OBSERVER-BASED ADAPTIVE SLIDING MODE CONTROL FOR UNCERTAIN SYSTEMS WITH DEAD-ZONE INPUT

Yu-Ting Kuo and Kuo-Ming Chang
Department of Mechanical Engineering, National Kaohsiung University of Applied Sciences, Kaohsiung, Taiwan

Keywords: Extension state observer, Adaptive control, Sliding mode control, Dead-zone, External disturbance.

Abstract: In this paper, an adaptive sliding mode control is proposed to address the tracking control objective of uncertain nonlinear system preceded by an unknown dead-zone and with unmeasurable system state. Based on the extension state observer, sliding mode control, and adaptive dead-zone inverse techniques, a robust observer-based adaptive sliding mode control scheme is developed without available system state. The proposed control scheme can ensure global stability of the controlled system subject to unknown nonlinear function and external disturbance and achieve the tracking control objective satisfactorily.

1 INTRODUCTION

Generally, due to physical constraints of the dynamical systems, it may exist some non-smooth nonlinear characteristics in the control input, such as backlash, saturation, dead-zone, which can severely limit system performance or even result in system unstability. Hence, the nonlinear effects should be considered and compensated in analysis or realization of a control system. Recently, non-smooth nonlinearitites have been drawn much attention in the control community.

Dead-zone is one of the most important non-smooth nonlinearities arisen in actuator, such as servo valves and DC servo motors. In recent years, dead-zone has been extensively discussed in the literature. In most practical motion systems, the dead-zone is usually unknown. To handle systems with unknown dead-zone, Tao and Kokotovic (1994; 1995) proposed continuous- and discrete-time adaptive dead-zone inverses for linear systems with unmeasurable dead-zone outputs to improve the tracking performance by using dead-zone inverse. Without constructing the dead-zone inverse, Wang et al. developed a new robust adaptive approach of a class of nonlinear system preceded by a dead-zone. Ma and Yang further exploded an adaptive output feedback control without the dead-zone inverse for uncertain nonlinear system with an unknown non-symmetric dead-zone. The considered system is dominated by a triangular system without zero dynamics satisfying polynomial growth in unmeasurable states. Selmic and Lewis employed neural networks to construct a dead-zone precompensator, which is used to improve the tracking performance of motion system in the presence of unknown dead-zone. For controlling a class of uncertain multi-input multi-output nonlinear state time-varying delay systems with unknown nonlinear dead-zone and gain signs, an adaptive neural control is proposed by Zhang and Ge. This control is designed based on the intuitive concept and piecewise description of dead-zone and the principle of sliding mode control and such this control scheme can guarantee that all signals are semi-globally uniformly ultimately bounded. Liu and Zhou used the universal approximation property of the fuzzy-neural networks to approximate unknown nonlinear function and then presented an observer-based adaptive fuzzy-neural control for a class of uncertain nonlinear systems with unknown dead-zone input to improve the control performance.

In this paper, an observer-based adaptive sliding mode control approach for uncertain systems with unknown dead-zone is proposed to achieve the tracking control objective in the presence of unknown system nonlinear function and external disturbance. The paper is organized as follows: Section 2 gives some descriptions of the system; Section 3 presents the controller design based on adaptive control, sliding mode control and extension state observer techniques; The stability of the controlled system is proved in Section 4 and conclusions are made in Section 5.

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2 SYSTEM DESCRIPTIONS

Consider a class of n th-order single-input and single-output uncertain nonlinear system with a dead-zone function, which is described in the following dynamical equation

$$x^{(n)} = f(x, \dot{x}, \cdots, x^{(n-1)}, t) + w(t) + d(t) \tag{1}$$

where x is the system output, f is an unknown system nonlinear function, $d(t)$ is an external disturbance, and $w(t)$ is a dead-zone nonlinear function. The dead-zone function with input $u(t)$ and output $w(t)$ is graphically shown in Fig. 1 for some unknown constants $0 < b_l, b_r, m_l, m_r < \infty$.

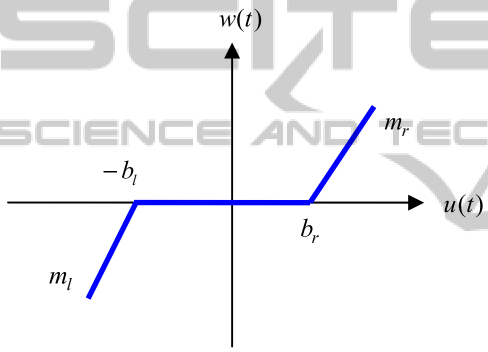

Figure 1: Dead-zone function.

As shown in Figure 1, the dead-zone function can be described mathematically by

$$w(t) = \begin{cases} m_r[u(t) - b_r], & \text{if } u(t) > b_r \\ 0, & \text{if } -b_l \le u(t) \le b_r \\ m_l[u(t) + b_l], & \text{if } u(t) < -b_l \end{cases} \tag{2}$$

Define a system state vector as

$$\begin{aligned} \boldsymbol{X}(t) &= \left[x(t) \quad \dot{x}(t) \quad \cdots \quad x^{(n-1)}(t)\right]^T \\ &= [x_1(t) \quad x_2(t) \quad \cdots \quad x_n(t)]^T \in R^n \end{aligned} \tag{3}$$

Then, the system in (1) can be expressed by a state space representation

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ &\vdots \\ \dot{x}_{n-1} &= x_n \\ \dot{x}_n &= f(\boldsymbol{X}, \ t) + w(t) + d(t) \\ &= a(t) \end{aligned} \tag{4}$$

In this paper, the following assumptions, which specify the class of uncertain nonlinear systems are made as follows:

Assumption 1. Uncertain external disturbance $d(t)$ is a bounded function. It means that there exists one positive constant c_1 such that $|d(t)| \le c_1$.

Assumption 2. Nonlinear function $a(t)$ is assumed to be differentiable with respect to time and its derivative with respect to time is bounded, i.e. $|\dot{a}(t)| \le c_2$ with $c_2 > 0$.

Let the desired state vector be

$$\begin{aligned} \mathbf{X}_d(t) &= [x_d(t) \quad \dot{x}_d(t) \quad \cdots \quad x_d^{(n-1)}(t)]^T \\ &= [x_{d1}(t) \quad x_{d2}(t) \quad \cdots \quad x_{dn}(t)]^T \in R^n \end{aligned} \tag{5}$$

Then, define the tracking error as

$$\begin{aligned} \boldsymbol{E}(t) &= \boldsymbol{X}(t) - \boldsymbol{X}_d(t) \\ &= [e_1(t) \quad e_2(t) \quad \cdots \quad e_n(t)]^T \end{aligned} \tag{6}$$

In this paper, the control objective is to design an observer-based adaptive sliding mode control to achieve $\boldsymbol{E}(t) \to 0$ as $t \to \infty$ under the condition that the system states are not available during the control process.

3 OBSERVER-BASED ADAPTIVE SLIDING MODE CONTROL

In this section, an observer-based adaptive sliding mode control scheme will be developed to achieve the state tracking control objective. Because system states are not available, a so-called extension state observer is constructed to obtain estimated system states. On the constructing process of extension state observer, an augmented state vector is given as follows:

$$\begin{aligned} \boldsymbol{X}_a(t) &= [x_1(t) \quad x_2(t) \quad \cdots \quad x_n(t) \quad a(t)]^T \\ &= [x_1(t) \quad x_2(t) \quad \cdots \quad x_n(t) \quad x_{n+1}(t)]^T \end{aligned} \tag{7}$$

Then, we have

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ &\vdots \\ \dot{x}_{n-1} &= x_n \\ \dot{x}_n &= a(t) \\ \dot{x}_{n+1} &= \dot{a}(t) \end{aligned} \tag{8}$$

In this paper, the extension state observer is given in the following form

$$\dot{\hat{x}}_1 = \hat{x}_2 - Lk_1(\hat{x}_1 - x_1)$$
$$\dot{\hat{x}}_2 = \hat{x}_3 - L^2k_2(\hat{x}_1 - x_1)$$
$$\vdots$$
$$\dot{\hat{x}}_{n-1} = \hat{x}_n - L^{n-1}k_{n-1}(\hat{x}_1 - x_1)$$
$$\dot{\hat{x}}_n = \hat{x}_{n+1} - L^nk_n(\hat{x}_1 - x_1)$$
$$\dot{\hat{x}}_{n+1} = -L^{n+1}k_{n+1}(\hat{x}_1 - x_1) \quad (9)$$

where L is a design positive constant, constants $k_1, k_2, \cdots, k_{n+1}$ are chosen according to the pole assignment method. Define a state error vector between the estimated augmented system state and augmented system state as

$$\begin{aligned}\tilde{\boldsymbol{X}}_a(t) &= \hat{\boldsymbol{X}}_a(t) - \boldsymbol{X}_a(t)\\ &= [\hat{x}_1(t) \quad \hat{x}_2(t) \quad \cdots \quad \hat{x}_{n+1}(t)]^T - [x_1(t) \quad x_2(t) \quad \cdots \quad x_{n+1}(t)]^T\\ &= [\tilde{x}_1(t) \quad \tilde{x}_2(t) \quad \cdots \quad \tilde{x}_{n+1}(t)]^T \end{aligned} \quad (10)$$

where $\hat{\boldsymbol{X}}_a(t)$ is the estimated state of the augmented system state. From (8) and (9), we can obtain the dynamic equation of state error expressed by

$$\begin{bmatrix}\dot{\tilde{x}}_1\\ \dot{\tilde{x}}_2\\ \vdots\\ \dot{\tilde{x}}_n\\ \dot{\tilde{x}}_{n+1}\end{bmatrix} = \begin{bmatrix}-Lk_1 & 1 & 0 & \cdots & 0\\ -L^2k_2 & 0 & 1 & \cdots & 0\\ \vdots & & & \vdots & \\ -L^nk_n & 0 & 0 & \cdots & 1\\ -L^{n+1}k_{n+1} & 0 & 0 & \cdots & 0\end{bmatrix}\begin{bmatrix}\tilde{x}_1\\ \tilde{x}_2\\ \vdots\\ \tilde{x}_n\\ \tilde{x}_{n+1}\end{bmatrix} + \begin{bmatrix}0\\ 0\\ \vdots\\ 0\\ -\dot{a}(t)\end{bmatrix} \quad (11)$$

or

$$\dot{\tilde{\boldsymbol{X}}}_a(t) = \boldsymbol{A}\tilde{\boldsymbol{X}}_a(t) + \boldsymbol{F}(t) \quad (12)$$

Wher $\mathbf{A} = \begin{bmatrix}-Lk_1 & 1 & 0 & \cdots & 0\\ -L^2k_2 & 0 & 1 & \cdots & 0\\ \vdots & & & \vdots & \\ -L^nk_n & 0 & 0 & \cdots & 1\\ -L^{n+1}k_{n+1} & 0 & 0 & \cdots & 0\end{bmatrix}$ and $\boldsymbol{F}(t) = \begin{bmatrix}0\\ 0\\ \vdots\\ 0\\ -\dot{a}(t)\end{bmatrix}$.

From the equation $|s\boldsymbol{I} - \boldsymbol{A}| = 0$, it yields that the characteristic equation of matrix $\boldsymbol{A}$ is given by

$$s^{n+1} + Lk_1s^n + L^2k_2s^{n-1} + \cdots + L^nk_ns + L^{n+1}k_{n+1} = 0 \quad (13)$$

Both sides of (13) are divided by L^{n+1}, then we have

$$L^{-(n+1)}s^{n+1} + L^{-n}k_1s^n + L^{-(n-1)}k_2s^{n-1} + \cdots + L^{-1}k_ns + k_{n+1} = 0 \quad (14)$$

Define a variable as

$$s_1 = L^{-1}s \quad (15)$$

From (15), Eq. (14) can be further represented as

$$s_1^{\,n+1} + k_1s_1^{\,n} + k_2s_1^{\,n-1} + \cdots + k_ns_1 + k_{n+1} = 0 \quad (16)$$

To yield that all the zero locations of (16) lie on the left-hand plane of s_1 plane, constants $k_1, k_2, \cdots, k_{n+1}$ can be given appropriately by using the pole assignment method. Setting suitable values $k_1, k_2, \cdots, k_{n+1}$ implies that all the eigenvalues of matrix $\boldsymbol{A}$ lie on the left-hand plane of s plane. Because all the eigenvalues of matrix $\boldsymbol{A}$ lie on the left-hand plane of s plane, it can be concluded that the error dynamic system in (12) is asymptotically stable. The solution of (12) is obtained as follows:

$$\tilde{\boldsymbol{X}}_a(t) = e^{At}\tilde{\boldsymbol{X}}_a(0) + \int_0^t e^{A(t-\tau)}\boldsymbol{F}(\tau)d\tau \quad (17)$$

Without loss of generality, we can set $\tilde{\boldsymbol{X}}_a(0) = 0$ in the design process. Hence, it yields from Eq. (17) and Assumption 2 that

$$\tilde{\boldsymbol{X}}_a(t) = \int_0^t e^{A(t-\tau)}\boldsymbol{F}(\tau)d\tau \quad (18)$$

and

$$\left\|\tilde{\boldsymbol{X}}_a(t)\right\| = \left\|\int_0^t e^{A(t-\tau)}\boldsymbol{F}(\tau)d\tau\right\| < c_3$$

where c_3 is a positive constant. To design a sliding mode controller, a sliding function formed in the space of state error can be defined as

$$S(t) = \boldsymbol{\Gamma}\boldsymbol{E}(t) \quad (19)$$

where $\boldsymbol{\Gamma} = [\gamma_1 \quad \gamma_2 \quad \cdots \quad \gamma_{n-1} \quad 1]$ is a constant vector. While an appropriate control law is applied and the sliding mode is occurred in finite time, the error dynamics in the sliding mode can be defined by

$$e_n + \gamma_{n-1}e_{n-1} + \cdots + \gamma_1e_1 = 0 \quad (20)$$

In (19), positive constants $\gamma_1, \gamma_2, \cdots, \gamma_{n-1}$ should be chosen such that $\lambda^{n-1} + \sum_{i=1}^{n-1}\gamma_i\lambda^{i-1}$ is a Hurwitz polynomial.

Because the system state is not available, the sliding function cannot be constructed by the system state. In this paper, a so-called almost sliding function is given as

$$\begin{aligned}\hat{S}(t) &= \boldsymbol{\Gamma}\hat{\boldsymbol{E}}(t)\\ &= \boldsymbol{\Gamma}[\hat{\boldsymbol{X}}(t) - \boldsymbol{X}_d(t)]\end{aligned} \quad (21)$$

where $\hat{\boldsymbol{X}}(t) = [\hat{x}_1(t) \quad \hat{x}_2(t) \quad \cdots \quad \hat{x}_n(t)]^T$.

Then, we have

$$\begin{aligned}\hat{S} &= \hat{S} - S + S\\ &= (\hat{e}_n - e_n) + \sum_{i=1}^{n-1}\gamma_i(\hat{e}_i - e_i) + S\\ &= \boldsymbol{\Lambda}\tilde{\boldsymbol{X}}_a(t) + S\end{aligned} \quad (22)$$

where $\boldsymbol{\Lambda} = [\gamma_1 \quad \gamma_2 \quad \cdots \quad \gamma_{n-1} \quad 1 \quad 0]$.
Substituting (18) into (22), it yields that

$$\hat{S} = \boldsymbol{\Lambda}\int_0^t e^{A(t-\tau)}\boldsymbol{F}(\tau)d\tau + S \tag{23}$$

From (23), the derivative of function $\hat{S}$ with respect to time is given by

$$\dot{\hat{S}} = \boldsymbol{\Lambda A}\int_0^t e^{A(t-\tau)}\boldsymbol{F}(\tau)d\tau + \boldsymbol{\Lambda F} + \dot{S} \tag{24}$$

Then, from (23) we can obtain

$$\begin{aligned}\dot{\hat{S}} &= \boldsymbol{\Lambda A}\int_0^t e^{A(t-\tau)}\boldsymbol{F}(\tau)d\tau + \boldsymbol{\Lambda F} + \dot{S} - \dot{\hat{S}} + \dot{\hat{S}} \\ &= \boldsymbol{\Lambda A}\int_0^t e^{A(t-\tau)}\boldsymbol{F}(\tau)d\tau + \boldsymbol{\Lambda F} + \dot{e}_n + \sum_{i=1}^{n-1}\gamma_i\dot{\hat{e}}_i + \sum_{i=1}^{n-1}\gamma_i(\dot{x}_i - \dot{\hat{x}}_i) \\ &= \boldsymbol{\Lambda A}\int_0^t e^{A(t-\tau)}\boldsymbol{F}(\tau)d\tau + \boldsymbol{\Lambda F} + \dot{e}_n + \sum_{i=1}^{n-1}\gamma_i\dot{\hat{e}}_i - \sum_{i=1}^{n-1}\gamma_i\tilde{x}_{i+1} \\ &\quad + \sum_{i=1}^{n-1}\gamma_i g^i k_i \tilde{x}_1 \end{aligned} \tag{25}$$

Define a constant and a constant vector, respectively as

$$c_4 = \sum_{i=1}^{n-1}\gamma_i g^i k_i \tag{26}$$

$$\boldsymbol{\Gamma}_1 = [c_4 \quad -\gamma_1 \quad \cdots \quad -\gamma_{n-1} \quad 0] \tag{27}$$

Then, from (26) and (27), (25) can be represented by

$$\dot{\hat{S}} = \boldsymbol{\Lambda A}\int_0^t e^{A(t-\tau)}\boldsymbol{F}(\tau)d\tau + \boldsymbol{\Lambda F} + \dot{e}_n + \sum_{i=1}^{n-1}\gamma_i\dot{\hat{e}}_i + \boldsymbol{\Gamma}_1\tilde{\boldsymbol{X}}_a \tag{28}$$

It follows that from (18)

$$\begin{aligned}\dot{\hat{S}} &= \boldsymbol{\Lambda A}\int_0^t e^{A(t-\tau)}\boldsymbol{F}(\tau)d\tau + \boldsymbol{\Gamma}_1\int_0^t e^{A(t-\tau)}\boldsymbol{F}(\tau)d\tau + \boldsymbol{\Lambda F} \\ &\quad + \dot{e}_n + \sum_{i=1}^{n-1}\gamma_i\dot{\hat{e}}_i \\ &= \boldsymbol{\Lambda A}\int_0^t e^{A(t-\tau)}\boldsymbol{F}(\tau)d\tau + \boldsymbol{\Gamma}_1\int_0^t e^{A(t-\tau)}\boldsymbol{F}(\tau)d\tau + \boldsymbol{\Lambda F} \\ &\quad + f + w + d - \dot{x}_{dn} + \sum_{i=1}^{n-1}\gamma_i\dot{\hat{e}}_i \end{aligned} \tag{29}$$

Suppose that $\int_0^t e^{A(t-\tau)}\boldsymbol{F}(\tau)d\tau = 0$ and $\boldsymbol{\Lambda F} = 0$, a so-called equivalent nonlinear input $w_e(t)$ can be obtained from $\dot{\hat{S}} = 0$ in (29)

$$w_e(t) = -\sum_{i=1}^{n-1}\gamma_i\dot{\hat{e}}_i + \dot{x}_{dn} - (f + d) \tag{30}$$

In addition the equivalent nonlinear input, for approaching the sliding surface, a switching nonlinear input is given as

$$w_s(t) = -k_d\hat{S} - \eta\, sat\left(\frac{\hat{S}}{\varepsilon}\right) \tag{31}$$

where k_d and η are two design positive constants, ε is a sufficient small positive constant, and $sat(\cdot)$ is a saturation function, which is represented by

$$sat\left(\frac{\hat{s}}{\varepsilon}\right) = \begin{cases} 1, & if\ \frac{\hat{s}}{\varepsilon} \geq 1 \\ \frac{\hat{s}}{\varepsilon}, & if\ -1 < \frac{\hat{s}}{\varepsilon} < 1 \\ -1, & if\ \frac{\hat{s}}{\varepsilon} \leq -1 \end{cases}$$

Hence, the ideal nonlinear input can be obtained in the following form.

$$\begin{aligned} w_{di}(t) &= w_e(t) + w_s(t) \\ &= -\sum_{i=1}^{n-1}\gamma_i\dot{\hat{e}}_i + \dot{x}_{dn} - (f + d) - k_d\hat{S} - \eta\, sat\left(\frac{\hat{S}}{\varepsilon}\right) \end{aligned} \tag{32}$$

In (32), since f and d are two unknown functions, we can not obtain the ideal nonlinear input in the practical control. From (1), the above input can be expressed as

$$w_{di}(t) = -\sum_{i=1}^{n-1}\gamma_i\dot{\hat{e}}_i + \dot{x}_{dn} - (x^{(n)} - w) - k_d\hat{S} - \eta\, sat\left(\frac{\hat{S}}{\varepsilon}\right) \tag{33}$$

Then, it yields that the desired nonlinear input can be designed in the following form

$$w_d(t) = -\sum_{i=1}^{n-1}\gamma_i\dot{\hat{e}}_i + \dot{x}_{dn} - \hat{x}_{n+1} + \hat{w}_d - k_d\hat{S} - \eta\, sat\left(\frac{\hat{S}}{\varepsilon}\right) \tag{34}$$

where $\hat{x}_{n+1}$ is the estimated value of $x^{(n)}$, which is obtained from the extension state observer and $\hat{w}_d$ is a filtered signal, which is given by

$$\dot{\hat{w}}_d = -\delta\hat{w}_d + \delta w_d \tag{35}$$

where δ is a design positive constant. Hence, the following result can be achieved.

$$\lim_{t\to\infty}\lim_{\delta\to\infty}\hat{w}_d = w_d = w$$

Because the considered system contains an unknown dead zone in this paper, an adaptive dead zone inverse is proposed. The objective of the dead zone inverse is to cancel the dead zone so that $w(t) = w_d(t)$ for any $w_d(t)$ which is the desired nonlinear input to the system. If dead zone parameters b_l, b_r, m_l, m_r are known, we can cancel the dead-zone effect. Mathematically, the relation between $u(t)$ and $w_d(t)$, which specifies the dead zone inverse, is defined as follows:

$$u(t)=\begin{cases}\dfrac{w_d+m_r b_r}{m_r}, & if\ w_d(t)>0\\ 0, & if\ w_d(t)=0\\ \dfrac{w_d+m_l b_l}{m_l}, & if\ w_d(t)<0\end{cases} \quad (36)$$

Define some constant vectors as

$$N=[n_r \quad n_l],\ M=[m_r \quad m_l]^T,$$
$$\theta=[m_r b_r \quad m_l b_l]^T=[\theta_1 \quad \theta_2]^T$$

where $n_r=\begin{cases}1, & if\ w_d>0\\ 0, & otherwise\end{cases}$ and $n_l=\begin{cases}1, & if\ w_d<0\\ 0, & otherwise\end{cases}$.
Then, (36) can be represented by

$$u(t)=\frac{1}{NM}(w_d+N\theta) \quad (37)$$

While we use the above dead zone inverse, it has a problem that the parameters b_l, b_r, m_l, m_r are unknown. In this section, the adaptive dead zone inverse based on the estimates to produce the control input is represented in the following form.

$$u(t)=\begin{cases}\dfrac{w_d+m_r b_r}{\hat{m}_r}, & if\ w_d(t)>0\\ 0, & if\ w_d(t)=0\\ \dfrac{w_d+m_l b_l}{\hat{m}_l}, & if\ w_d(t)<0\end{cases}$$
$$=\frac{1}{N\hat{M}}(w_d+N\hat{\theta}) \quad (38)$$

where $\hat{M}=[\hat{m}_r \quad \hat{m}_l]^T$ and $\hat{\theta}=[\hat{\theta}_1 \quad \hat{\theta}_2]^T$ are the estimates of M and θ, respectively. Define parameter error, slope ratio, and estimated slope ratio, respectively as

$$\tilde{\theta}=\hat{\theta}-\theta=[m_r b_r \quad m_l b_l]^T-[m_r b_r \quad m_l b_l]^T \quad (39)$$

$$\phi=[\phi_r \quad \phi_l]^T=\begin{bmatrix}\dfrac{m_r}{m_r} & \dfrac{m_l}{m_l}\end{bmatrix}^T=[1 \quad 1]^T \quad (40)$$

$$\hat{\phi}=[\hat{\phi}_r \quad \hat{\phi}_l]^T=\begin{bmatrix}\dfrac{m_r}{\hat{m}_r} & \dfrac{m_l}{\hat{m}_l}\end{bmatrix}^T \quad (41)$$

Then, we have the estimate error of the slope ratio as

$$\tilde{\phi}=\hat{\phi}-\phi=[\tilde{\phi}_r \quad \tilde{\phi}_l]^T \quad (42)$$

Define a function as

$$S_\varepsilon=\hat{S}-\varepsilon\cdot sat\left(\frac{\hat{S}}{\varepsilon}\right) \quad (43)$$

The adaptation laws are given by

$$\dot{\hat{\theta}}=-\alpha S_\varepsilon N^T \quad (44)$$

$$\dot{\hat{\phi}}=-\beta S_\varepsilon N^T(w_d+N\hat{\theta}) \quad (45)$$

$$\hat{m}_{j,n+1}=\hat{\phi}_{j,n}\,\hat{m}_{j,n},\ j=r,l \quad (46)$$

where α and β are positive constants to determine the adaptation rate. Since $w(t)=w_d(t)$, we have

$$w=NMu-N\theta \quad (47)$$

Then, from (29) and (47), it yields that

$$\dot{\hat{S}}=\boldsymbol{\Lambda A}\int_0^t e^{A(t-\tau)}\boldsymbol{F}(\tau)d\tau+\boldsymbol{\Gamma}_1\int_0^t e^{A(t-\tau)}\boldsymbol{F}(\tau)d\tau+\boldsymbol{\Lambda F}$$
$$+f+NMu-N\theta+d-\dot{x}_{dn}+\sum_{i=1}^{n-1}\gamma_i\dot{\hat{e}}_i \quad (48)$$

Substituting (38) into (48), it is obtained that

$$\dot{\hat{S}}=\boldsymbol{\Lambda A}\int_0^t e^{A(t-\tau)}\boldsymbol{F}(\tau)d\tau+\boldsymbol{\Gamma}_1\int_0^t e^{A(t-\tau)}\boldsymbol{F}(\tau)d\tau+\boldsymbol{\Lambda F}+f$$
$$+\frac{NM}{N\hat{M}}(w_d+N\hat{\theta})-N\theta+d-\dot{x}_{dn}+\sum_{i=1}^{n-1}\gamma_i\dot{\hat{e}}_i$$
$$=\boldsymbol{\Lambda A}\int_0^t e^{A(t-\tau)}\boldsymbol{F}(\tau)d\tau+\boldsymbol{\Gamma}_1\int_0^t e^{A(t-\tau)}\boldsymbol{F}(\tau)d\tau+\boldsymbol{\Lambda F}+f$$
$$+(1+N\tilde{\phi})(w_d+N\hat{\theta})-N\theta+d-\dot{x}_{dn}+\sum_{i=1}^{n-1}\gamma_i\dot{\hat{e}}_i$$
$$=\boldsymbol{\Lambda A}\int_0^t e^{A(t-\tau)}\boldsymbol{F}(\tau)d\tau+\boldsymbol{\Gamma}_1\int_0^t e^{A(t-\tau)}\boldsymbol{F}(\tau)d\tau+\boldsymbol{\Lambda F}+f$$
$$-\hat{x}_{n+1}+\hat{w}_d-k_d\hat{S}-\eta sat\left(\frac{\hat{S}}{\varepsilon}\right)+N\tilde{\theta}+d$$
$$+N\tilde{\phi}(w_d+N\hat{\theta}) \quad (49)$$

4 STABILITY ANALYSIS

Consider a Lyapunov function candidate as

$$V=\frac{1}{2}\left(S_\varepsilon^2+\frac{1}{\alpha}\tilde{\theta}^T\tilde{\theta}+\frac{1}{\beta}\tilde{\phi}^T\tilde{\phi}\right) \quad (50)$$

Then, the time derivative of function V is given by

$$\dot{V}=S_\varepsilon\dot{S}_\varepsilon+\frac{1}{\alpha}\dot{\tilde{\theta}}^T\tilde{\theta}+\frac{1}{\beta}\dot{\tilde{\phi}}^T\tilde{\phi}$$

1. As $|\hat{S}|<\varepsilon$, from (43), we have $S_\varepsilon=0$. It follows that $\dot{V}=0$.

2. As $|\hat{S}|\geq\varepsilon$, from (43), we have $\dot{S}_\varepsilon=\dot{\hat{S}}$.

From (44), (45), and (49), it is obtained that

$$\dot{V}=S_\varepsilon\Big[\boldsymbol{\Lambda A}\int_0^t e^{A(t-\tau)}\boldsymbol{F}(\tau)d\tau+\boldsymbol{\Gamma}_1\int_0^t e^{A(t-\tau)}\boldsymbol{F}(\tau)d\tau+\boldsymbol{\Lambda F}$$

$$+ f - \hat{x}_{n+1} + \hat{w}_d - k_d\hat{S} - \eta sat\left(\frac{\hat{S}}{\varepsilon}\right)\Bigg]$$

$$\dot{V} \le |S_\varepsilon|\left[f + d - \hat{x}_{n+1} + \hat{w}_d - k_d\varepsilon - \eta + \eta_1 + \eta_2 + \eta_3\right] - k_d S_\varepsilon^2 \quad (51)$$

where η_1, η_2, and η_3 are three given positive constants such that $\left\|\Lambda A\int_0^t e^{A(t-\tau)}\boldsymbol{F}(\tau)d\tau\right\| \le \eta_1$, $\left\|\boldsymbol{\Gamma}_1\int_0^t e^{A(t-\tau)}\boldsymbol{F}(\tau)d\tau\right\| \le \eta_2$, and $\|\Lambda \boldsymbol{F}\| \le \eta_3$.

According to the design in (9), (34), and (35), it can be obtained that

$\lim_{t\to\infty}(f + d + \hat{w}_d) = \lim_{t\to\infty}\hat{x}_{n+1}$ and

$$|f + d + \hat{w}_d - \hat{x}_{n+1}| \le |c_5 e^{-c_6 t}| + c_7 = c_8 \quad (52)$$

From (51) and (52), we obtain

$$\dot{V} \le |S_\varepsilon|\left[c_8 - k_d\varepsilon - \eta + \eta_1 + \eta_2 + \eta_3\right] - k_d S_\varepsilon^2 \quad (53)$$

where c_5, c_6, c_7, and c_8 are given positive constants.

When design parameters k_d, η, and ε in (31) are chosen and satisfy the following condition.

$$\eta_1 + \eta_2 + \eta_3 + c_8 \le k_d\varepsilon + \eta \quad (54)$$

From (53) and (54), it yield that

$$\dot{V} \le -k_d S_\varepsilon^2 \le 0 \quad (55)$$

From the above analysis, it can be concluded that $\dot{V} \le 0$ for all time. Therefore, V is an non-increasing function so that S_ε, $\hat{\theta}$, and $\hat{\phi}$ are bounded, i.e. S_ε, $\hat{\theta}$, and $\hat{\phi} \in L_\infty$. From (55), we have

$$k_d\int_0^t S_\varepsilon^2(t)dt \le V(0) - V(t) \le V(0) \le \infty$$

The above inequality means $S_\varepsilon \in L_2$. Since $\dot{S}_\varepsilon = \dot{\hat{S}}$, from (48), it follows that $\dot{S}_\varepsilon \in L_\infty$. According to Barbalat Lemma, it is concluded that $\lim_{t\to\infty} S_\varepsilon(t) = 0$, and then it yields from (43) that $\hat{S}(t)$ is a bounded signal and within bounded by

$$|\hat{S}(t)| < \varepsilon \quad \text{for all } t > t_1,\ t_1 > 0.$$

The above inequality means that $\hat{X}(t)$ can asymptotically follow reference signal $X_d(t)$ and also implies that system state $X(t)$ can asymptotically follow reference signal $X_d(t)$ by using extension state observer.

5 CONCLUSIONS

Without the requiqement of available system state, the main contribution of this paper is to develop an observer-based adaptive sliding mode control scheme to achieve the tracking control objective for an uncertain system which is preceded by an unknown dead-zone and subject to unknown system nonlinear function and external disturbance. In this paper, it is proved that the proposed control scheme can ensure global stability of the controlled system and can achieve the tracking control objective satisfactorily.

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