RESOURCE ALLOCATION PROBLEMS ON NETWORKS
Maximizing Social Welfare using an Agent-based Approach

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Abstract: Numerous applications can be formulated as an instance of resource allocation problems. Different kinds of solving techniques have been investigated, but the theoretical results cannot always be applied in practice due to inappropriate assumptions. Indeed, in these studies, agents are most of the time omniscient and/or have complete communication abilities. These hypotheses are not satisfied real life applications. practice. We propose in this paper a distributed mechanism leading to optimal solutions with respect to a more realistic environment. Agents only have limited perceptions and knowledge. Using local negotiations, they elaborate themselves optimal allocations, which can be viewed as emergent phenomena. We show that negotiations between individually rational agents lead to sub-optimal states in the society, and we propose a more suitable decision-making criterion, the sociability, leading to socially optimal solutions. Our method provides a sequence of transactions leading to optimal allocations, according to any communication networks, when four different welfare objectives are considered.

1 INTRODUCTION

Resource allocation problems arouse a great interest in the computer science community since such problems can be encountered through countless applications in real life. Centralized approaches as well as distributed approaches have been investigated to solve efficiently resource allocations problems. According to centralized solving techniques, all information are gathered in a single place in order to determine the best allocation. These kind of techniques suit well to the solving of applications like combinatorial auction whereas distributed solving methods, on which we focus in this paper, suit better to applications where privacy or dynamism is required for instance. Many studies on distributed frameworks have been performed (e.g. (Sandholm, 1998; Dunne et al., 2005; Chevaleyre et al., 2010)) in which authors aim to characterize the solutions that can be achieved. Indeed, these studies focus on the existence of transaction paths, on their length or on the properties satisfied by a solution depending on the settings of the solving method. However, these studies do not consider restrictions on agent communications but it is not satisfied in many applications, like the ones based on peer-to-peer networks or on social networks.Former studies focus on the characterization of solutions but did not focus on the mechanism required to achieve such solutions. Restricted contact networks have been considered in different contexts (de Weerdt et al., 2007)but our objective also differs. Indeed, we seek to design the negotiation settings that should be used in order to achieve a socially optimal state within agent societies.

We propose in this paper a solving method based on more realistic assumptions. Agents initially have very limited knowledge: their preferences, their resource bundle and a list of neighbors. Starting from this state, agents negotiate and try to trade their resources thanks to local transactions satisfying their acceptability criterion. We design the settings leading negotiation processes to optimal solutions using local transactions between agents, according to any kind of contact networks. Four different welfare functions have been considered. In each case, we first evaluate the quality of our solutions compared to solutions provided by centralized techniques. Then, we estimate the impact of networks topology in order to determine the characteristics penalizing or favoring the efficiency of agent negotiations.

This paper is organized as follows. Section 2 describes the basic parameters on which are based our model. Section 3 presents our solving approach while Section 4 describes the experimental protocol. Fi-

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nally, Section 5 successively details the results we obtain for utilitarian and egalitarian societies.

2 ISSUES ON AGENT NEGOTIATIONS

2.1 Definitions and Assumptions

We focus on distributed mechanisms to solve reallocation problems. A distributed solving process starts from an initial allocation, which evolves, step by step, thanks to local negotiation between agents, until the achievement of optimal allocations

Related to the resource nature, we choose to consider unique and atomic resources which are not shareable. Agents cannot alter the resources they own, they are only able to trade them. Let $\mathcal{A}$ be the set of all possible allocations.

We propose to consider several parameters, on which is based the definition of agent. An agent is defined with a **bundle** describing the owned resources, the **preferences** used to evaluate the agent satisfaction, a **behavior** specifying how agents interact, an **acceptability criterion** on which the agent determines if a deal is profitable, and a **neighborhood** representing the communication abilities.

Agents express preferences over the resource set, which are used to determine their individual welfare (Doyle, 2004). We choose to usean additive utility function. The satisfaction of an agent a_i to own a set ρ of resources can be computed as: $u_{a_i}(\rho) = \sum_{r_a \in \rho} u_{a_i}(r_a)$.

Essential notions have been defined in this section. Based on them, we can design the behaviors leading agents to trade efficiently their resources. A question can be raised: how can we evaluate a negotiation process?

2.2 Evaluation of a Negotiation Process

Since an objective is to identify the negotiation settings leading agent negotiations to optimal solutions, the absolute efficiency must be evaluated.The quality of two allocations can be compared thanks to the notions of the social choice theory (Arrow et al., 2002).

Maximizing the *utilitarian welfare* is equivalent to maximize the average individual welfare in a population. The utilitarian welfare associated with an allocation $A \in \mathcal{A}$ can be defined as $sw_u(A) = \sum_{a_i \in \mathcal{P}} u_{a_i}(\mathcal{R}_{a_i})$. The maximization of the *egalitarian welfare* tends to reduce inequalities in the population. It can be defined as $sw_e(A) = \min_{a_i \in \mathcal{P}} u_{a_i}(\mathcal{R}_{a_i})$. The Nash product considers the welfare of the whole population and reduces the inequalities among agents at the same time. It can be viewed as a compromise between the utilitarian and the egalitarian welfare: $sw_n(A) = \prod_{a_i \in \mathcal{P}} u_{a_i}(\mathcal{R}_{a_i})$. Finally, the elitist welfare, which only considers the welfare of the richest agent in the population, is defined as follows: $sw_{e\ell}(A) = \max\limits_{a_i \in \mathcal{P}} u_{a_i}(\mathcal{R}_{a_i})$.

For each social welfare notions, the optimal value can be determined or estimated by means of centralized algorithms, as suggested in (Nongaillard et al., 2008) but they are not detailed here. The optimal values, provided by these algorithms, are used as references to determine the absolute efficiency of a negotiation process.

Other facets of negotiations must also be considered. The impact of the social graph topology can be evaluated, in order to determine the cost of considering restrictions on agent communication abilities.

The topological sensitivity can be evaluated thanks to the standard deviation among the social values achieved at the end of negotiation processes. A large deviation means that the negotiation process is very sensitive to the graph topology, and thus the quality of provided solutions significantly varies according to the initial conditions.

2.3 The Social Graph: An Important Issue?

Since agents communication abilities are usually not restricted in allocation problems, it is legitimate to investigate the importance of such a parameter. Negotiation processes, which lead to optimal solutions according to complete communication abilities (i.e. based on complete contact networks), may only lead to solutions far from the optimum, when communications are restricted.

Proposition 1 (Social graph impact)**.** Independently of the objective function considered, the achievement of optimal resource allocations cannot be guaranteed if a restricted social graph is considered.

Proof. Let us prove this proposition using a counter-example, based on a population of 3 agents $\mathcal{P} = \{a_0, a_1, a_2\}$ and a set of 3 resources $\mathcal{R} = \{r_1, r_2, r_3\}$, where the aim is to maximize the utilitarian welfare. The agents preferences and the social graph with the initial resource allocation are described in Figure 1. For instance, agent a_1 associates the utility values 1 with resources r_1 and r_3. Let us consider a topology where agent a_1 can communicate with agents a_0 and a_2 while they can only negotiate with him. The initial allocation is $A = [\{r_1\}\{r_2\}\{r_3\}]$.

Table 1: Example of agents preferences.

Population $\mathcal{P}$	Resource Set $\mathcal{R}$		
	r_1	r_2	r_3
a_0	3	1	9
a_1	1	4	1
a_2	10	2	3

Agents only perform transactions increasing their own utility. According to such conditions and to the social graph, no transaction can be performed. Only two exchanges are possible, but both lead to a decrease of the individual welfare of at least one participant. The exchange of r_1 and r_2, or the exchange of r_2 and r_3 penalizes both participants. However, the current allocation is suboptimal. The exchange of r_1 and r_3, which leads to an increase of both participants' utility, is not possible since agent a_0 and agent a_2 cannot communicate. Hence, restrictions on agents communication abilities may prevent the achievement of optimal solutions. □

Proposition 2 (Negotiation order)**.** Independently to the objective function which is considered, the order in which agents negotiate with each other can prevent the achievement of optimal resource allocations.

Proof. Let us prove this proposition using a counter-example, based on a population of 3 agents $\mathcal{P} = \{a_0, a_1, a_2\}$ and a set of 3 resources $\mathcal{R} = \{r_1, r_2, r_3\}$, where the aim is to maximize the utilitarian welfare. The agents preferences are described in Table 2.Let us consider a topology where agent a_1 can communicate with agents a_0 and a_2 while they can only negotiate with him. The initial allocation is $A = [\{r_1\}\{r_2\}\{r_3\}]$.

Table 2: Example of agents preferences.

Population $\mathcal{P}$	Resource Set $\mathcal{R}$		
	r_1	r_2	r_3
a_0	2	10	4
a_1	5	3	9
a_2	2	7	1

Let us assume that agent a_1 initiates a negotiation. Depending on which neighbor the initiator selects to negotiate first, the negotiation process can end with sub-optimal allocations instead of optimal ones. If agent a_1 first chooses agent a_0 as partner, the exchange leads to a sub-optimal allocation from which the negotiation process cannot leave. However, if agent a_2 is selected first, the negotiation process ends on a socially optimal allocation. Hence, the optimum can only be achieved using a specific order of negotiation. □

Thus, the social graph represents an important issue since its topology may prevent the achievement of optimal allocations in practice. The influence of the contact network on the efficiency of negotiation processesmust not be omitted as it has been done in former studies.

3 SOLVING APPROACH CHARACTERISTICS

3.1 Transaction

During negotiation processes, the resource allocation evolves step by step by means of local transactions among agents. Only bilateral transactionsare considered in this paper. A transaction can be viewed as the association of the two participants' offer. It is a pair $\delta_{a_i}^{a_j}\langle u,v\rangle = (\rho_{a_i}^{\delta}, \rho_{a_j}^{\delta})$, where the initiator a_i offers a set $\rho_{a_i}^{\delta}$ of u resources and its partner a_j offers a set $\rho_{a_j}^{\delta}$ of v resources.

This representation can model transactions from any class, like the ones described in (Sandholm, 1998), using restrictions on the number of offered resources. During a *gift*, the initiator offers one resource and its partner provides nothing: it is then equivalent to a $\langle 1,0\rangle$-deal.

3.2 Acceptability Criterion

The acceptability criterion is used to determine whether a deal is profitable or not. The individual rationality is the most widely used criterion in the literature. It specifies that agents can only accept transactions (transforming their bundle of resources $\mathcal{R}_{a_i}$ into $\mathcal{R}_{a_j}$) increasing their individual welfare:$u_{a_i}(\mathcal{R}_{a_j}) > u_{a_i}(\mathcal{R}_{a_i})$.

With respect to the social criterion, agents accept transactions (changing the initial allocation A into A') that will not harm the society $sw(A') > sw(A)$

The sociability is a criterion centered on the social welfare value, which is a global notion. This value can only be determined thanks to the welfare of all agents. Such conditions cannot be satisfied since agents have only local information. However, to know the evolution of the welfare value is sufficient to determine if a transaction penalize the society. These computations can be restricted to the local environment of agents, by considering the remaining population as a constant.

3.3 Agent Behavior

Behaviors define agents from an external point of view. They describe how agents interact with each other, i.e. how they negotiate. During a negotiation, each agent makes and receives offers, checks their acceptability according to its own criterion. If a transaction is acceptable for every participant, it is performed. Otherwise, agents have to decide who has to modify its offer according to their behavior, and thus the negotiation continues.

Let us assume that agent $a_i \in \mathcal{P}$ initiates a negotiation and proposes an offer to of its partner $a_j \in \mathcal{N}_{a_i}$ previously selected. Both offers correspond to a bilateral transaction $\delta_{a_i}^{a_j}$. If both agents consider this transaction acceptable, it is performed. However, if one participant rejects the offer, three alternatives can then be considered:

- agent a_i gives up and ends the negotiation;
- agent a_i changes the selected partner;
- agent a_i changes its offer or asks to change its partner's offer.

Determining the order of these actions is an important issue. Many behaviors have been implemented and tested, but only the most efficient one is presented here. Agents always sorts the list of possible subsets they can offer according to their preferences. The initiator can then offer the least penalizing subset first. The initiator $a_i \in \mathcal{P}$ can change partners as well as offers during a negotiation process. Such an agent behavior is called *frivolous flexible*.

According to this behavior, if an acceptable transaction exists somewhere in the neighborhood, it will necessarily be identified. The neighborhood should be shuffled between two negotiations in order to modify the order in which neighbors are considered, and thus avoid a bias.

4 SIMULATIONS AND PROTOCOL

Simulations are characterized by the number of agents and by the mean number of resources per agent. During the experiments presented in this paper, 50 agents are negotiating 250 resources according to different settings. Agents can be either *rational* or *social*. Agents negotiate according to a negotiation policy, which is characterized by the size of agents' offers: $\langle 1,1 \rangle$ means that agents can only perform swaps whereas "up to $\langle 2,2 \rangle$" means that agents can propose up to two resources. It can also be explicitly written as: $\mathcal{T} = \{\langle 1,0\rangle, \langle 0,1\rangle, \langle 1,1\rangle, \langle 2,1\rangle, \langle 1,2\rangle, \langle 2,2\rangle\}$. Simulations are performed on social graphs that belong to different classes: complete, grids, Erdős-Rényi and small worlds.In this study, the link probability p varies from 0.05 up to 1.0. Each simulation is iterated 100 times from different initial resource allocations randomly generated, in order to evaluate the topological sensitivity. Utility functions and initial resource allocations are randomly generated according to a uniform probability distribution.

5 BILATERAL NEGOTIATIONS

This section is dedicated to the evaluation of negotiation processes according to two welfare notions. First, for each welfare notions, the efficiency is evaluated, by a comparison with the optimal social value, as well as the topological sensitivity. Tables 3 and 4 presents the efficiency of negotiation processes based on different negotiation policy and on different classes of social graphs. These tables contain the proportion of the optimal welfare value that can be achieved (left-side of the cells). The greater is the proportion, the closer to optima are the resulting allocations. The deviation (right-side of the cells) shows the proportion according to which may vary the solution quality. For instance, in Table 3, negotiation processes based on a grid where rational agents negotiate using $\delta\langle 1,1 \rangle$ transactions only end on social values representing 79.0% of the optimumDepending on the initial resource allocation, the welfare value achieved may vary of 1.6%.

Then, the impact of the graph connectivity is evaluated. The topology of a contact graph greatly affects the resource traffic and the negotiation efficiency. The larger are agent neighborhoods, the denser are social graphs, and the easier is the resource traffic. The probability p for a link to exist between nodes from any pair can be modified. Figures 1 to 2 show the evolution of the welfare value in time.

5.1 Utilitarian Case

Independently of the contact network's topology, rational negotiation processes always lead to weaker allocations than social negotiation processes. The restrictive character of the acceptability criterion affects the quality of the provided solution. When considering complete social graphs, different negotiation policy always lead to optimal resource allocations. However, the use of large offers leads to important additional costs.

Negotiation processes lead to allocations associated with up to 98.9% of the optimal welfare value when Erdős-Rényi graphs are considered. Only 91.4% of the optimum is achieved when small-worlds are considered. In an Erdős-Rényi graph, the probability for a link to exist between any pair of nodes

Table 3: Utilitarian efficiency (%) and its deviation (%) according to the class of social graphs.

Social graph class	Rational policy $\langle 1,1\rangle$		Rational policy up to $\langle 2,2\rangle$		Social policy $\langle 1,0\rangle$		Social policy $\langle 1,1\rangle$		Social policy up to $\langle 1,1\rangle$		Social policy up to $\langle 2,2\rangle$	
Full	96.6	0.3	97.0	0.2	100	0	98.3	0.2	100	0	100	0
Grid	79.0	1.6	81.3	1.3	86.2	0.9	85.3	1.1	86.1	0.9	86.1	0.9
Erdős-Rényi	94.8	0.5	95.0	0.4	98.9	0.1	97.1	0.2	98.9	0.1	98.9	0.1
Small World	80.8	2.0	84.8	1.3	91.4	0.8	90.0	1.0	90.2	0.8	90.3	0.8

is constant, while in small-worlds, the larger is the number of neighbors, the higher is the probability to link this agent. Many agents have only one neighbors, and the resource traffic is unequally distributed. Then, bottlenecks, i.e., agents that block the resource circulation, may appear. When grids are considered, social negotiation processes achieve up to 86.2% of the optimum. A weak mean connectivity handicaps the resource traffic and hence the achievement of socially efficient allocations.

The more restricted are social graphs, the weaker is the negotiation efficiency, and the higher is the deviation. In all cases, the standard deviation observed among the social values achieved remains small. It means that when the utilitarian welfare is considered, the topology has not a significant impact for a given class. The deviation is higher with rational negotiations since they restrict more the resource traffic, which then influences on the solution quality. The more restricted is the resource traffic, the higher is the standard deviation, and thus more important become the initial resource allocation.

Figure 1 shows the impact of the social graph connectivity on the efficiency within a population negotiating using social gifts. It represents the evolution of the utilitarian welfare value in time.

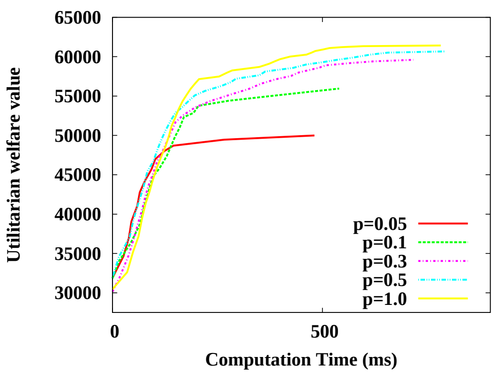

Figure 1: Utilitarian value vs. computation time according to the mean connectivity.

This figure shows that a weak linking probability, which corresponds to small neighborhoods, leads to a welfare value far from the optimum. The gradual increase of the probability p leads to larger welfare values and to more time-consuming negotiations. Larger neighborhoods facilitate the resource circulation by offering a larger number of possible transactions to all agents. The impact becomes really significant when $p < 0.3$. Above this threshold, the resource circulation is sufficient to achieve socially interesting allocations.

5.2 Egalitarian Case

Table 4 shows that, generally, negotiations among rational agents achieve unfair allocations. Indeed, rational negotiations end quite far (only 20%) from the optimal welfare value. The standard deviation is also very important (up to 73% of deviation on small-worlds). Thus, the rationality criterion is definitively not well-adapted to solve egalitarian problems efficiently. It restricts the set of possible transactions too much and throws negotiation processes into local optima. Generosity is hence an essential feature in order to achieve fair allocations.

Even using on complete graphs, no social negotiation policy can guarantee the achievement of egalitarian optima. Whereas social gifts are well adapted to the solution of utilitarian problems, they do not suit to egalitarian negotiations. Only 78.5% of the optimum can be achieved in the best cases. Indeed, after a finite number of transactions, agents can not give any additional resource without becoming poorer than their partners. Negotiations based on social swaps lead to severely sub-optimal resource allocations with an efficiency of at most 24.1% on complete social graphs. The inherent constraints of swap transactions prevent the modification of the resource distribution, which penalizes a lot egalitarian negotiations. When both gifts and swaps are allowed, the negotiation efficiency is really close to the optimum. Larger bilateral transactions improve only a little the fairness among agents, but are much more expensive to determine.

Social graphs of weaker mean connectivity like grids lead negotiation processes to socially weaker allocations. When small-worlds are considered, the resource traffic is restricted by the large number of agent leaves, leading to a larger deviation. Indeed, some resources may be trapped in the bundle of agent leaves.

Similarly to utilitarian negotiations, Figure 2

Table 4: Egalitarian efficiency (%) and its deviation (%) according to the class of social graphs.

Social graph class	Rational policy ⟨1,1⟩		Rational policy up to ⟨2,2⟩		Social policy ⟨1,0⟩		Social policy ⟨1,1⟩		Social policy up to ⟨1,1⟩		Social policy up to ⟨2,2⟩	
Full	19.3	62.9	20.8	73.9	78.5	1.8	24.1	28.7	99.9	0.3	99.9	0.3
Grid	13.9	71.3	14.6	80.2	66.2	4.1	23.6	29.6	80.2	1.8	80.6	1.7
Erdős-Rényi	17.4	71.9	20.2	76.8	77.3	2.2	23.8	27.3	96.1	6.8	96.6	6.5
Small World	13.1	73.0	13.9	77.5	63.8	10.4	23.4	27.8	78.1	9.4	78.2	10.5

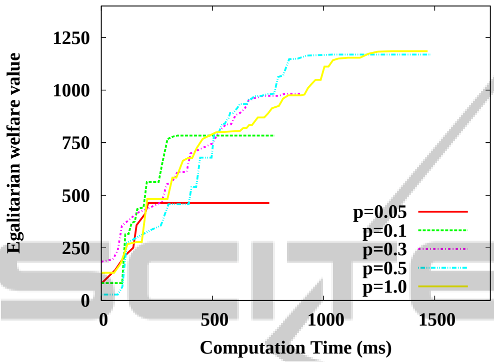

Figure 2: Egalitarian value vs. computation time according to the mean connectivity.

shows that a large linking probabilityleads to larger welfare value. Indeed, larger neighborhoods facilitate the resource circulation by offering larger numbers of possible transactions to all agents. The impact of the connectivitybecomes really significant below $p \leq 0.5$. Thus, the impact of the mean connectivity in a contact graph is more important in a egalitarian society than in a utilitarian one.

6 CONCLUSIONS

Many applications can be represented by an allocation problem but only a limited number of studies focus on the mechanism require to achieve the greatest results in a realistic context. Centralized solving techniques may be really inefficient, due to scalability issues or unadapted as soon as restrictions on agent communications are considered. Studies investigating distributed methods remain often theoretical and do not consider plausible assumptions. Indeed, agents are omniscient most of the time and are able to negotiate with all agents within the population. Nevertheless, such an ideal context is not satisfied in many real life applications.

In this paper, we propose a distributed mechanism based on local interactions, leading a negotiation process to socially optimal allocations, according to a more realistic context. Agents have only limited perceptions and knowledge. They have to reorganize themselves the resource allocation using local transactions, achieving socially optimal allocations (or socially close allocations if the need arises) by emergence. All kinds of contact network can be used to restrict agents' communication abilities. We identify the characteristics favoring and penalizing the negotiation efficiency according to different negotiation settings and different welfare notions. We show that negotiations between rational agents (widely used in literature) are not efficient, and we provide the negotiation settings to use in order to achieve allocation that are optimal for the society when four different welfare notions are considered.

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