Park, NC: Instrument Society of America.

Baranyi, P., Korondi, P., Hashimoto, H., Wada, M., 1997. Fuzzy inversion and rule base reduction. In *Proceedings of IEEE International Conference on Intelligent Engineering Systems (INES'97)*. Budapest, Hungary, 301-306.

Bellomo, D., Naso, D., Babuška, R., 2008. Adaptive fuzzy control of a non-linear servo-drive: theory and experimental results. *Engineering Applications of Artificial Intelligence*. 21, 846-857.

Blažič, S., Škrjanc, I., Matko, D., 2003. Globally stable direct fuzzy model reference adaptive control. *Fuzzy Sets and Systems*. 139, 3-33.

Ferreira, P. M., Ruano, A. E., 2009. On-line sliding-window methods for process model adaptation. *IEEE Transactions on Instrumentation and Measurement*. 58, 3012-3020.

Haber, R. E., Haber-Haber, R., Jiménez, A., Galán, R., 2009. An optimal fuzzy control system in a network environment based on simulated annealing. An application to a drilling process. *Applied Soft Computing*. 9, 889-895.

Hladek, D., Vaščák, J., Sinčák, P., 2009. Multi-robot control system for pursuit-evasion problem. *Journal of Electrical Engineering*. 60, 143-148.

Johanyák, Z. C., 2010. Student evaluation based on fuzzy rule interpolation. *International Journal of Artificial Intelligence*. 5, 37-55.

Juang, C.-F., Chang, P.-H., 2011. Recurrent fuzzy system design using elite-guided continuous ant colony optimization. *Applied Soft Computing*. 11, 2687-2697.

Landau, Y.-D., 1979. *Adaptive Control: The Model Reference Approach*. New York: Marcel Dekker.

Leva, A., Maggio, M., 2011. A systematic way to extend ideal PID tuning rules to the real structure. *Journal of Process Control*. 21, 130-136.

Ling, S. H., Iu, H. H. C., Chan, K. Y., Lam, H. K.., Yeung, C. W., Leung, F. H. F., 2008. Hybrid particle swarm optimization with wavelet mutation and its industrial applications. *IEEE Transactions on Systems, Man, and Cybernetics, Part B: Cybernetics*. 38, 743-763.

Onieva, E., Naranjo, J. E., Milanés, V., Alonso, J., García, R., Pérez, J., 2011. Automatic lateral control for unmanned vehicles via genetic algorithms. *Applied Soft Computing*. 11, 1303-1309.

Precup, R.-E., David, R.-C., Petriu, E. M., Preitl, S., Rădac, M.-B., 2011a. Fuzzy control systems with reduced parametric sensitivity based on simulated annealing. *IEEE Transactions on Industrial Electronics*. DOI 10.1109/TIE.2011.2130493.

Precup, R.-E., David, R.-C., Petriu, E. M., Rădac, M.-B., Preitl, S., Fodor, J., 2011b. Evolutionary optimization-based tuning of low-cost fuzzy controllers for servo systems. *Knowledge-Based Systems*. DOI 10.1016/j.knosys.2011.07.006.

Precup, R.-E., Preitl, S., Fur, G., 2003. PI predictive fuzzy controllers for electrical drive speed control: Methods and software for stable development. *Computers in Industry*. 52, 253-270.

Preitl, S., Precup, R.-E., 1999. An extension of tuning relations after symmetrical optimum method for PI and PID controllers. *Automatica*. 35, 1731-1736.

Rashedi, E., Nezamabadi-pour, H., Saryazdi, S., 2009. GSA: A gravitational search algorithm. *Information Sciences*. 179, 2232-2248.

Sala, A., Guerra, T. M., Babuška, R., 2005. Perspectives of fuzzy systems and control. *Fuzzy Sets and Systems*. 156, 432-444.

Valdez, F., Melin, P., Castillo, O., 2011. An improved evolutionary method with fuzzy logic for combining particle swarm optimization and genetic algorithms. *Applied Soft Computing*. 11, 2625-2632.

Visioli, A., 2004. A new design for a PID plus feedforward controller. *Journal of Process Control*. 14, 457-463.