AN APPROXIMATED EXPRESSION FOR THE CONVERGENCE TIME OF ADAPTIVE BLIND EQUALIZERS

Monika Pinchas
Department of Electrical and Electronic Engineering, Ariel University Center, 40700 Ariel, Israel

Keywords: Blind deconvolution, Blind equalization, Acquisition time.

Abstract: In this paper, closed-form approximated expressions are proposed for the convergence time (or number of iterations required for convergence) and for the Intersymbol Interference (ISI) as a function of time valid during the stages of the iterative deconvolution process. The new derivations are valid for the noiseless, real and two independent quadrature carrier case and for type of blind equalizers where the error that is fed into the adaptive mechanism which updates the equalizer's taps can be expressed as a polynomial function of order three of the equalized output like in Godard's algorithm. Up to now, the equalizer's performance (convergence time and ISI as a function of time) could be obtained only via simulation when the channel coefficients were known. The new proposed expressions are based on the knowledge of the initial ISI and channel power (which is measurable) and eliminate the need to carry out any more the above mentioned simulation.

1 INTRODUCTION

It is well known that ISI (Intersymbol Interference) is a limiting factor in many communication environments where it causes an irreducible degradation of the bit error rate (BER) thus imposing an upper limit on the data symbol rate. In order to overcome the ISI problem, an equalizer is implemented in those systems.

The paper is organized as follows: After having described the system under consideration in Section 2, the closed-form approximated expression for the ISI as a function of time is introduced in Section 3. In Section 4 simulation results are presented and the conclusion is given in Section 5.

2 SYSTEM DESCRIPTION

The system under consideration is illustrated in Fig.1, where we make the following assumptions:

1. The input sequence $x[n]$ belongs to a real or two independent quadrature carrier case constellation input with variance σ_x^2 where $x_1[n]$ and $x_2[n]$ are the real and imaginary parts of $x[n]$ respectively.

2. The unknown channel $h[n]$ is a possibly nonminimum phase linear time-invariant filter in which the transfer function has no "deep zeros", namely, the zeros lie sufficiently far from the unit circle.

3. The equalizer $c[n]$ is a tap-delay line.

4. The noise $w[n]$ is an additive Gaussian white noise with zero mean and variance $\sigma_w^2 = E[w[n]w^*[n]]$ ($E[\cdot]$ is the expectation operator). The sequence $x[n]$

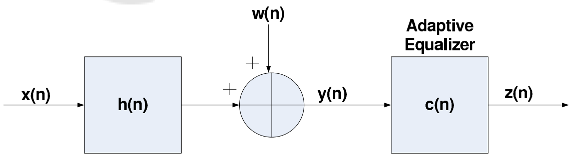

Figure 1: Block diagram of a baseband communication system.

is transmitted through the channel $h[n]$ and is corrupted with noise $w[n]$. The equalized output is defined as: $z[n] = x[n] + p[n] + \tilde{w}[n]$ where $p[n]$ is the convolutional noise, namely, the residual intersymbol interference (ISI) arising from the difference between the guess $c_g[n]$ and ideal value $c[n]$ and $\tilde{w}[n] = w[n] * c_g[n]$. The ISI is often used as a measure of performance in equalizers' applications, defined by: $ISI[n] = \frac{\sum_{\tilde{m}} |\tilde{s}[\tilde{m}]|^2 - |\tilde{s}|^2_{max}}{|\tilde{s}|^2_{max}}$ where $|\tilde{s}|_{max}$ is the component of $\tilde{s}$, given by $\tilde{s}[n] = c_g[n] * h[n]$, having the maximal absolute value. Next we turn to the adaptation mechanism of the equalizer which is based on a predefined cost function $F[n]$ that characterizes the intersymbol interference, see (Godard, 1980), (Pinchas, 2011) and (Shalvi and Weinstein, 1990). In this paper we assume that $\frac{\partial F[n]}{\partial z[n]}$ can be expressed as a polynomial function of order three of the equalized output

Pinchas M..
AN APPROXIMATED EXPRESSION FOR THE CONVERGENCE TIME OF ADAPTIVE BLIND EQUALIZERS.
DOI: 10.5220/0003529204110414
In *Proceedings of the 8th International Conference on Informatics in Control, Automation and Robotics* (ICINCO-2011), pages 411-414
ISBN: 978-989-8425-74-4

namely as $P(z[n])$. Thus we may write: $\underline{c}_{eq}[n+1] = \underline{c}_{eq}[n] - \mu\frac{\partial F[n]}{\partial z[n]}\underline{y}^*[n] = \underline{c}_{eq}[n] - \mu P(z[n])\underline{y}^*[n]$ where μ is the step-size parameter, $\underline{c}_{eq}[n]$ is the equalizer vector where the input vector is $\underline{y}[n] = [y[n]\ldots y[n-L+1]]^T$ and L is the equalizer's tap length. The operator $()^T$ denotes for transpose of the function (). The real part of $P(z[n])$ may be expressed as: $P_r(z[n]) = \Big(a_1(x_r + p_r[n]) + a_3(x_r + p_r[n])^3 + a_{12}(x_r + p_r[n])(x_i + p_i[n])^2\Big)$ where $x_r = x_1[n]$, $x_i = x_2[n]$, $p_r[n]$ and $p_i[n]$ are the real and imaginary parts of $p[n]$ respectively and a_1, a_{12}, a_3 are parameters of the chosen equalizer. In the latter stages where the blind equalizer has converged we may write that $E[p^2[n+1]] \cong E[p^2[n]]$. Since we deal with the real or two independent quadrature carrier case we may assume as was done in (Pinchas, 2009) that $E[p_r^2[n]] = E[p_i^2[n]]$. Thus, in the latter stages where the blind equalizer has converged we have $E[p_r^2[n+1]] \cong E[p_r^2[n]]$. Recently, an expression for $E[\Delta p_r^2] = E[p_r^2[n+1] - p_r^2[n]]$ was derived in (Pinchas, 2009):

$$E[\Delta p_r^2] \cong B(D_1 B m_p^3[n] + A_1 m_p^2[n] + B_1 m_p[n] + C_1 B) \tag{1}$$

where B_1, B, D_1, A_1 and C_1 are defined in (Pinchas, 2009). $E[p_r^2[n]] = m_p[n]$, $E[(x_1[n])^2] = \sigma_{x_r}^2$, $E[(x_2[n])^2] = \sigma_{x_i}^2$ and R is the channel length. With the help of $E[\Delta p_r^2]$ defined in (1), the ISI as a function of time can be obtained.

3 ISI AS A FUNCTION OF TIME

We start our derivations for the very low ISI case where the eye diagram is almost open or already open and then turn to the more general case where the initial ISI can have much higher values (where the eye diagram is very closed). Since we deal with the real and two independent quadrature carrier case, we start our derivations first with the real valued case and then turn to the two independent quadrature carrier case.

The following (additional) assumptions are made:

1. The convolutional noise $p[n]$, is a zero mean, white Gaussian process with variance $\sigma_p^2[n] = E[p[n]p^*[n]]$.

2. The source signal $x[n]$ is an independent non-Gaussian signal with known variance and higher moments.

3. The convolutional noise $p[n]$ and the source signal are independent. Thus,

$$\sigma_z^2[n] = E[z[n]z^*[n]] = E[(x[n] + p[n])(x[n] + p[n])^*] = E[x[n]x^*[n]] + E[p[n]p^*[n]]$$

For the very low ISI case ($B(D_1 B m_p^3[n] + A_1 m_p^2[n]) \ll B(B_1 m_p[n] + C_1 B)$) we may approximate (1) as follows:

$$\frac{E[\Delta p_r^2]}{\Delta t} \cong \frac{BB_1}{\Delta t} m_p[n] + \frac{B^2 C_1}{\Delta t} \tag{2}$$

Note that for the real valued and two independent quadrature carrier case we may write: $m_p[n] = \sigma_{x_r}^2 \cdot ISI[n]$ for $|\tilde{s}|_{max}^2 = 1$. By using $ISI[n] = \frac{m_p[n]}{\sigma_{x_r}^2}$ (for $|\tilde{s}|_{max}^2 = 1$), the solution of (2) is given by:

$$\tilde{m}_p(t) \cong \sigma_{x_r}^2\left(\widetilde{ISI}(0) + \frac{BC_1}{B_1\sigma_{x_r}^2}\right)e^{\frac{BtB_1}{\Delta t}} - \frac{BC_1}{B_1} \tag{3}$$

where $\widetilde{ISI}(0)$ is the ISI for the continues time case obtained at $t = 0$. Now, by using again that $ISI[n] = \frac{m_p[n]}{\sigma_{x_r}^2}$ (for $|\tilde{s}|_{max}^2 = 1$) and (3) we obtain:

$$\widetilde{ISI}(t) = \frac{\tilde{m}_p(t)}{\sigma_{x_r}^2} \cong \left(\widetilde{ISI}(0) + \frac{BC_1}{B_1\sigma_{x_r}^2}\right)e^{\frac{BtB_1}{\Delta t}} - \frac{BC_1}{B_1\sigma_{x_r}^2} \tag{4}$$

As it was already implied, the obtained expression for the ISI as a function of time given by (4) is only valid for the very low ISI case where the eye-diagram is almost open or already open. Obviously, this is not a case of interest. But in order to obtain a practical approximated expression for the ISI as a function of time valid during the whole convergence process of the equalizer, the expression of (4) was modified as follows:

$$\widetilde{ISI}(t) \cong \left(\widetilde{ISI}(0) - 10^{\frac{ISI_r}{10}}\right)e^{\frac{\gamma BtB_1}{\Delta t}} + 10^{\frac{ISI_r}{10}} \tag{5}$$

where γ is given by: $\gamma = \left(\min\left[\frac{1}{\sigma_{x_r}^2}\sqrt{\left|\frac{B_1}{BD_1}\right|}, \frac{1}{\sigma_{x_r}^2}\left|\frac{B_1}{A_1}\right|\right]\right)\frac{1}{\widetilde{ISI}(0)L}$ and ISI_r is the residual ISI expressed in dB units and is defined for $|\tilde{s}|_{max}^2 = 1$ in (Pinchas, 2009). It should be pointed out that (5) is only ad-hoc approximation. Although this expression (5) was not obtained based on strong mathematical foundations, it is still interesting to see the steps that lead to (5). This is exactly what is done in the following. Since it was already implied in (Pinchas, 2009) that the expression for the residual ISI from (4) defined by $\frac{-BC_1}{B_1\sigma_{x_r}^2}$ is less accurate than ISI_r, it was reasonable to use in (5) the most accurate expression for the residual ISI that approximately is known. Next we go back to (1) and derive some conditions that may lead approximately to (2). Note that (4) was obtained by assuming the approximation of (2). In order to get approximately the expression of (2) from (1), the following conditions should hold: $|B^2 D m_p^3[n]| << |BB_1 m_p[n]|$ and $|A_1 B m_p^2[n]| << |BB_1 m_p[n]|$ which lead by using the relation of $ISI[n] = \frac{m_p[n]}{\sigma_{x_r}^2}$ to:

$$ISI[n] << \min\left[\frac{1}{\sigma_{x_r}^2}\sqrt{\left|\frac{B_1}{BD}\right|}, \frac{1}{\sigma_{x_r}^2}\left|\frac{B_1}{A_1}\right|\right] \qquad (6)$$

Now, we may say that if the above condition holds, the obtained expression in (4) for the ISI as a function of time is approximately valid. Please note that for the very low ISI case, the convergence time of an equalizer is much faster compared to the case where the initial ISI is considered high. In addition, according to (Lee and Messerschmitt, 1997), the best rate of convergence is dependent on the number of filter coefficients. The more coefficients (in the equalizer), the longer it takes for the coefficients to converge. The more coefficients there are, the more ”noise” is introduced into the adaptation of each coefficient by the simultaneous adaptation of the other coefficients (Lee and Messerschmitt, 1997). Now, let us go back to the function of γ in (5). It can be seen that γ functions as an compensation factor between the very low ISI condition (6) and any other given initial ISI. Therefore, when the initial ISI is much higher than given in (6), γ will slow down the convergence rate.

Next we turn to calculate the total iteration number that takes to enter the convergence state. The exponent from (5) can be written as follows: $e^{\gamma\frac{BtB_1}{\Delta t}} = e^{\overline{\left(\frac{t}{\frac{\Delta t}{\gamma BB_1}}\right)}} = e^{-\frac{t}{\tau}}$ where $\tau = |\frac{\Delta t}{\gamma BB_1}|$. Next we assume that for $t = 8\tau$ the equalizer has approximately reached its steady state position. Note that for a simple RC circuit (one capacitor and one resistor), it is often assumed that the capacitor is approximately full charged or discharged after $5\tau = 5RC$ seconds. Since we are looking for a more accurate solution we choose instead of 5τ, 8τ which was found by simulation trials leading to satisfying results. Thus we may write: $t = 8\tau = 8|\frac{\Delta t}{\gamma BB_1}|$ Now let the sampling time be Δt. Thus we may write that $t = n\Delta t = 8|\frac{\Delta t}{\gamma BB_1}|$ from which we obtain (for $\Delta t \neq 0$) the total number of iteration required for convergence:

$$n = \frac{\widetilde{ISI}(0)L}{\min\left[\frac{1}{\sigma_{x_r}^2}\sqrt{\left|\frac{B_1}{BD}\right|}, \frac{1}{\sigma_{x_r}^2}\left|\frac{B_1}{A_1}\right|\right]}\frac{8}{B}\left|\frac{1}{B_1}\right| \qquad (7)$$

4 SIMULATION

In the following we use Godard’s equalizer (Godard, 1980) and the 16QAM constellation (a modulation using $\pm$ {1,3} levels for in-phase and quadrature components) as the source. The equalizer taps for Godard’s equalizer (Godard, 1980) were updated according to: $c_l[n+1] = c_l[n] - \mu_G\left(|z[n]|^2 - \frac{E[|x[n]|^4]}{E[|x[n]|^2]}\right) z[n] y^*[n-l]$ where μ_G is the step-size and l is the equalizer’s tap length. The values for a_1, a_{12} and a_3 corresponding to Godards’s (Godard, 1980) algorithm were defined as a_1^G, a_{12}^G and a_3^G respectively and were given by: $a_1^G = -\frac{E[|x[n]|^4]}{E[|x[n]|^2]}$, $a_{12}^G = 1$ and $a_3^G = 1$. Two different channels were considered.

Channel1 (initial ISI = 0.44): The channel parameters were determined according to (Shalvi and Weinstein, 1990): $h_n = 0$ for $n < 0$; -0.4 for $n = 0$ $\quad 0.84 \cdot 0.4^{n-1}$ for $n > 0$.

Channel2 (initial ISI = 0.5): The channel parameters were determined according to (Fiori, 2001):
$h_n = (-0.0144, 0.0006, 0.0427, 0.0090, -0.4842, -0.0376, 0.8163, 0.0247, 0.2976, 0.0122, 0.0764, 0.0111, 0.0162, 0.0063)$
For Channel1 and Channel2 an equalizer with 13 and 21 taps was used respectively. In the simulation, the equalizer was initialized by setting the center tap equal to one and all others to zero. Fig. 2 and Fig. 3 show the simulated performance of Godard’s equalization method for the 16QAM input case, namely the ISI as a function of iteration number for various step-size parameters, channel characteristics and equalizer’s tap length, compared with the calculated ISI as a function of iteration number (5) proposed in this paper. According to Fig. 2 and Fig. 3, the approximated closed-form expression for the ISI as a function of time (or iteration number) (5), fits very well the simulated results. Next, the expression for the total number of iteration required for convergence (7) was calculated for each simulation:

Case I – Described in Figure 2. The calculated number of iteration required for convergence according to (7) is 3135.

Case II – Described in Figure 3. The calculated number of iteration required for convergence according to (7) is 1788.

According to Fig. 2 and Fig. 3, there is a high correlation between the simulated and calculated (7) results for the number of iteration required for convergence.

Next we turn to the noisy case situation. Fig.4 shows the simulated performance of Godard’s equalization method for the 16QAM input case, namely the ISI as a function of iteration number for various SNR values, compared with the calculated ISI as a function of iteration number (5) proposed in this paper.

According to Fig.4, the approximated expression for the ISI as a function of iteration number (5) is valid also for the noisy case.

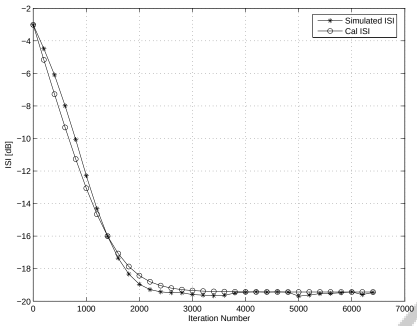

Figure 2: A comparison between the simulated (with Godard's algorithm) and calculated ISI as a function of time for the 16QAM source input going through channel2. The averaged results were obtained in 100 Monte Carlo trials for the noiseless case. The equalizer's length was set to 21 and $\mu_G = 0.00003$.

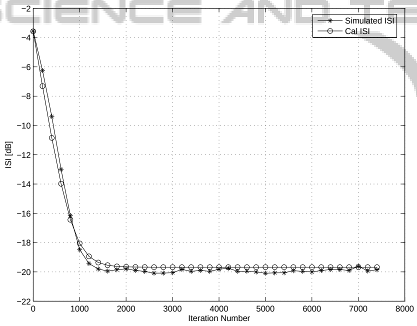

Figure 3: A comparison between the simulated (with Godard's algorithm) and calculated ISI as a function of time for the 16QAM source input going through channel1. The averaged results were obtained in 100 Monte Carlo trials for the noiseless case. The equalizer's length was set to 13 and $\mu_G = 0.00005$.

5 CONCLUSIONS

In this paper, a closed-form approximated expression was proposed for the (ISI) as a function of time for type of blind equalizers where the error that is fed into the adaptive mechanism which updates the equalizer's taps can be expressed as a polynomial function of the equalized output of order three. Based on the closed-form approximated expression for the ISI as a function of time, an approximated closed-form expression for the convergence time (or number of iteration required for convergence) as a function of initial ISI, step-size parameter, equalizer's tap length, input signal statistics and channel power was derived. The new closed-form approximated expressions were tested via simulation where high correlation was found between the calculated and simulated results. These results indicate that the need to simulate the whole system in order to find the convergence time or the ISI as a function of time for each different step-size parameter is eliminated. Although the approximated expression for the ISI as a function of time was derived for the noiseless case, simulation results have shown that it is valid also for the noisy condition.

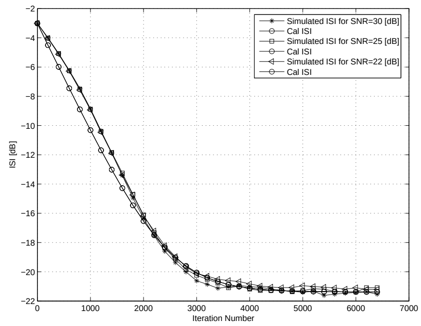

Figure 4: A comparison between the simulated (with Godard's algorithm) and calculated ISI as a function of time for the 16QAM source input going through channel2. The averaged results were obtained in 100 Monte Carlo trials for the noisy case. The equalizer's length was set to 21 and $\mu_G = 0.00002$.

REFERENCES

Fiori, S. (2001). A contribution to (neuromorphic) blind deconvolution by flexible approximated bayesian estimation. In *Signal Processing 81, 2131–2153*. Elsevier.

Godard, D. (1980). Self recovering equalization and carrier tracking in two-dimentional data communication system. In *IEEE Transaction Communication 28 (11) 1867-1875*. IEEE.

Lee, E. A. and Messerschmitt, D. G. (1997). *Adaptive Equalization, in: E. A. Lee and D. G. Messerschmitt, Digital Communication*. Kluwer Academic Publisher, third printing, 2nd edition.

Pinchas, M. (2009). A closed approximated formed expression for the achievable residual intersymbol interference obtained by blind equalizers. In *Signal Processing Journal (Eurasip), DOI: 10.1016/j.sigpro.2009.12.014*.

Pinchas, M. (2011). A mse optimized polynomial equalizer for 16qam and 64qam constellation. In *Signal, Image and Video Processing, Volume 5, Issue 1, DOI 10.1007/s11760-009-0138-z*.

Shalvi, O. and Weinstein, E. (1990). New criteria for blind deconvolution of nonminimum phase systems (channels). In *IEEE Trans. Information Theory 36 (2), 312-321*.