Control and Model Parameters Identification of Inertia Wheel Pendulum

Paweł Drapikowski, Jarosław Gośliński and Adam Owczarkowski
Institute of Control and Information Engineering, Poznań University of Technology, Piotrowo 3a, 60-965 Poznań, Poland

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Abstract: This paper presents control method of an inverted pendulum with an inertial drive (*IWP Inertia Wheel Pendulum*). This is a non-linear, underactuated mechanical system and therefore it has more degrees of freedom than control variables. In application, electric motor has been placed on the pendulum top and it is a source of torque, which accelerates the flywheel. Position of the pendulum depends on the acceleration of the flywheel. This paper shows algorithm for the real object to keep the inertia wheel pendulum in the vertical position at equilibrium point. In order to achive this aim, authors decided to concentrate on modern and advanced techniques of control and estimation such as LQR regulator, sensor fusion, extended Kalman Filter and model parameters identification.

1 INTRODUCTION

Control of an inverted pendulum is a well known problem (Bradshaw and Shao, 1996). There are many control methods and different types of inverted pendulum. One of the most interesting pendulum construction is the inertia wheel pendulum (Spong and P. Corke, 1999). It has got motor with attached flywheel, located on the top. Many times, this kind of pendulums are only subject of simulation. In this paper, control problem as well as hardware implementation is presented. For the future research purpose, authors developed *IWP* with addidional wheels on the bottom (mobile robot), as shown in Fig. 1.

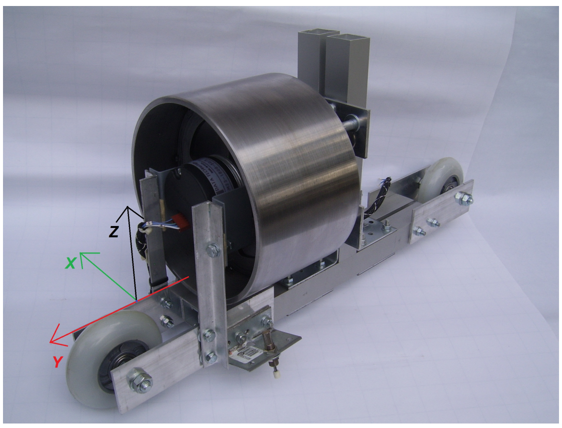

Figure 1: IWP - real photo.

In this independend IWP platform, absolute position and velocity sensor were needed as well as proper control algorithm. For needs of this application, sensor fusion algorithm was used (given in section 2) and also Extended Kalman Filter (stated in section 3). It can be seen that with filter algorithm of Kalman function, control will be based on Linear-Quadratic problem, which is true and described in section 4. During further steps, authors decided to involve model parameters identificator in order to keep control gain up to date and to track for object parametrs change. Identifcation methods are stated in section 5. The overall control scheme of the IWP is shown in Fig.2.

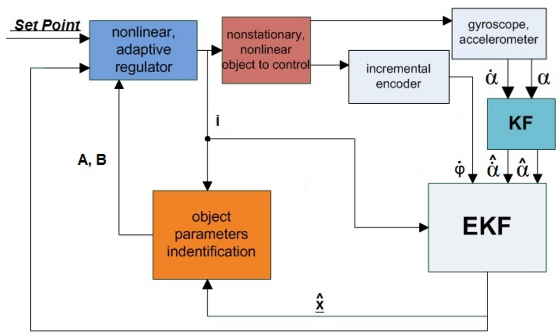

Figure 2: Overall control scheme.

2 SENSORS AND FUSION ALGORITHM

Measurement unit consists of absolute sensors i.e. gy-

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roscope and accelerometer. Problem of noisy signals has led to choose uncommon version of the Kalman filter for sensor fusion. It takes simultaneously two signals: from the gyroscope and from the accelerometer. This sensors fusion has two major advantages. Firstly, it gives better angle estimation when integrating gyroscope result than taking raw data directly from accelerometer. Secondly, it provides estimation without bias, when taking into account accelerometer measurements (as a reset function for the qyroscope integrator bias).

For better understanding fusion algorithm, short description about sensors location must be given. The main gyrocope axis lies in line with Y IWP's axis and active accelerometer axis is parallel to X pendulum's axis. This allows to concatenate measurements results from gyroscope and accelerometer sensors. Accelerometer works here as an inclinometer and gyroscope as an angle velocity meter. In Kalman Filter configured for a sensor fusion given by following equations:

Prediction

$$\begin{aligned} \hat{\underline{x}}_k^- &= A \cdot \hat{\underline{x}}_{k-1} + B \cdot u_{k-1} \\ P_k^- &= A \cdot P_{k-1} \cdot A^T + Q \end{aligned} \tag{1}$$

Correction

$$\begin{aligned} K_k &= P_k^- \cdot C^T \cdot (C \cdot P_k^- \cdot C^T + R)^{-1} \\ \hat{\underline{x}}_k &= \hat{\underline{x}}_k^- + K_k \cdot (z_k - C \cdot \hat{\underline{x}}_k^-) \\ P_k &= (I - K_k \cdot C) \cdot P_k^- \end{aligned} \tag{2}$$

matrices A, B, C shape mathematical model of the measurement system (fusion). For matrices:

$$A = \begin{bmatrix} 1 & 0 & -dt \\ 0 & 0 & -1 \\ 0 & 0 & 1 \end{bmatrix}, B = \begin{bmatrix} dt \\ 1 \\ 0 \end{bmatrix} \tag{3}$$

$$C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \tag{4}$$

* where dt is equal to integration interval

and input signal u_k (equal to gyroscope measurement) as well as input z_k (equal to accelerometer declination) it is possible to perform sensor fusion. This algorithm has two inputs: angle velocity and declination. In Eqns. (1) and (2), $\hat{\underline{x}}_k^-$ and $\hat{\underline{x}}_k$ stand for estimation vector (*a priori* and *a posteriori*, respectively). During the process of fusion, three values are estimated:

$$\hat{\underline{x}} = [\hat{\alpha}, \hat{\dot{\alpha}}, \hat{\zeta}]^T \tag{5}$$

$\hat{\alpha}$ stands for an IWP angle, $\hat{\dot{\alpha}}$ is equal to angular velocity and finally $\hat{\zeta}$ is an angular velocity bias (offset), which must be known in order to improve angle velocity estimation.

3 EXTENDED KALMAN FILTER

During the work also has taken advantage of extended Kalman filter. It uses two groups of input signals: measurements (with fusion results) and excitation of the object (which is equal to current). By using Kalman Filter it is possible to predict next state of the IWP. For this purpose desired values (angle, angular velocity of the pendulum and the angular velocity of the flywheel) and input signal are needed. This Kalman Filter is based on IWP's mathematical model. Model can be given by IWP matrices: (6), (7),(8) and (9).

$$A_k = \begin{bmatrix} 1 & T_p & 0 \\ \frac{m_c g l_p sin(\alpha)}{I_r + I_k} T_p & 1 - \frac{b_\alpha}{I_r + I_k} T_p & -\frac{b_\phi}{I_r + I_k} T_p \\ 0 & 0 & 1 - \frac{b_\phi}{I_k} T_p \end{bmatrix} \tag{6}$$

$$B_k = \begin{bmatrix} 0 \\ \frac{k}{I_r + I_k} T_p \\ \frac{k}{I_k} T_p \end{bmatrix} \tag{7}$$

$$C_k = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{8}$$

$$D_k = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{9}$$

As it has already been stated excitation signal for the IWP is current, which flows through the DC motor. To make this current controllable, it was obligatory to use current regulator. It has been completed with current regulator directly in the plant. Thus, model above is current driven.

IWP's model is nonlinear (matrix 6). Therefore, in order to estimate state vector, authors had to use extended Kalman filter. This estimator can be stated as a two-phase algorithm. In first phase IWP's model is stimulated by current value, (calculated from the IWP's position regulator), then covariance matrix (P_k^-) projection is made. First phase is given by group of equations (10):

$$\begin{aligned} \hat{\underline{x}}_k^- &= f_k(\hat{\underline{x}}_{k-1}, u_{k-1}) \\ F_k &= \frac{\partial f(\hat{\underline{x}}_{k-1}, u_{k-1})}{\partial \hat{\underline{x}}|_{\hat{\underline{x}} = \hat{\underline{x}}_{k-1}}} = \frac{\partial [A_k \cdot \hat{\underline{x}}_{k-1} + B_k \cdot u_{k-1}]}{\partial \hat{\underline{x}}|_{\hat{\underline{x}} = \hat{\underline{x}}_{k-1}}} \\ P_k^- &= F_k \cdot P_{k-1} \cdot F_k^T + Q_k \end{aligned} \tag{10}$$

In the second phase, whole state ($\hat{\underline{x}}_k^-$), estimated in the first phase is corrected. During this stage angular velocity from IWP's wheel and estimates from sensor fusion are taken into account as a vector $\underline{z}_k$. Correction phase is as follows:

$$\begin{array}{c} H_k = \frac{\partial h(\hat{\underline{x}}_{k-1})}{\partial \hat{\underline{x}}|_{\hat{x}=\hat{x}_{k-1}}} = \frac{\partial [C_k \cdot \hat{\underline{x}}_{k-1}]}{\partial \hat{\underline{x}}|_{\hat{x}=\hat{x}_{k-1}}} \\ K_k = P_k^- \cdot H_k^T \cdot (H_k \cdot P_k^- \cdot H_k^T + R_k)^{-1} \\ \hat{\underline{x}}_k = \hat{\underline{x}}_k^- + K_k \cdot (\underline{z}_k - C_k \cdot \hat{\underline{x}}_k^-) \\ P_k = (I - K_k \cdot H_k) \cdot P_k^- \end{array} \tag{11}$$

In correction phase Kalman gain (K_k) is calculated as well as an ultimate state vector ($\hat{\underline{x}}_k$). Finally, covariance matrix is updated.

4 OBJECT PARAMETERS IDENTIFICATION

Identification process was introduced in order to track change of the model parameters. Among many system identification methods, there are a few which can be used in this work. Before selecting indetification method it is necessary to choose proper structure for the model. One of the best structure (due to the simplicity of its implementation) is ARX (Autoregressive Exogenous model). Frame for this structure is given by following equation:

$$y_k = \frac{B(q^{-1})}{A(q^{-1})} u_{k-d} + \frac{1}{A(q^{-1})} e_k \tag{12}$$

* where $d > 0$ and:
$A(q^{-1}) = 1 + a_1 q^{-1} + ... + a_n q^{-n}$
$B(q^{-1}) = b_0 + b_1 q^{-1} + ... + b_m q^{-m}$

For the model description, ARX structure (12) can be taken into consideration but only under the assumption that IWP's model is linearized. As has already been stated, IWP's model is nonlinear (*sin* function in (6)). This must be eliminated from model. Model also has to take shape of transitional equations which can be derived from IWP's matrices (6), (7),(8) and (9):

$$\alpha_k = \alpha_{k-1} + \dot{\alpha}_{k-1} T_p \tag{13}$$

$$\begin{array}{c} \dot{\alpha}_k = \frac{m_c g l_p}{I_r + I_k} sin(\alpha_{k-1}) + (1 - \frac{b_\alpha T_p}{I_r + I_k}) \dot{\alpha}_{k-1} + \\ + (-\frac{b_\phi T_p}{I_r + I_k}) \phi_{k-1} + \frac{k T_p}{I_k + I_r} i \end{array} \tag{14}$$

$$\phi_k = (1 - \frac{b_\phi T_p}{I_k}) \phi_{k-1} + \frac{k T_p}{I_k} i_{k-1} \tag{15}$$

As it can be seen, Eqn. (13) is only additional and there are no parametres to be identified. Second equation (14) which forms the model cannot be written in a ARX structure directly, without modifications. To do so, it is necessary to eliminate term with *sin* function (which depends on α parameter). In order to this, whole term with *sin* function has to be included to a known disturbance e_k:

$$e_k = \frac{m_c g l_p}{I_r + I_k} sin(\alpha_{k-1}) + \xi \tag{16}$$

* where ξ is additional noise

Furthermore, equation (14) has two inputs i.e. variable ϕ and i. For needs of the ARX structure it is desirable to have only one input variable and one output variable. This can be achieved by proper substitution of equation (15) into equation (14). From (15) following equation can be derived:

$$\frac{I_k}{I_k + I_r} \phi_k - \frac{I_k}{I_k + I_r} \phi_{k-1} = -\frac{b_\phi T_p}{I_k + I_r} \phi_{k-1} + \frac{k T_p}{I_k + I_r} i \tag{17}$$

which can be substituted to (14). After substituting, new system of equations will be obtained:

$$\phi_k = (1 - \frac{b_\phi T_p}{I_k}) \phi_{k-1} + \frac{k T_p}{I_k} i_{k-1} \tag{18}$$

$$\dot{\alpha}_k = (1 - \frac{b_\alpha T_p}{I_r + I_k}) \dot{\alpha}_{k-1} + \frac{I_k}{I_k + I_r} \phi_k - \frac{I_k}{I_k + I_r} \phi_{k-1} + e_k \tag{19}$$

From this point, system consists of two equations ((18) and (19)). These equations are suitable for ARX structure. Only in equation (19) it might be seem, that the delay parameter d is not greater then zero (ARX structure, eqn. (12)), which does not allow the process to be correct. Nevertheless, in this particular case ϕ_k can be taken directly from the eqn. (18) which is normally calculated before $\dot{\alpha}_k$.

In order to perform identification process least squares method has been selected. Identification process must be executed as an online algorithm. This can be accomplished by use of the recursive least squares method (*RLS*) which is given by equations (20)-(23)

$$\hat{\underline{\theta}}_k = \hat{\underline{\theta}}_{k-1} + \frac{P_{k-1} \underline{\varphi}_k}{\lambda_k + \underline{\varphi}_k^T P_{k-1} \underline{\varphi}_k} \varepsilon_k \tag{20}$$

$$P_k = (P_{k-1} - \frac{P_{k-1} \underline{\varphi}_t \underline{\varphi}_k^T P_{k-1}}{\lambda_k + \underline{\varphi}_k^T P_{k-1} \underline{\varphi}_k}) \frac{1}{\lambda_k} \tag{21}$$

$$\varepsilon_k = y_k - \underline{\varphi}_k^T \hat{\underline{\theta}}_{k-1} \tag{22}$$

$$\lambda_k = \lambda^0 \lambda_{k-1} + (1 - \lambda^0) \tag{23}$$

For each of the final model equations (i.e. (18) and (19)) procedure of the RLS must be carried out. After each iteration of the RLS, results are available in $\hat{\underline{\theta}}_k$ vector. For the equation (18) it can be written that:

$$\hat{\underline{\theta}}_k = \begin{bmatrix} (1 - \frac{b_\phi T_p}{I_k}) & \frac{k T_p}{I_k} \end{bmatrix}^T \tag{24}$$

and for equation (19) it is true that:

$$\hat{\underline{\theta}}_k = \begin{bmatrix} (1 - \frac{b_\alpha T_p}{I_r + I_k}) & \frac{I_k}{I_k + I_r} & \frac{I_k}{I_k + I_r} \end{bmatrix}^T \tag{25}$$

At this point very important is to set proper values for the forgetting factor λ_0 and λ^0. For maximum value of 1, whole algorithm will be not sensitive for parameters change. On the other hand, small values for λ's can make RLS method internally unstable, which could be observed as a oscillation in parameters.

5 LQR REGULATOR

The control system uses a linear quadratic regulator LQR. It includes changing in time model which is linearized in every point of trajectory. This solution is suitable for stabilizing non-linear systems and it has adaptive nature.

The LQR regulator can be applied to non-linear system which is linerized along the trajectory. Then the system matrices change in time.

The control aim is to make every state variable as a desired value which is zero. The feedback uses the whole state vector. Regulation system is in Fig. 3.

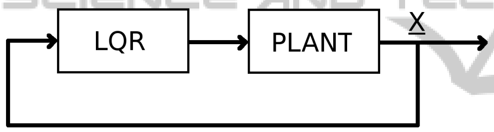

Figure 3: The LQR regulation system.

The control law is based on the optimal criterion given by equation:

$$J = \sum_{i=0}^{n} \left[\underline{x}_i^T Q_{LQR} \underline{x}_i + R_{LQR} \underline{u}_i^2 \right], \tag{26}$$

where $Q_{LQR} = Q_{LQR}^T$, J is quadratic cost function, $\underline{x}_i$ is a state vector, $\underline{u}_i$ is a control vector, n is a number of iteration in regulator calculation process, Q_{LQR} is a state weight matrix and R_{LQR} is weight of control signal. The aim is to minimize that quadratic criterion. R_{LQR} and Q_{LQR} have constant values ((Wang et al., 2010), (Zhang and Wang, 2010)):

$$R_{LQR} = 100, \tag{27}$$

$$Q_{LQR} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{28}$$

This paper shows discrete time form of LQR equations with finite horizon which made it easer to implement it in the real application. In (Zabczyk, 1991) it is shown how difficult solving continuous time LQR equations is. For discrete problem it is needed to have discrete state space model:

$$\underline{x}_{k+1} = A_d \underline{x}_k + B_d \underline{u}_k, \tag{29}$$

$$\underline{y}_k = C_d \underline{x}_k + D_d \underline{u}_k, \tag{30}$$

where A_d, B_d, C_d and D_d are discrete model matrices. The author of (Sauer, 2011) describes how to obtain them having linear model matrices. Here was used approximate iterative method.

The control law is given by:

$$\underline{u}_{k+1} = -K_k \underline{x}_k, \tag{31}$$

where K_k is a gain vector which equals to:

$$K_k = (R_{LQR} + B_d^T P_k B_d)^{-1} B^T P_k A_d. \tag{32}$$

The P_k is only unknown value which is the solution of an iterative backward discrete time Riccati equation ((Yishao, 1996)):

$$P_{k-1} = Q_{LQR} + A_d^T (P_k - P_k B_d (R + B_d^T P_k B_d)^{-1} B_d^T P_k) A_d, \tag{33}$$

with initial value $P_n = Q_{LQR}$. Sometimes it is called ILQR (Iterative Linear Quadratic Regulator) like in (In-Won et al., 2010) and (Li and Todorov, 2004).

6 RESULTS

In order to check every mentioned algorithms a prototype of an IWP was built. Using the computer simulation acceptable mechanical parameters were found. As a measurement unit, inertial sensor ADIS16355 from Analog Devices Company was used. It has high precision tri-axis gyroscope and accelerometer. The second important thing was the implemented – high torque, DC current motor. It was direct drive for the flywheel. Direct drive generates much less vibrations than any mechanical transmissions. The last essential thing was well-balanced flywheel. Every calculations were made by using a personal computer.

Sensor fusion was the first thing, that authors had to prepare and then check. In Fig. 4 comparison of results of the Kalman Filter fusion is shown.

As it can be seen, accelerometer generates signal with additional noise which is (i.a.) dynamic acceleration. In normal operation, it is impossible to estimate declination from the pivot point by using only accelerometer. By using fusion with gyroscope good estimation can be obtained. In this example, authors used proper settings for the fusion algorithm which is: small variance (Q=0.0001) for gyroscope signal and large variance (R=0.1) for accelerometer signal.

Right after fusion, algorithm of Extended Kalman Filter performs his action. One of the estimates is shown in Fig. 5. It is easy to see that Extended Kalman Filter gives the opportunity to make estimates robust for the noise, even from optical encoder. The

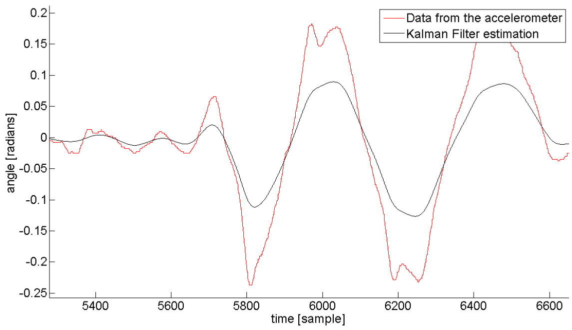

Figure 4: Measurement from accelerometer (red line) and fusion result (black line) - comparison.

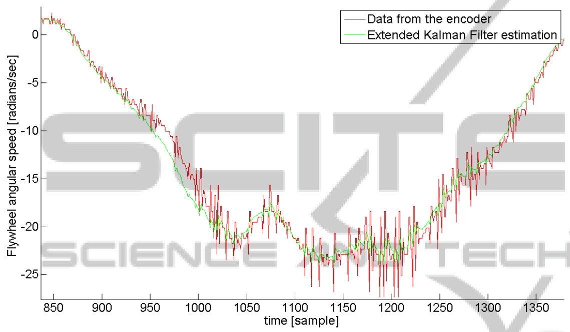

Figure 5: Measurement from the encoder (red line) and its Extended Kalman Filter estimation (green line) - comparison.

estimations from identification module are as important as those from Kalman filter. In Fig. 6 and 7 identification results are shown. First figure shows results for the vector (24) and the second one, shows results for (25) (with addional error estimation - red line).

As it can be seen, identification gives good results (blue line is the reference, theoretical value for each of parameter), but for the RLS algorithm, where there are more then two parameters for the identifiacation it is necessary to wait about 16 seconds (4000 samples with rate of 250 sample/sec).

Last thing was the regulator. The LQR regulation worked properly, because it was possible to stabilize the pendulum in vertical position. Every three state variables were close to zero value during the experiments. Trajectory of the first state variable is shown in Fig. 8.

Finally the pendulum was able to stand in the vertical position in infinite time horizon. The maximum amplitude of the first state variable was 2.5 degree. The angle of 9 degrees was the biggest possible external disturbance that has not led to the collapse.

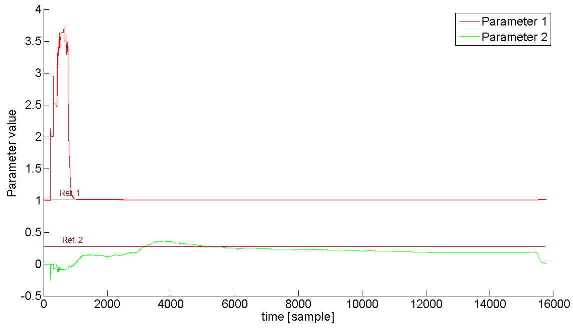

Figure 6: Identification results (1). Two parameters from Eq. (24) with their reference values.

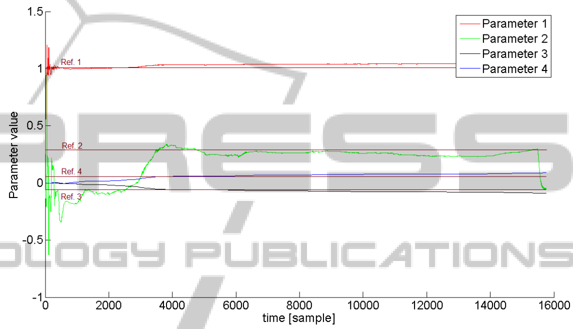

Figure 7: Identification results (2). Four parameters from Eq. (25) with their reference values.

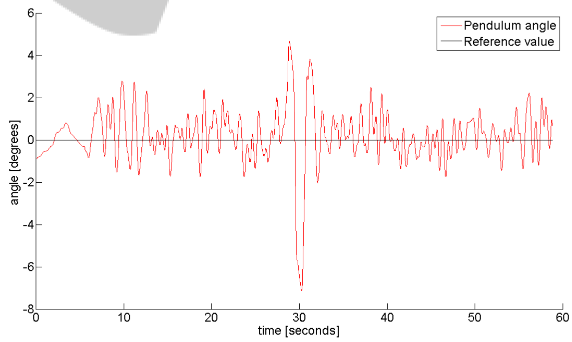

Figure 8: Experimental trajectory of the first state variable (angle from vertical position).

7 CONCLUSIONS

The experiment confirmed the theoretical considerations. It was possible to achieve satisfactory results through a combination of Kalman filter, identification and LQR regulation. Though, whole application working fine, results of algorithm of the parameters identification are not as good as they were expected to be. In this case, changes track of the parameters can lead to dangerous situations, such as sudden angle growth or decrease (LQR depends on identification process). Therefore it is recommended to take some precautions for example: software conditions that make the identificator supervised. However, the above solution shows that the creation of innovative electric vehicles with inertia stabilization is possible. In the future personal computer will be replaced by DSP microcontroller or FPGA device. This may help in creating a fully independent stabilization unit.

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