

STUDY ON THE LONG-TERM INCENTIVE MECHANISM OF THE LARGE-SCALE DREDGING PROJECT

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Abstract: Through the principal-agent theory and game theory, this article has established the long-term income model of the large-scale dredging project, which has obtained the solution of the long-term incentive model, analyzed the impact of the dynamic consistency as well as pledge and negotiating cost to incomes of the principal and agent by means of increasing different constraint conditions. Furthermore, the study also shows that the long-term incentive model can provide the agent with stronger incentive.

1 INTRODUCTION

The incentive is essential for the management, and the principal-agent theory is widely applied for the analysis on the incentive problem. And, the principal-agent theory deems the management problem as the fact that how the principal designs the incentive mechanism to seduce the agent to take behaviors optimum to the principal from his own interests. In 1981, Lazear put forward the game theory, and he thought that the large salary gap can reduce the monitoring cost, seduce efforts of the agent, and highly motivate the consistent interests of the principal and the agent. (Lazear E, Rosen S., 1981). Furthermore, Holmstrom & Milgrom put forward the output share incentive mechanism that the purely selfish and risk-neutral principal shall employ the agent with the jealousy and pride preference and risk avoidance. (Holmstrom B, Milgrom P., 1987). And, Aoki pointed out that, the long-term employment can motivate the agent to accumulate human capitals special for the enterprise. (Aoki, M.,1988). Also, Yong Zhang established the two-stage model for the manager, that is the long-term and short-term income incentives as well as obtained solutions and analyzed relevant conclusions. (Yong Zhang. 2004). The study of Debing Ni revealed that the optimal sharing proportion will increase along with the increased effort cost of the agent while reduce along with the increased expected growth rate of the market price and effort output. (Debing Ni and Xiaowo Tang.,

2005). Besides, Zongjun Wang obtained the optimal income combination of the manager through the long-term and short-term incentive models. (Zongjun Wang, Chongshuai Qian, and Tian Xia, 2008). Lijun Li studied the problem of how to motivate the producer to reduce costs as well as pointed out the cost difference between the asymmetric information and the symmetric information, namely the incentive cost; only if the agent shares the cost-saving income, can the principal realize his expected income; in the premise of incomplete and asymmetric information of the dredging project. (Lijun Li, Xiaoyuan Huang, etc., 2003). Bin Zhou put forward the relationship between the agent's incentive coefficient and the project cost; that is, the higher the agent's incentive coefficient, the lower the project cost; in addition to the above, it also established the incentive mechanism based on the equity preference. (Bin Zhou, Zigang Zhang, 2010).

Above scholars have studied the income of the manager from different views; however the actual income of the agent will be restricted by numerous kinds of factors. This article, from the long-term incentive view, has corrected relevant study assumptions, increased the long-term incentive constraint and the agent's capability constraint, and compared the impact of the long-term and short-term incentives to the agent's income to make the model be close to actual conditions and obtain corresponding conclusions.

2 MODELING

In order to facilitate the study, following assumptions are introduced:

Assumption 1: Both the principal and the agent pursue maximizing own benefits.

Assumption 2: In a certain period, the agent's current income will be related to that of previous period, and his ability will be enhanced along with the increased service year.

Assumption 3: The agent's capability can be applied to pursue the short-term income and long-term income, and the required negotiating cost for signing one contract is r .

Supposing the payment made by the principal to the agent is $y_i = k_N^i v_N^i$, $i = 1, 2, \dots, N$; i refers to the stage i project, and the total of the stage i project is Q_i cube (earthwork); the bid award price of the unilateral dredged soil of the stage i project is x_i yuan/cube; the project agent's oil consumption and material consumption of the stage i are t_i yuan/cube, with the fixed cost of c_{0i} yuan/hour; the income of the stage i project is v_N^i yuan/cube; the principal's income of the stage i project is v_{N1}^i yuan/cube; the payment made by the principal to the agent of the stage i project is y_i yuan/cube. When the agent's effort cost is $c_i(a)$ yuan/hour, c_{ai} refers to the agent's payroll in the competitive market; the agent's income of the stage i project is v_{N2}^i yuan/cube; when the maximum rated hourly output of the engineering ship applied by the agent is p_m cube/hour, the output when the project income is zero; namely the critical output is p_0 cube/hour.

According to above assumptions, the model to maximize the principal's income is as follows:

$$\text{Max} \sum_1^N v_{N1}^i = \sum_1^N [v_N^i - y(v_N^i)] \quad (1)$$

$$\sum_1^N v_N^i = \sum_1^N [p(x_i - t_i) - c_{0i}] \geq 0 \quad (2)$$

$$\sum_1^N v_{N2}^i = \sum_1^N [y(v_N^i) - c_i(a)] \geq 0 \quad (3)$$

$$\sum_1^N k_1^i \leq \frac{N}{2}, k_N^{i+1} \geq k_N^i, (p = p_m) \quad (4)$$

$$\sum_1^N v_{N2}^i = \sum_1^N [y(v_N^i) - c_i(a)] \geq Ny_d$$

$$y_d = \frac{1}{2} - \frac{(\frac{1}{2} - \lambda^d)(p_m - p_d)}{p - p_0}, \lambda^d = 0.48 \quad (5)$$

$$i = 1, 2, \dots, N$$

In the model, the objective function (1) formula means the principal's utility function of the stage N project; constraint (2) means the total utility function of the stage N project; constraint (3) means the agent's utility function of the stage N project; when the agent pursues maximize his own utility, the formula is objective function, and the formula (1) which is larger than or equals to 0 is the constraint condition; the (4) formula is the rigid constraint condition, when the agent enhances the hourly output to the rated output, the game pricing incentive coefficient is 1/2, and the incentive coefficient of the stage N project will not be larger than the sum of N short-term game incentive coefficients; or else, the principal is inclined to sign the short-term contract. According to the assumption 2, when the agent is engaged in some work for a long time, his capability will be improved gradually; therefore, his income shall be increased correspondingly; the constraint (5) expresses that, the income of the long-term contract signed by the agent shall be higher than that of the short-term contract; y_d refers to the income of the short-term game pricing formula.

3 MODEL SOLUTION AND ITS ANALYSIS

3.1 Solution to Maximize the Principal's Income in Stage N

Firstly, give up the constraint (4) and the constraint (5), and directly solve the problem of maximizing the principal's income in the stage N

Establish Lagrange function and obtain the agent's reaction function in the stage N:

Derive the $k_N^1, k_N^2, \dots, k_N^N, \lambda$ in turn and eliminate λ , including

$$k_N^i = \frac{v_N^{i-1}}{v_N^i} k_N^{i-1} \quad i = 2, 3, \dots, N \quad (6)$$

The principal knows the agent's reaction function in the stage N; then, the principal's optimal incentive coefficient in the stage I is as follows

$$\sum_1^N v_{1i} = (1 - k_N^1)v_N^1 + \dots + (1 - k_N^N)v_N^N + c_{a1} + \dots + c_{aN} \quad (7)$$

Substitute the (7) formula by (6); make the first derivation to k_N^1 , and make the derivative as 0, including

$$k_N^1 = \frac{1}{N} - \frac{c_N}{p - p_0} \quad (8)$$

As $\frac{\partial k_N^1}{\partial p} > 0$, $\frac{\partial^2 k_N^1}{\partial p^2} < 0$, when $p = p_m$, make $k_N^1 = \omega$; that is, when the agent increases the output to the rated one of the ship, the corresponding incentive coefficient will reach the maximum value ω , including:

$$k_N^1 = \frac{1}{N} - \frac{(\frac{1}{N} - \omega)(p_m - p_0)}{(p - p_0)} \quad (9)$$

Viewing from the (9) formula, the ω value shall be larger than 0, which shall be related to the validity of the contract signed between the principal and the agent.

$$\text{Making } \omega = \frac{1}{N+1} \quad (10)$$

$$k_N^1 = \frac{1}{N} - \frac{(p_m - p_0)}{N(N+1)(p - p_0)} \quad p > p_0 \quad (11)$$

Such formula meets the monotonic increasing requirements of the incentive coefficient k to the hourly output p as well as demands that the second derivative shall be smaller than 0. As $\frac{\partial k_N^1}{\partial N} = -\frac{1}{(N+1)^2} < 0$ fails to meet conditions of the assumption (2), this formula shall be converted. Supposing that the agent increases the hourly output to p_m in whole N cooperation periods of the principal and the agent, the agent's optimal incentive coefficient in the stage I is $k_N^1 = \frac{1}{N+1}$. In addition to the above, with the increased cooperation period, the

incentive coefficient will increase till $k_N^N = \frac{1}{2}$ in the last period. During N stages of the contract, the agent's incentive coefficient will be increased by $\frac{1}{N}$ for each increasing cooperation. Therefore, the agent's optimal incentive coefficient shall be as follows:

Table 1: Calculation Sheet of the Principal's and Agent's Income when $p=p_m$.

i	1	2	3	...	N
k_N^i	$\frac{1}{N+1}$	$\frac{1}{N}$	$\frac{1}{N-1}$...	$\frac{1}{2}$

According to above reasonings, the (11) formula can be rewritten as

$$k_N^i = \frac{1}{N+1-i} - \frac{p_m - p_0}{(N+1-i)(N+2-i)(p - p_0)} \quad (12)$$

3.2 Solution to Maximize the Agent's Income in Stage N

Firstly, give up the constraint (4) and the constraint (5), and directly solve the problem of maximizing the agent's income in the stage N. here, the agent's income is the objective function, and the principal's income is larger than 0, which is the participation constraint.

Establish Lagrange function and obtain the principal's reaction function in the stage N, including:

Derive the $k_N^1, k_N^2, \dots, k_N^N, \lambda$ in turn and eliminate λ , including

$$k_N^i = 1 - \frac{v_N^{i-1}}{v_N^i} (1 - k_N^{i-1}) \quad i = 2, 3, \dots, N \quad (13)$$

The agent knows the principal's reaction function in the stage N; then, the agent's optimal incentive coefficient in the stage I is as follows

Substitute the (13) formula by (3); make the first derivation to k_N^1 , including:

$$k_N^1 = 1 - \frac{1}{N} - \frac{c_N}{N^2(p - p_0)} \quad (14)$$

As $\frac{\partial k_N^1}{\partial p} > 0$, $\frac{\partial^2 k_N^1}{\partial p^2} < 0$, when $p = p_m$, k_N^1 will be maximum; therefore, $c_N = p_m - p_0$; that is, when the

agent increases the output to the rated one of the ship, the corresponding incentive coefficient will reach the maximum value, including:

$$k_N^1 = 1 - \frac{1}{N} - \frac{p_m - p_0}{N^2(p - p_0)} \quad (15)$$

$\frac{\partial k_N^1}{\partial N} = \frac{1}{N^2} + \frac{2}{N^3} > 0$, which meets the constraint (2); that is, during the contract period, the agent's incentive coefficient will be increased along with the increased working time. Accordingly, the (15) formula can be rewritten as: $k_N^i = 0 \quad i = 1$

$$k_N^i = 1 - \frac{1}{i} - \frac{(p_m - p_0)}{i^2(p - p_0)} \quad i = 2, 3, \dots, N \quad (16)$$

3.3 Solution of the Long-term Incentive Model

When both the principal's income and the agent's income are optimal, the optimization solution can be obtained without the constraint (4) and the constraint (5). Compare the (12) and (16) formulas, the agent's incentive coefficient will meet $\frac{\partial k_N^i}{\partial i} > 0, \frac{\partial^2 k_N^i}{\partial i^2} < 0$; that

is, the agent's incentive coefficient will be increased along with the increased working time. The long-term cooperation will be good to the agent; however the principal's long-term optimization is not the same as the agent's long-term optimization; accordingly, the feasible solution of the long-term optimization is between the principal's long-term income optimization solution and the agent's long-term income optimization solution. Viewing from the figure 1, we find out that the feasible solution is located in the area between line I and line III. However, after increasing the constraint (4) and constraint (5), the principal's income optimization solution (12) formula and the agent's income optimization solution (16) formula can't meet the constraint (4) constraint (5); therefore, solutions meeting all constraint conditions of the long-term incentive shall be obtained. According to the constraint conditions (4), when $p = p_m$,

$$k_N^N > k_N^{N-1} > \dots > k_N^2 > k_N^1, \text{ supposing}$$

$$k_N^i = [1 + (i - 1)\phi]k_N^1 \quad i = 1, 2, \dots, N, \text{ then}$$

$$k_N^1 \leq \frac{1}{2[1 + \frac{N-1}{2}\phi]} \quad (17)$$

According to the agent's incentive coefficient solution when the principal income is optimal, when $p = p_m$:

$$k_N^1 = \frac{1}{N+1} \quad (18)$$

Making $\frac{1}{N+1} = \frac{1}{2[1 + \frac{N-1}{2}\phi]}$, $\phi = 1$; therefore,

when $p = p_m$,

$$k_N^i = \frac{i}{N+1}, \text{ then, when } p_0 \leq p \leq p_m,$$

$$k_N^i = \frac{i}{N} - \frac{i(p_m - p_0)}{N(N+1)(p - p_0)} \quad (19)$$

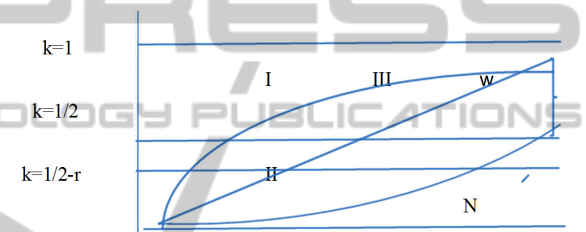


Figure 1: Comparison Chart of the Agent's Incentive Coefficient.

$k = 1$ —Total incentive coefficient of the project

$k = \frac{1}{2}$ —Agent's incentive coefficient line in the

dynamic game pricing formula ($p = p_m$)

$k = \frac{1}{2} - r$ —Agent's incentive coefficient after

deducting the negotiating cost in the dynamic game pricing formula ($p = p_m$)

I---Agent's incentive coefficient line when the long-term contract agent is optimal ($p = p_m$),

$$k_N^i = 1 - \frac{1}{i} - \frac{1}{i^2}, \quad i = 1, 2, \dots, N$$

II---Agent's long-term incentive coefficient line

$$(p = p_m), \quad k_N^i = \frac{i}{N+1}, \quad i = 1, 2, \dots, N$$

III---Agent's incentive coefficient line when the long-term contract principal is optimal ($p = p_m$),

$$k_N^i = \frac{1}{N+2-i}, \quad i = 1, 2, \dots, N$$

3.4 Comparative Analysis on Three Solutions

3.4.1 Long-term and Short-term Dynamic Consistency

Determine the agent's incentive coefficient based on the maximization of the principal's long-term income; namely the (12) formula.

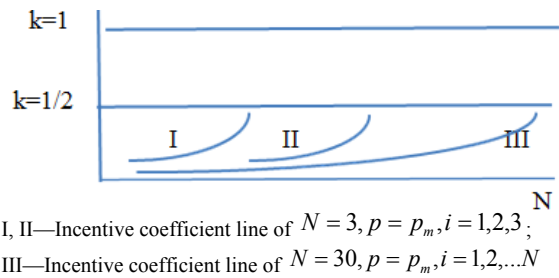
Without considering discounts, suppose that the agent tries to increase the output of the ship to the rated one during the cooperation period, the sum of incentive coefficients of three stages shall be $\sum_1^3 k_N^i = \frac{1}{4} + \frac{1}{3} + \frac{1}{2} = \frac{13}{12}$ and agent's total incentive coefficient sum during 30-year career shall be $\frac{130}{12}$.

However, if the agent and the principal sign the 30-year contract, the agent's 30-year incentive coefficient sum shall be $\sum_1^{30} k_N^i = \frac{1}{31} + \frac{1}{30} + \dots + \frac{1}{3} + \frac{1}{2} \approx 2.9$. Therefore, the longer contract period will be more beneficial to the principal; however, for the agent, the higher expectations for the future, the shorter-term contract will be.

Determine the agent's incentive coefficient based on the maximization of the agent's long-term income; namely the (16) formula or the comparison between 3-period and 30-period. Without considering discounts, the incentive coefficient sum of three-period contract is: $\sum_1^3 k_N^i = 0 + \frac{1}{4} + \frac{1}{9} = \frac{13}{36}$; that of ten 3-period contracts within 30 years is $\frac{130}{36}$, and that of 30-period is:

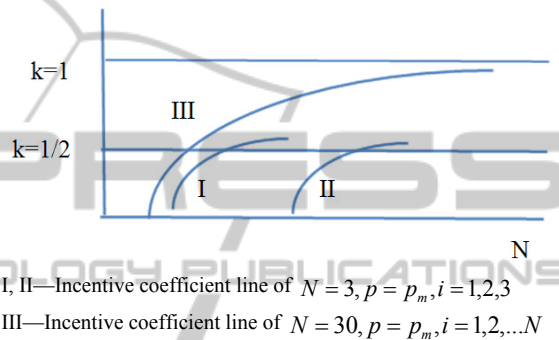
$$\sum_1^{30} k_N^i = 0 + \frac{1}{4} + \frac{5}{9} + \dots + \frac{811}{841} + \frac{869}{900} > \frac{30}{2}$$

Obviously, the long-term contract is more beneficial to the agent. Under such condition, the principal will choose to sign the contract with short cooperation period. Therefore, under these two conditions, the principal and the agent are in the bargaining game process; namely, the principal and agent are dynamically inconsistent in the long-term and short-term incomes, see figure 2 [A,B,C].



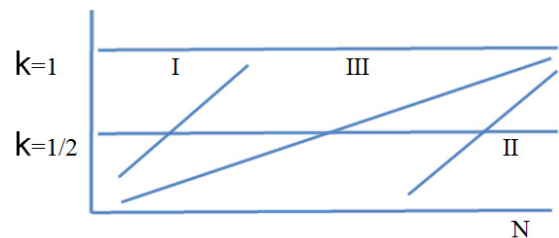
I, II—Incentive coefficient line of $N = 3, p = p_m, i = 1, 2, 3$;
 III—Incentive coefficient line of $N = 30, p = p_m, i = 1, 2, \dots, N$

Figure 2A: Comparison Between the Long-term and Short-term Incentive Coefficients in the Principal's Long-term Income Optimization.



I, II—Incentive coefficient line of $N = 3, p = p_m, i = 1, 2, 3$;
 III—Incentive coefficient line of $N = 30, p = p_m, i = 1, 2, \dots, N$

Figure 2B: Comparison Between the Long-term and Short-term Incentive Coefficients in the Agent's Long-term Income Optimization.



I, II—Incentive coefficient line of $N = 3, p = p_m, i = 1, 2, 3$;
 III—Incentive coefficient line of $N = 30, p = p_m, i = 1, 2, \dots, N$

Figure 2C: Comparison the Long-term and Short-term Incentive Coefficients in the Long-term Incentive Model.

Determine the agent's incentive coefficient based on the long-term incentive model solution; namely the (19) formula. No matter how long the contract period is, when the agent tries to increase it to the rated output p_m , it can certify that the incentive coefficient sum meets $\sum_1^N k_N^i = \frac{N}{2}$; namely the (19)

formula meets the dynamic consistency of the long-term and short-term incomes of the principal and the agent.

3.4.2 Comparison between the Long-term Income and the Short-term Income

As the long-term optimization solutions of the agent and the principal can't meet the dynamic consistency, it is not a kind of stable solution, which will change along with both parties' negotiating skills. When the principal takes the priority, he hopes to sign the long-term contract based on his long-term income optimization; however, the agent prefers to the dynamic game pricing. When the agent takes the priority, he hopes to sign the long-term contract based on his long-term income optimization; however, the principal prefers to the dynamic game pricing. Accordingly, the analysis on the dynamic game pricing and long-term incentive model solutions will be more significant. Taking the 3 stages as the example, the negotiating cost will not be considered and the agent's calculation data in the 3 stages are the same.

The income of 3 contracts signed by the agent according to the short-term game pricing coefficient is:

$$3v_2 = \int_{p_0 + \frac{1}{2}(p_m - p_0)}^{p_m} \left[\frac{3}{2} - \frac{(\frac{3}{2} - 3\lambda)(p_m - p_0)}{(p - p_0)} \right] [p(x-t) - c_0] dp$$

The agent signs a 3-period contract based on the long-term incentive model:

$$k_3^i = \frac{i}{3} - \frac{i(p_m - p_0)}{12(p - p_0)} \quad N = 3, i = 1, 2, 3$$

$$\sum_1^3 v_3^i = \int_{p_0 + \frac{1}{4}(p_m - p_0)}^{p_m} \left[2 - \frac{(p_m - p_0)}{2(p - p_0)} \right] [p(x-t) - c_0] dp$$

$$\sum_1^3 v_3^i - 3v_2 = \int_{p_0 + \frac{1}{2}(p_m - p_0)}^{p_m} \left[\frac{1}{2} - \frac{0.44(p_m - p_0)}{(p - p_0)} \right] [p(x-t) - c_0] dp$$

$$+ \int_{p_0 + \frac{1}{4}(p_m - p_0)}^{p_0 + \frac{1}{2}(p_m - p_0)} \left[2 - \frac{(p_m - p_0)}{2(p - p_0)} \right] [p(x-t) - c_0] dp > 0$$

3.4.3 Negotiating Cost r

Costs are required for facts that the principal searches for the agent and the agent signs the agreement with the principal; compared with the scale advantages of the principal, the proportion of the agent's negotiating cost of its own income will be higher. As for the agent, if there is no negotiating cost r , the incentive coefficient of N 1-period

contract is the same as the sum of the incentive coefficient of the N-period contract. When the negotiating cost $r > 0$, and $\sum_1^N k_N^i - r > (\frac{1}{2} - r)N$, the

income of the N-period contract will be higher than that of N 1-period contracts. As for the principal, though the negotiating cost proportion of the income is not high, the long-term contract will be more beneficial.

3.4.4 Pledged Capital w or "Hostage"

When calculating from the figure 1,

$$w = \int_1^{N/2} \left[\left(\frac{1}{2} - k_N^i \right) v_N^i - c_{ai} \right] dt = \int_{N/2}^N (k_N^i v_N^i - c_{ai}) dt$$

The income of the agent's first half of the career which is deducted by the principal has been compensated in his later half of the career, which can be deemed as the investment or savings made by the agent to the principal. Due to the pledged capital, the agent's and the principal's goals are further harmonized, and if the agent be fired because of effortless, or lead to the principals' fewer income; the agent will also have corresponding loss. And, the longer the worktime is, the larger the corresponding loss will be. Accordingly, the long-term contract incentive to the agent is larger.

4 CONCLUSIONS

From the standpoint of the long-term incentive, the article has studied the income of the agent of the large-scale dredging project, established the long-term incentive model, obtained the optimization solutions of the principal's long-term income and the agent's long-term income; in addition to the above, it has obtained the long-term incentive model solution through increasing various kinds of constraints (long-term incentive constraint, agent's capability constraint, and short-term income constraint). The main conclusions cover: I. As for the principal's long-term optimization solution and the agent's long-term optimization solution; while as the long-term incentive model solution can meet the dynamic consistency requirements, it is a kind of stable solution; II. In the long-term incentive model, the agent's long-term income is higher than the short-term income, which has provided the agent with stronger incentive; III. If consideration is not given to the negotiating cost, the sum of the coefficient of the short-term game price equals to

that of the long-term incentive model. If there is the negotiating cost, the coefficient sum of the long-term contract is larger than that of the short-term contract; accordingly, the long-term contract will be beneficial to the principal and the agent; IV. As the existence of the pledge, the objection of the agent and the principal will be more gradually-consistent. the agent's incentive shall be further strengthened; V. The long-term incentive coefficient is similar to the seniority pay while the seniority pay also has differences. The classification of the long-term incentive coefficient is the sharing proportion of the project income; thus the principal shall also assess the agent's hourly output. When the income of the project under the management of the agent is lower, the principal shall not increase the agent's incentive; namely, coefficient by years, that is, the interior promotion system shall be established. After increasing the constraint, the long-term incentive model will be more practicable and convenient operation and application. And, the disadvantage is that, as the article fails to give the agent capability classification incentive mode, which shall be further studied.

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