A COMPREHENSIVE AND COMPARATIVE SURVEY OF THE SIFT ALGORITHM

Feature Detection, Description, and Characterization

L. Younes, B. Romaniuk and E. Bittar

SIC - CReSTIC, University of Reims Champagne Ardenne, rue des Crayères, 51687 Reims Cedex 2, France

Keywords: SIFT, Computer Vision, Feature Points, Descriptors, Comparison.

Abstract: The SIFT feature extractor was introduced by Lowe in 1999. This algorithm provides invariant features and the corresponding local descriptors. The descriptors are then used in the image matching process. We propose an overview of this algorithm: the methodology and the tricky steps of its implementation, properties of the detector and descriptor. We analyze the structure of detected features. We finally compare our implementation to others, including Lowe's.

1 INTRODUCTION

Computer vision systems explore image content in search for distinctive invariant features, serving a wild variety of applications such as object detection and recognition, 3-D reconstruction or image stitching.

The feature points of an image are extracted in two steps. Feature points are first detected, then a descriptor is computed in their neighboring region in order to locally characterize them. Whereas many methods exist in the literature that enable the extraction of feature points of different types (Harris and Stephens, 1988; Smith and Brady, 1997; Bay et al., 2008; Ke and Sukthankar, 2004), we limit our study to the detectors based on a shape description of a point using size, thickness or principle directions to characterize wispy/misty forms. The most widely used algorithm in this context is the SIFT one (Scale Invariant Feature Transform) (Lowe, 2004). Lowe uses a difference of Gaussian function for the identification of extrema in an image pyramid constructed at different smoothing scales. A comparative study (Mikolajczyk and Schmid, 2005) shows that the choice of the detector depends on the image type and so of the application field. The algorithm of Lowe is nowadays widely recommended in the literature (Juan and Gwun, 2009) for its repeatability and robustness.

In this paper, we propose a SIFT algorithm summary specifying the tricky steps of its implementation. We then present an analysis of the structure and localization of the selected feature points. We end up with a comparison of the results of our implementation to others including Lowe's.

2 SIFT FEATURES: DETECTION and DESCRIPTION

The main steps of the SIFT algorithm are the followings:

- 1. Extrema Detection. Interest points are computed as local extrema along a scale space pyramid. They satisfy the property of invariance to scale and rotation. Once localized, their coordinates and the scale factor at which they were detected are stored.
- Keypoints Selection. For every candidate point a complementary process is executed in order to achieve a more accurate localization of the point. The localized points are then filtered in order to reject low contrasted points and the ones on low curvatures edges.
- 3. **Orientation Assignment.** For rotation invariance purposes, each keypoint is associated with an orientation, which corresponds to the direction of the most significant gradient of the neighborhood. Hence, additional keypoints may be generated if several significant gradients arise.
- 4. **Descriptor Computation.** For every interest point, a numerical descriptor is computed from

Younes L., Romaniuk B. and Bittar E.

A COMPREHENSIVE AND COMPARATIVE SURVEY OF THE SIFT ALGORITHM - Feature Detection, Description, and Characterization. DOI: 10.5220/0003864604670474

In Proceedings of the International Conference on Computer Vision Theory and Applications (VISAPP-2012), pages 467-474 ISBN: 978-989-8565-03-7

the gradient vectors of its neighborhood. The descriptor is conceived to be robust to affine transformation and illumination changes.

2.1 Extrema Detection

For the aim of the detection of candidate keypoints that are invariant to scale, a pyramid of images at different resolutions, denoted octaves, is constructed from the initial image (Figure 1).

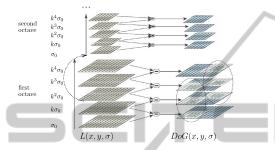


Figure 1: Two octaves of the image pyramid, each made of five images *i* of scale factor $k^i \sigma_0$.

Gaussian smoothing, at uniformly increasing scale factors ($\sigma_0, k\sigma_0, k^2\sigma_0, ...$) are then applied to each octave. Lowe recommends a three octaves pyramid, each octave constituted of five smoothed scale factors images. Every smoothed image $L(x, y, \sigma)$ of its octave is computed as the result of the Gaussian convolution at scale factor σ of the image I(x, y):

$$L(x, y, \sigma) = G(x, y, \sigma) * I(x, y), \tag{1}$$

where * represents the convolution operator.

According to Lowe, the first image of each octave must be smoothed at a scale factor $\sigma_0 = 1.6$. In the following we explain how this implementation can be done. The initial image has an intrinsic smoothing estimated empirically to $\sigma_{init} = 0.5$. The first image of the first octave is created by doubling the size of the initial image. Its smoothing factor is thus estimated to $\sigma_{init} * 2 = 1$. The scale factor to be applied to the image in order to achieve the desired scale σ_0 is computed according to the formula which indicates the smoothing scale σ_{final} obtained after applying two successive smoothings at scales σ_1 and σ_2 :

$$\sigma_{final} = \sqrt{\sigma_1^2 + \sigma_2^2}.$$
 (2)

In our case, $\sigma_{final} = \sigma_0$ is required, with $\sigma_1 = 1$. It is then necessary to smooth the image at a scale $\sigma_2 = 1.26$. Similarly, this formula serves for the computation of the smoothing factors to apply consecutively to the image in order to build the octaves. Between two consecutive octaves the image is rescaled to its half. As the first image of octave n + 1 should be at scale factor σ_0 , the image at scale factor $k^2 \sigma_0$ of the octave *n* is chosen to be rescaled. Though, with $k = \sqrt{2}$, $k^2 \sigma_0 = 2\sigma_0$, this image will lead to one at a scale σ_0 , thereby simplifying the process.

Afterwards, extrema have to be detected. For this, Lowe uses an approximation of the Laplacian of the image with a finite difference of Gaussian DoG, which is of low computation time. The pyramid of DoG is derived from the image pyramid of the smoothed images $L(x, y, k^n \sigma)$. Every DoG is the result of the subtraction of two Gaussian smoothed images at successive scales $(k^n \sigma_0 \text{ and } k^{n+1} \sigma_0)$. It can be defined as:

$$D(x, y, \sigma) = L(x, y, k\sigma) - L(x, y, \sigma).$$
(3)

The *DoG* pyramid is used for the identification of the candidate keypoints. They correspond to extrema identified over three consecutive *DoG* images within one octave. An extremum is a maximum or minimum in its 26-neighborhood: the intensity of the point in the *DoG* image is compared to its 8-neighbors, then to 9-neighbors at superior and inferior scales. The value of $k = \sqrt{2}$ has lead, according to (Lowe, 2004), to a robustness in the detection and localization of the extrema, even in the presence of significant differences in resolution.

2.2 Keypoints Selection

The second step consists in accurately localizing the points, and proceeding to the rejection of low contrasted points and the ones on edges of small curvature. This step leads to the selection of stable and well localized keypoints.

A sub-pixel precision is sought for the localization of the keypoints, to enhance the matching quality between keypoints. Given a keypoint $C(x_C, y_C, \sigma_C)$, we define $\mathbf{x} = (x, y, \sigma)^T$ as the offset from *C*. A second order Taylor approximation is used to compute the local extremum of the function D(x) in the neighborhood of *C*. Let *D* be the value of $D(\mathbf{C})$.

$$D(\mathbf{x}) = D + \frac{\partial D^T}{\partial \mathbf{x}} \mathbf{x} + \frac{1}{2} \mathbf{x}^T \frac{\partial^2 D}{\partial \mathbf{x}^2} \mathbf{x}$$
(4)

The local extremum is the point $\hat{\mathbf{x}}$ for which the derivative of the function D(x) is equal to zero:

$$\hat{\mathbf{x}} = (\hat{x}, \hat{y}, \hat{\mathbf{\sigma}})^T = -\frac{\partial^2 D^{-1}}{\partial \mathbf{x}^2} \frac{\partial D}{\partial \mathbf{x}}$$
(5)

Let $(D_x, D_y \text{ and } D_{\sigma})$ be the first derivatives and $(D_{\alpha\beta}, (\alpha, \beta) \in \{x, y, \sigma\}^2)$ be the second derivatives of D with respect to x, y and σ , $\hat{\mathbf{x}}$ is the solution of the following equation system:

$$\begin{pmatrix} D_{xx} & D_{xy} & D_{x\sigma} \\ D_{xy} & D_{yy} & D_{y\sigma} \\ D_{x\sigma} & D_{y\sigma} & D_{\sigma\sigma} \end{pmatrix} \begin{pmatrix} x \\ y \\ \sigma \end{pmatrix} = \begin{pmatrix} -D_x \\ -D_y \\ -D_{\sigma} \end{pmatrix}$$
(6)

The derivatives are approximated by the differences of Gaussian in the 4-neighborhood of the keypoint. When the value of one component of \mathbf{x} is greater than 0.5 in one of the 3 dimensions, it means that the extremum is closer to a neighbor of \mathbf{C} than to \mathbf{C} itself. In this case \mathbf{C} is changed to this new point and the computation is performed again from its coordinates. By at most five iterations of the process, the obtained value of the offset will be considered as the most accurate localization of the candidate keypoint. It is then possible to compute the value of *DoG* at the extremum $\hat{\mathbf{x}}$.

In order to reject the low contrasted points, the point is rejected if the value of $|D(\hat{\mathbf{x}})|$ is less than a threshold equal to 0.03. For all the process the pixel values are normalized to the range [0...1].

In the following step, the candidate keypoints localized on edges of small curvature will be rejected as their localization along the edge is difficult to estimate precisely. Hence, solely corners and points on highly curved edges, like corners, will be kept as interest points. In this goal, the principle curvatures are estimated. These are proportional to the eigenvalues α and β of the Hessian matrix.

Let *r* be the ratio between the greatest and lowest eigenvalues $\alpha = r\beta$. The use of the determinant and trace values of the Hessian matrix avoids the explicit computation of the eigenvalues. Lowe suggests the rejection of candidate keypoints having

$$\frac{Tr(H)^2}{Det(H)} \ge \frac{(r_s+1)^2}{r_s}$$
 where $r_s = 10.$ (7)

2.3 Orientation Assignment

The detected keypoints are characterized by their coordinates and the scale under which they were extracted. It is necessary to assign a consistent orientation to each detected point to obtain an invariance to rotation. For each keypoint (x, y), the closest scale factor (σ) is chosen and the associated image $L(x, y, \sigma)$ at this scale is used for the computation of the magnitude m(x, y) and the orientation $\theta(x, y)$ of the gradient: An histogram of orientations is established from the image L, computed over a window centered at (x, y, σ) , of diameter $c * \sigma$, where c is a constant. Each point of this windows contributes to the histogram bin corresponding to the orientation of its gradient, by adding a value of wm(x, y). This quantity is the magnitude of the gradient weighted by a Gaussian function of the distance to the keypoint, of standard deviation one and half the scale factor of the keypoint:

$$wm(x,y) = m(x,y) \frac{1}{2\pi(1.5\sigma)^2} e^{-\frac{dx^2+dy^2}{2(1.5\sigma)^2}}$$
 (8)

m(x, y) is the magnitude of the gradient at the location (x, y), dx and dy are the distances in x and y directions to the keypoint, and σ is the scale factor of this latter. The histogram of orientations is subdivided into 36 bins, each covering an interval of 10 degrees. The bin of maximal value characterizes the main orientation of the interest point. If other bins have a value greater than 80% of the maximal value, new interest points are created and are associated with these orientations. The value of the main orientation is refined from the peak bin of the histogram by detecting the maximum of a parabola which fits the main orientation and its adjacent bins. This maximum is evaluated as the angle for which the value of the derivative of the parabola is zero. The histogram is used in a circular order such as the successor of the last orientation is the first one. The keypoints are represented by four values, (x, y, σ, θ) , which denote respectively the position, the scale and the orientation of the keypoint, granting its invariance to these parameters.

2.4 Descriptor Computation

The computation of a numerical descriptor for every keypoint is the ultimate step of the SIFT algorithm. A descriptor is a vector elaborated from the magnitudes and orientations of the gradients in the neighborhood of the point. It is computed from the image $L(x, y, \sigma)$ at the scale factor at which the point was detected. As in section 2.3 the gradients magnitudes in the studied region are weighted (equation 8) by a Gaussian function of standard deviation 1.5 s. This gives less emphasize to gradients far from the keypoint and hence yields to a certain tolerance to small shifts in the window position. To grant invariance to rotation, all the gradient orientations inside the descriptor window are rotated relatively to the dominant orientation. Practically, the keypoint orientation is subtracted from every gradient orientation to reach this result. Furthermore the descriptor window is rotated in the direction of the keypoint orientation (Figure 2). This window has the same size as in section 2.3.

The descriptor window is then subdivided into 16 regions (Figure 2), and an eight bin histogram of orientations is computed for each. Each point contributes to the bin of the histogram corresponding to the orientation of its gradient. Its contribution is the product of its weighted magnitude $wm(x, y, \sigma)$ (equation 8) multiplied by an additional coefficient (1 - d),

IN

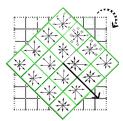


Figure 2: The descriptor computed in the neighborhood of the interest point is a concatenation of 16 8-D histograms.

where *d* is the distance of the gradient orientation to the central orientation of the histogram bin. The values of the 8-D histograms of the 16 regions are packed in a predefined order in an 4x4x8 = 128 dimensional vector leading to a unique identification of the feature point. The descriptor is normalized to ensure invariance to illumination changes. Large gradients, i.e. greater than an empirical threshold of 0.2, are reset to this value and the normalization is done again.

3 VISUAL ANALYSIS

The aim of this section is to understand the properties of the feature points retained by the SIFT algorithm. For this, we will focus on their position and the scale on which they were detected. Lowe¹ proposed an executable program achieving the detection of the feature points. We used this code to generate the results presented in this section. We will analyze the results on three different nature images: a synthetic image containing simple geometric objects, an image characterized by a repetitive content and finally a natural image. For the upcoming examples presented in this paper, the origin of an arrow corresponds to the localization of a detected feature point. If many arrows have the same origin, this illustrates the case when the histogram of orientations presents many peaks. The arrows orientation correspond to the most significant gradients orientations in the neighborhood of the feature point and their magnitudes reflect the scale at which this point was detected.

In the synthetic image (Figure 3) containing simple geometric objects, we obviously notice that feature points are not exactly located over the edges and that these are detected at different scales, *i.e* the length of the arrows differ relative to the points. At lower scales, only keypoints of high curvature and contrast are detected. In this image keypoints are situated in the vicinity of the corners of the pentagon. Whereas these feature points are detected at different scales, they are not exactly localized the same. The greater

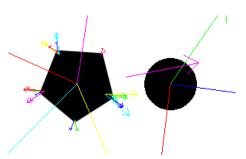


Figure 3: Detected feature points in a synthetic image containing simple geometric objects.

the scale of detection is, the farther the location of the detected feature point is from the corner of the pentagon. This is due to the smoothing of the image that makes the edges diffuse in the differences of Gaussian images. The main orientations associated to these points are similar according to the different scales. Deprived of high curvatures, the edges of the pentagon do not hold any feature points at lower scales. It's also the case of the disk. Furthermore, we assume that at a high scale, the center of the geometric shape characterizes it. In this case, the main orientations are not linearly distributed over the spatial plan. This nonlinearity is due to the fact that at high scales images are strongly smoothed and figures merge partially in the image modifying its global spatial organization. We also notice that a feature point was detected on high scales between the two geometric objects. This point hold on the information of the spatial organization of the scene.

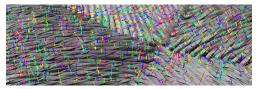


Figure 4: Detected feature points on an image characterized by a repetitive content.

Figure 4 presents a globally contrasted image with a repetitive content corresponding to straw. We notice that most of the keypoints are detected in highly contrasted region. Their number decreases in low contrasted bottom right region. We observe that the scale factor of the detected features is proportional to the width of each straw. The main orientation associated to each feature point is the direction of significant gradients: the orientation is perpendicular to the edge of a straw.

Figure 5 corresponds to a region of interest extracted from an old postcard of the Reims Circus $(France)^2$. On the left we show the original image

¹http://www.cs.ubc.ca/~lowe/keypoints/

²http://amicarte51.free.fr/reims/carte.php3?itempoint=8

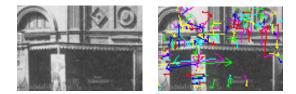


Figure 5: Detected feature points on an image of an old postcard of the Reims Circus (France).

and on the right the different feature points detected on with the SIFT algorithm. We mention one more time many detections at different scales and principally in the highly contrasted zones. Details in the images are represented at lower scales, *i.e* façade decorative architectural details of the circus. At higher scales feature points illustrate the spatial organization of the coarse elements present in the image (openings, windows....)

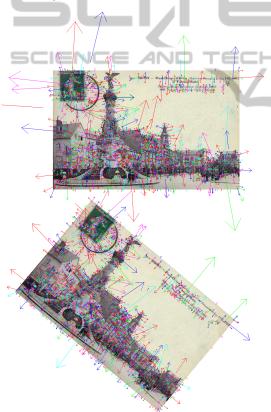


Figure 6: Detected feature points on an image of an old postcard of the Reims Erlon Place (France). On the top the detected feature points on the original image, on the bottom the detected feature points on a rotated image.

Figure 6 presents the features detection obtained on an Reims Erlon Place old postcard³. This figure illustrates the features detected on a correctly oriented image and the ones obtained on a rotated image. We can observe here that the detected features are similar in the two images even if some features detected near the borders are not the same. This is due to the fact both of the images contain white borders allowing the rotation (the image submitted to the algorithm must be rectangular).

4 COMPARISON

It exists many implementations of the SIFT algorithm, we selected here two of them⁴. A *Matlab* executable implementation is proposed by Lowe on his website (*cf.* part 3.) Another implementation computed in C++ using the *OpenCV* library for image processing is provided by Hess⁵. As we wanted to control the parameterization of the algorithm, we also computed our own implementation in C++ using *OpenCV*. Our implementation results in this section respect the parameters suggested by Lowe in 2004.

In this section we will discuss the parameters suggested by Lowe in 2004 analyzing the results we obtain with his executable implementation. We will compare his results to those obtained with the Hess implementation and to ours.

4.1 Parametrization of the Algorithm

In section 2 we have described the SIFT algorithm and the parameters that Lowe recommended after some empiric tests. In (Lowe, 2004) Lowe suggest to build a pyramid of three octaves, each of them composed of five smoothed images. The smoothing scale factors increase uniformly with $\sqrt{2}$ frequency between each image. In theory the highest scale factor that can be reached on the last image of an octave is $4\sigma_0 = 4 * 1.6 = 6.4$. Thereby, at same resolutions, the largest scale factor σ under which a feature point can be detected is 6.5 * 2 = 12.8 at the last image of the third octave. We analyze the features points extracted with the implementation of Lowe. We observe that this value is exceeded. We then conclude that the Lowe's implementation increases the number of octaves of the constructed pyramid or the number of images in each octave comparing to the values suggested in (Lowe, 2004).

[&]amp;ref=diversreims/0003.jpg

³http://amicarte51.free.fr/reims/carte.php3?itempoint=6

[&]amp;ref=erlon/0004.jpg

⁴A third interesting one is http://www.vlfeat.org/ overview/sift.html

⁵http://blogs.oregonstate.edu/hess/code/sift/

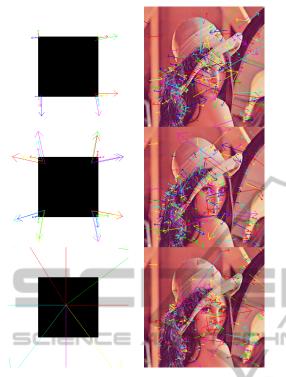


Figure 7: SIFT feature points detection obtained with three different implementations: our's (top), Lowe's (center) and Hess's(bottom.)

4.2 Feature Points Validation

In figure 7 we present feature point detection results on a synthetic image on the left and Lena image on the right. The first line of this figure corresponds to the results we obtained with our implementation of the algorithm, the middle line with the executable Lowe's implementation and finally the bottom line with the Hess implementation.

The synthetic image represents a black square over a white background. We observe that feature points associated to low scale factors, *i.e* points with high curvatures as the square corners, are not detected by the Hess implementation. Such points are normally detected in the first steps of the process. We can then conclude that the implementation of Hess do not consider low scale or that it rejects edge points of high curvature. We notice that our implementation detects features with low scale factors. However, Lowe detects more feature points. They correspond to higher scales (greater than 12.8). Similarly, the points detected by Hess represent really high scale features.

The results obtained with Lena's image shows that more feature points were detected in highly contrasted zones. We can notice that the different structures described in section 3 are characterized here. An obvious similarity of the detected features is denoted on the feather, the details of the hat, the highly curved architectural structures.... Once more, less feature points appears within Hess results. The main dissimilarity between our implementation and the two other implementations concerns the survey of the high scales. This study validates the feature points detection process for the three implementations.

A validation of the quality of the detected feature point descriptors is fundamental since these are crucial for the matching process.

4.3 Validation of the Descriptors

The robustness of the computed descriptors can be validated through the matching process. The method we use to match two different images is the one proposed by Lowe in (Lowe, 2004). For each feature point in the original image, the Euclidien distance is computed between the associated descriptor and all the feature points descriptors present in the second image. A distance ratio of the two closest neighbors is then compared to a threshold of 0.6. The closest descriptor is considered as a candidate match if the ratio do not exceed this threshold. To validate the robustness of the computed descriptors, we test the results of the matching between an image and its associated transformed image obtained by a known transformation. Thus, once a match is found, we compute the distance between the identified feature point and the real match obtained applying the transformation. If this distance is higher than 3 or 5 pixels, the candidate match is rejected.

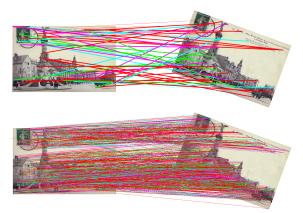


Figure 8: Matches identified without rotation of the descriptor (top) and with its rotation (bottom). The thick lines correspond to wrong matches while thin lines represent correct matches.

This approach proves the importance of the rotation of the descriptor window relatively to the dominant orientation of the feature point as described in the section 2.4. Figure 8 shows results of the matching process obtained for the rotated (20°) Reims Erlon Place postcard. Wrong matches are here represented by thick lines while thin lines correspond to correct matches considering a 5 pixels tolerance. The top part of the figure was obtained with a static descriptor window. In this case we notice that few matches were detected, and that most of them are wrong. The bottom part of the figure illustrates the results obtained with a mobile (rotary) descriptor window. In this case matches are numerous and they are roughly correct.

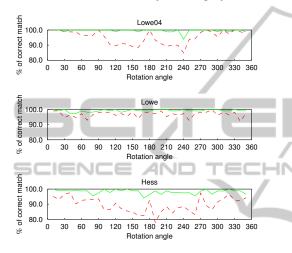


Figure 9: Percentage of correct matches depending on the rotation angle of the image. Results are obtained one the fragment of the Reims Circus old postcard using the three implementations: our's (Lowe 2004), Lowe's and Hess's. The dotted line was obtained with a maximal error tolerance of 3 pixels, the continuous one with a 5 pixels tolerance.

As (Morel and Yu, 2011) have proven the scaleinvariance of the SIFT method, we will focus our study on invariance to rotation. A series of tests is performed for the two tolerance levels (3 and 5 pixels). Images are successively rotated by an angle of 10° until reaching 350°. Figure 9 shows the variations of the percentage of correct matches relatively to the angle of the rotation, the two levels of distance error tolerance and using the three different implementations of the SIFT algorithm. These results are obtained for the fragment of the Reims Circus old postcard. The same tests computed for the Reims Erlon Place old postcard (presented in Figure 8) show a stable percentage of 96% for all the implementations and for both tolerance levels. In figure 9 Lowe detected 157 feature points on the original image and obtained a mean value of 110 matches overall the rotated images according to the two error tolerances. Hess detected 111 feature points on the original image and obtained a mean value of 84 matches. We detected 128 feature points on the original image and obtained a mean

value of 77 matches. In the image of the Reims Erlon Place, Lowe detected 1973 feature points on the original image and obtained a mean value of 1500 matches according to the 3 pixels error tolerance and 1513 according to the 5 pixels one. Hess detected 1463 feature points on the original image and obtained a mean value of 1004 matches according to the 3 pixels error tolerance and 1016 according to the 5 pixels one. We detected 1510 feature points on the original image and obtained a mean value of 937 matches according to the two error tolerances. We can notice that on these two figures that the results are obviously very performing when the error tolerance is of 5 pixels for both images and the three implementations. When the error tolerance is of 3 pixels, the results are preforming and comparable for the Reims Erlon Place old postcard. We notice a lower performance for the fragment of the Reims Circus old postcard. Even if Hess found more matches in this case than us, his implementation presents the worst results cause he detected more wrong matches, the Lowe's one is the best even if for some angles our implementation parametrized according to Lowe 2004 shows better matching percentage.

	Reims Circus		Erlon Place	
	3 px	5 px	3 px	5 px
Lowe04	95.05	99.56	97.64	99.41
Lowe	96.87	99.26	99.41	99.96
Hess	89.65	98.23	97.86	99.23

Figure 10: Evaluation of the matching process on the Reims Circus postcard extract and the Reims Erlon Place postcard for 3 and 5 pixels tolerance levels.

Figure 10 presents the means percentage of correct matches for all the angles of rotation for both levels of tolerance computed for the fragment of the Reims Circus old postcard and the Reims Erlon Place old postcard. The three aforementioned implementations where tested: our (Lowe04) implementation, Lowe's executable *Matlab* implementation and Hess's implementation. We notice performing results for all the implementation with an advantage for the Lowe's implementation. Our results are situated between the Hess's and Lowe's while being dependent of the test image.

5 CONCLUSIONS

In this paper, we presented a synthetic view of the SIFT algorithm insisting on the tricky steps of its implementation. We analyzed and discussed the properties of the detected feature points and their corresponding descriptors. We presented a survey of three implementations of the algorithm: Lowe's, Hess's and our's implementation respecting the parametrization suggested by Lowe in 2004. We studied the differences between these implementation and conclude on the robustness of all of them with an advantage for the Lowe's implementation. Our implementation will be used in the context of the spatio temporal 3-D reconstruction of the city of Reims using old postcards as data. The challenge is to take into account the architectural evolution across the years in order to match the different views of the buildings.

REFERENCES

- Bay, H., Ess, A., Tuytelaars, T., and Van Gool, L. (2008). Speeded-up robust features (surf). *Computer Vision* and Image Understanding, 110(3):346–359.
- Harris, C. and Stephens, M. (1988). A combined corner and edge detector. In *Alvey Vision Conf.*, pages 147–151. Manchester, UK.
- Juan, L. and Gwun, O. (2009). A comparison of sift, pca-sift and surf. *Int. Journal of Image Processing*, *IJIP'09*, 3(4):187–152.

y public

- Ke, Y. and Sukthankar, R. (2004). Pca-sift: A more distinctive representation for local image descriptors. Computer Society Conf. on Computer Vision and Pattern Recognition, CVPR'04, 2:506–513.
- Lowe, D. (2004). Distinctive image features from scaleinvariant keypoints. *Int. Journal of Computer Vision*, 60(2):91–110.
- Mikolajczyk, K. and Schmid, C. (2005). A performance evaluation of local descriptors. *IEEE trans. on Pattern Analysis and Machine Intelligence*, 27(10):1615– 1630.
- Morel, J. and Yu, G. (2011). Is sift scale invariant? *Inverse Problems and Imaging*, 5(1):115–136.
- Smith, S. and Brady, J. (1997). Susan a new approach to low level image processing. *Int. Journal of Computer Vision*, 23(1):45–78.