IMPROVED COMPRESSED GENETIC ALGORITHM: COGA-II

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Keywords: Genetic algorithm, Multi-objective optimization, Objective compression.

Abstract: This paper presents an improved version of compressed objective genetic algorithm to solve problems with a large number of objectives. The improved compressed objective genetic algorithm (COGA-II) employs a rank assignment for the screening of non-dominated solutions that best approximate the Pareto front from vast numbers of available non-dominated solutions. Since the winning non-dominated solutions are heuristically determined from the survival competition, the procedure is referred to as a winning-score based ranking mechanism. In COGA-II, an m-objective vector is transformed to only one criterion, the winning score of which assignment is improved from that of the previous version, COGA. COGA-II is subsequently benchmarked against a non-dominated sorting genetic algorithm II (NSGA-II) and an improved strength Pareto genetic algorithm (SPEA-II), in seven scalable DTLZ benchmark problems. The results reveal that for the closeness to the true Pareto front COGA-II is much better than NSGA-II, and SPEA-II. For diversity of solutions, the diversity of the solutions by COGA-II is comparable to that of SPEA-II, while NSGA-II has poor diversity. COGA-II can also prevent solutions diverging from true Pareto solutions that occur on NSGA-II and SPEA-II for problems with more than 4 objectives. Thus, it can be concluded that COGA-II is suitable for solving an optimization problem with a large number of objectives.

1 INTRODUCTION

In multi-objective optimization, an increase in the number of conflicting objectives significantly raises the difficulty level in multi-objective optimization problems (Deb et al., 2005). The total number of non-dominated solutions inevitably explodes owing to the way that a non-dominated solution is defined. When two candidate solutions are compared, a solution **a** does not dominate another solution **b** unless all objectives from **a** satisfy the domination condition. With a large number of objectives, the chance that two solutions cannot dominate one another is unsurprisingly high. Since a genetic algorithm is only capable of reporting a finite set of solutions, a large number of possible non-dominated solutions have to be screened for a good approximation of Pareto front (Pierro et al., 2007, Purshouse and Fleming, 2007)

This investigation also focuses on the screening procedure that assigns "preference" levels to non-dominated solutions in view that the higher the preference level of non-dominated solutions, the better the Pareto front approximation. A compressed-objective genetic algorithm (COGA) is an MOEA that successfully integrates this procedure into the multi-objective search framework (Maneeratana et al., 2006). The introduction of two conflicting preference objectives during the

Boonlong K., Chaiyaratana N. and Maneeratana K..
IMPROVED COMPRESSED OBJECTIVE GENETIC ALGORITHM: COGA-II.
DOI: 10.5220/0003086700950103
In *Proceedings of the International Conference on Evolutionary Computation* (ICEC-2010), pages 95-103
ISBN: 978-989-8425-31-7

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Here, a similar screening approach is introduced. Instead of two conflicting preference objectives, only one preference objective, a winning score, is implemented. The winning score directly influences the rank and hence survival chance of a solution. With only one preference objective, the problem representation transformation is simpler than that in COGA. New MOEA with the winning score based ranking mechanism is thus proposed: an improved compressed-objective genetic algorithm (COGA-II). COGA-II involves the assignment of preference levels to non-dominated solutions for selection of winning solutions that best describe the Pareto fronts in problems with a large number of objectives. It is benchmarked against the non-dominated sorting genetic algorithm II, NSGA-II (Deb et al., 2002) and the improved strength Pareto evolutionary algorithm, SPEA-II (Zitzler et al., 2002). The chosen test suites are DTLZ1-7 (Deb et al., 2005) with three to six objectives. The organization of this paper is as follows. In section 2, the proposed algorithm, COGA-II is described. In section 3, the performance evaluation criteria, covering an existing measure for closeness to the Pareto front (Zitzler et al., 2000) and a modified index for solution distribution, are explained. Next, the results from benchmark trials against NSGA-II, and SPEA-II are compared in section 4 with the conclusions in section 5.

2 IMPROVED COMPRESSED-OBJECTIVE GENETIC ALGORITHM

Optimization problems usually arise when limited resources are available for existing demands. If multiple conflicting objectives are required in the problem formulation, the problem is multi-objective. Various techniques have been proposed for solving these multi-objective problems. Among these, the genetic algorithm has been established as one of the most widely used methods for multi-objective optimization (Li and Zhang 2006, Igel et al. 2007, Zhou et al. 2007). Due to the parallel search nature of the algorithm, the approximation of multiple optimal solutions – the Pareto optimal solutions, comprised of non-dominated individuals – can be effectively executed. The performance of the algorithm always degrades as the search space or problem size gets bigger. An increase in the number of conflicting objectives has also significantly raised the difficulty level (Deb et al., 2005). Thus, the non-dominated solutions may deviate from the true Pareto front; the coverage of the Pareto front by the solutions generated may be affected.

A number of strategies have been successfully integrated into genetic algorithms to solve these problems, including a direct modification of selection pressure (Fonseca and Fleming, 1993), (Srinivas and Deb, 1994) and elitism (Zitzler and Thiele, 1999), (Keerativuttitumrong et al., 2002). Although they have been proven to significantly improve the search performance of genetic algorithms, virtually all reported results deal with only few objectives. In reality, the possibility that a candidate solution is not dominated always increases with objective numbers, leading to an explosion in the total number of non-dominated solutions. This difficulty stems from the way that a non-dominated solution is defined. By domination definition, a candidate solution **x** is dominated by another solution **y** if and only if (a) all objectives from **y** are either better than or equal to the corresponding objectives from **x** and (b) at least one objective from **y** is better than the corresponding objective from **x**. Hence, if one single objective from **y** does not satisfy the conditions, **y** would not dominate **x**. In a problem with a large number of objectives, the chance that two solutions cannot dominate one another is inevitably high. A genetic algorithm must be able to pick out a well chosen solution set from a vast number of non-dominated solutions in order to successfully approximate the Pareto front.

In order to properly approximate such Pareto fronts, a number of non-dominated solutions must be excluded from the search target. One possible technique is to assign different preference levels to non-dominated solutions under consideration. It is hypothesized that a set containing highly preferred solutions would reflect a close approximation of the true Pareto front. The original compressed-objective genetic algorithm (COGA) (Maneeratana et al., 2006) employed two conflicting criteria in the preference assignment. This can be viewed as a transformation from an *m*-objective problem to a two-objective problem during the survival competition between two non-dominated solutions. This thesis will present the improved version of compressed-objective genetic algorithm. The improved compressed objective genetic algorithm (COGA-II) is quite different from the original COGA in which three-or-more objectives is transformed to only one preference objective during the survival competition of two non-dominated solutions. The preference objectives of the COGA are winning score and vicinity index, in the other hand, the COGA-II has only one preference objective, winning score. Although the COGA-II employs winning score as the COGA, its winning score assignment is not the same as that of the original algorithm. The winning score assignment of and the main procedure of the COGA-II will be described in the following topics.

2.1 Winning Score Assignment

The winning score is heuristically calculated from the numbers of superior and inferior objectives between a pair of two non-dominated individuals. Let $\sup_{ij}$, inj_{ij} and eq_{ij} be the number of objectives in the individual *i* which are superior to the corresponding objectives in the individual *j*, the number of objectives in *i* which are inferior to that in *j*, and the number of objectives in *i* which are equal to that in *j*, respectively. For an objective *k* in an m-objective problem, ρ_{ijk} is defined as

$$\rho_{ijk} = \begin{cases} (\sup_{ij} + \inf_{ij})/2\sup_{ij}, \text{if } i \text{ is superior to } j \\ (\sup_{ij} + \inf_{ij})/2\inf_{ij}, \text{if } i \text{ is inferior to } j \\ 1, \quad \text{if } i \text{ is equal to } j \end{cases} \quad (1)$$

The equation yields

$$\sum_{k=1}^{m} \rho_{ijk} = m \quad (2)$$

for any individual pairs *i* and *j*. The manner at which ρ_{ijk} deviates from one depends on the ratio between the numbers of superior and inferior objectives during each solution comparison. This reflects the dependency and correlation among objectives of interest.

Next, the summation of ρ_{ijk} for the objective *k* over all possible individual pairs is given by

$$V_k = \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \rho_{ijk} \quad (3)$$

where *N* is the total number of non-dominated individuals. The winning score of the individual *i* in a non-dominated individual set or WS_i is given by

$$WS_i = \sum_{j=1}^{N} w_{ij}, \text{ where } w_{ij} = \sum_{j=1}^{m} WF_k q_{ijk}, \quad \text{and } WF_k = V_k / \sum_{l=1}^{m} V_l \quad (4)$$

The q_{ijk} is a competitive score. $q_{ijk} = 1$ if the objective *k* of the individual *i* is superior to that of the individual *j* but $q_{ijk} = -1$ if the objective *k* of *i* is inferior to that of *j*. Obviously, $q_{ijk} = 0$ if the objective *k* from both individuals is equal. The w_{ij} is a weighted sum of competitive scores from all objectives. For any individual pair *i* and *j*, $w_{ji} = -w_{ij}$, $w_{ii} = 0$ and $-1 < w_{ij} < 1$. This leads to $-N < WS_i < N$. It is noted that the winning score assignment is not an objectives weighted sum method such as Soylu and Köksala (2010), Zhang and Li (2007). The assignment is used for only non-dominated solutions.

It is required that an individual with a high winning score must be close to the true Pareto front. This requirement is satisfied if the relationship between the winning score and the distance from a non-dominated individual to the true Pareto front in the objective space can be described by a decreasing function. This relationship can be identified using the following multi-objective problem scenario. Consider a multi-objective minimization problem in which the *i*-th objective or f_i equals to the decision variable x_i where $x_i \in [0,1]$. This problem has only one true optimal solution with all objectives equal to zero. First, a random solution is generated and placed in an arc-hive. Another random solution is then generated and compared with the archival solution. The archive is appended if the new solution is neither dominated by nor a duplicate of the existing archival solution. At the same time, if the new solution dominates the archival solution, the dominated solution will be expunged from the archive. The process of creating random solutions and archive updating is repeated until the archive is full.

Figure 1 displays the relationship between the winning score of each solution and its distance to the true optimal solution for problems with a different number of objectives. The archive size is set to 200 while the illustrated problems contain 3–6, 8, 10, 15 and 20 objectives. In Figure 1, the relationship can be described by a decreasing function especially when the number of objectives is large.

Table 1 shows the average of percent correct from comparison of all pairs of 200 non-dominated solutions, of which the number of possible solution pairs is equal to C(200,2) = (200!)/(198!2!) = 19,900, from 30 runs. For any solution pair comparison, the comparison by winning score is correct if the solution with a higher winning score is closer to the true Pareto solution than the other. The data in Table 1 show that winning score can accurately identify different levels of the non-dominated solution with more than 78% correctness of comparison and its effectiveness is increased when the number of objectives increases. Hence, the winning score can be used to estimate the quality of a solution in a non-dominated solution set.

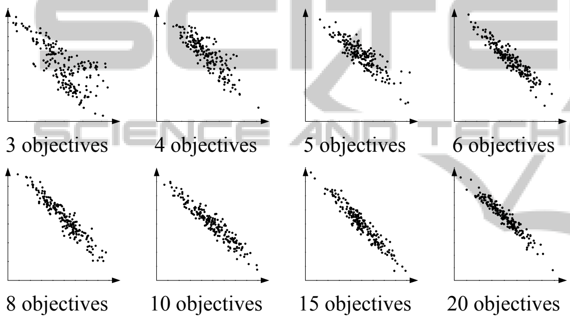


Figure 1: Relationship between the winning score (horizontal axis) and the distance from the true optimal solution (vertical axis).

Table 1: Average of Percent Correct by Winning Score Comparison of Various Numbers of Objectives.

No. of Objectives	3 Obj	4 Obj	5 Obj	6 Obj
% Correct by Winning Scores	78.08%	80.71%	85.60%	87.69%
No. of Objectives	**8 Obj**	**10 Obj**	**15 Obj**	**20 Obj**
% Correct by Winning Scores	89.82%	90.85%	91.55%	91.54%

2.2 Rank Assignment

With the use of the winning score, a rank value can be assigned to an individual in COGA-II as follows.

1. Evaluate the winning score of each individual in the non-dominated individual set.
2. Find extreme individuals among N non-dominated individuals. The number of extreme individuals is equal to E, which does not exceed $2m$ (two individuals with the minimum and maximum values of each objective).
3. Sort E extreme non-dominated individuals in descending order of the winning score. The firstly sorted individual is assigned rank 1. The secondly sorted individual is assigned rank 2 and so forth. Therefore, the lowest rank of extreme individuals is E. In the same way, $N - E$ non-extreme non-dominated individuals are also sorted in descending order of the winning scores. However, the ranks of these non-dominated individuals vary from $E + 1$ to N. This rank assignment guarantees that a rank of an extreme individual is always higher than that of a non-extreme individual.
4. Assign a rank to each dominated individual. The rank of a dominated individual is given by N plus the number of its dominators. The addend N ensures that the rank of a non-dominated individual is higher than that of a dominated individual.

2.3 Main Procedure

The main algorithm for COGA-II is as follows.

1. Generate an initial population P_0 and an empty archive A_0. Initialize the generation counter $t = 0$.
2. Merge the population P_t and the archival population A_t together to form the merged population R_t. Then assign ranks to individuals in R_t.
3. Put all N non-dominated individuals from R_t. into the archive A_{t+1}. If N is larger than the archive size Q, truncate the non-dominated individual set using the operator described in the next sub-section. On the other hand, if $N < Q$ then $Q - N$ dominated individuals with the least number of their dominators are filled to the archive A_{t+1}.
4. Perform binary tournament selection with replacement on archival individuals in order to fill the mating pool with special attention to both the rank and the summation of distances between an individual i or SDT_i in the archive and all individuals in the current mating pool.
   At the beginning, SDT_i is set to zero. In the first selection iteration, two individuals from the archive are randomly picked; the individual with higher rank will be selected. Otherwise, if their ranks are equal, one individual is selected at random. After the selection, SDT_i of any remaining individual i in the archive is updated by adding its current value with the distance between the individual i and the selected individual. In the next selection iteration, the winning individual is also determined from the rank. However, if the ranks of two competing individuals are equal, the individual with more

SDT is selected. Subsequently, SDT_i of an individual *i* in the archive is again updated. The binary tournament selection is then repeated until the mating pool is fulfilled.

5. Apply crossover and mutation operations within the mating pool. Then place the offspring into the population P_{t+1} and increase the generation counter by one ($t \leftarrow t + 1$).
6. Go back to step 2 until the termination condition is satisfied. Report the final archival individuals as the output solution set.

A truncation operator for maintaining individuals in the archive is introduced next.

2.4 Truncation

Only the winning score assignment may does not guarantee diversity of solutions. The truncation is therefore used to maintain the diversity of an archive of any generation. With the use of the truncation operator, *Q* non-dominated individuals are extracted from *N* available individuals and placed into the archive A_{t+1}. The truncation operation is as follows

1. Find extreme individuals among *N* non-dominated individuals. The number of extreme individuals is equal to *E*, which does not exceed $2m$. If there is only one individual with the minimum/maximum value of objective *k*, this is the extreme individual. In contrast, if there are multiple individuals with the minimum/maximum value of objective *k*, an individual is chosen at random to be the extreme individual.
2. Place all *E* extreme individuals in the archive. Set the number of archival individuals $L = E$ and remaining individuals $R = N - E$. Then, calculate the Euclidean distance d_i^{RL} between the individual *i* in *R* and its nearest neighbor in *L*. If there are two-or-more individuals in L that have the same objective vector as that of the individual j in *R*, d_j^{RL} is set by

$$d_j^{RL} = -(c_j - 1)\sqrt{m} \tag{5}$$

3. Select $Q - L$ individuals with the highest values of d_i^{RL} from *R*. Then, move the candidate with the highest winning score among these $Q - L$ selected individuals to the archive. If there are more than one individual with the highest winning score, the chosen candidate is the one with the highest value of d_i^{RL}.
4. Increase the counter for the number of archival individuals ($L \leftarrow L + 1$) and decrease the counter for the number of remaining individuals ($R \leftarrow R - 1$). Then, update d_i^{RL} for the remaining individual *i*.
5. Go back to step 3 until the archive is fulfilled.

3 PERFORMANCE EVALUATION CRITERIA

Good non-dominated solutions should be close to the true Pareto front and uniformly distributed along the front. From many available performance metrics for MOEAs evaluation (Deb and Jain 2003), two performance metrics are used here: the average distance between the non-dominated solutions to the true Pareto optimal solutions, or M_1 metric (Zitzler et al., 2002) and a newly proposed clustering index *CI* for the description of solution distribution.

The clustering index *CI* is a diversity metric which indicates the distribution of non-dominated solutions on a hyper-surface. The proposed *CI* differs from the grid diversity metric (Deb and Jain, 2002) such that the calculation of *CI* does not require a grid division of each objective, at which the suitable number of divided grids is difficult to identify.

For a non-dominated solution set *A* of size *Q*, *CI* of *A* is evaluated from a derived non-dominated solution set *A*' of size *Q*' where $Q' >> Q$. The evaluation of *CI* from *A* is as follows from which the range of *CI* is between 0 and 1 where a higher *CI* value indicates a better solution distribution. The *CI* evaluation is as follows.

1. Copy all solutions in set *A* to the derived solution set *A*'.
2. Randomly select the first parent p_1 and parent p_2 from *A* and *A'* respectively.
3. Perform crossover and mutation operations on both parents to obtain two children c_1 and c_2. Then, calculate objectives of both children.
4. Check whether each child neither dominates nor is dominated by any solutions in *A*. If both children do not satisfy this condition, go back to step 3. If only one child satisfies this condition, put it in *A*'; if both children satisfy the condition, randomly pick a child and put it in *A*'.
5. Update the number of members in *A*'. If *A*' is not completely filled, go back to step 2. Otherwise, go to step 7.
6. Divide *Q*' solutions in *A*' into *Q* groups by a clustering method [4]. Then, find the number of groups *G* that contains the first *Q* solutions, which are identical to solutions in *A*. *CI* of *A* is equal to G/Q.

In this study, an acceptable value for *Q*', which should be large as possible, is determined empirically from all bench-mark problems and is subsequently set at 4,000.

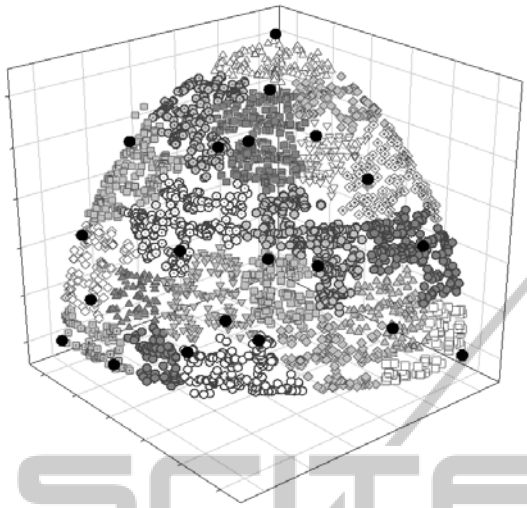


Figure 2: An example of *CI* evaluation of non-dominated solutions from the three-objective DTLZ2.

The demonstration of *CI* calculation is given in the following example in Figure 2. Let *A* contain 20 non-dominated solutions which are represented by black circles of a three-objective DTLZ2 problem as shown in the figure. After the derivation of *A*' with 1,000 non-dominated solutions, solution clusters are created. Each cluster is illustrated in the figure using a unique marker. *CI* is equal to the number of clusters containing solutions from *A*, 16, divided by the total number of solutions in *A*, which is equal to 20. Thus, *CI* of *A* is equal to 0.80.

4 RESULTS AND DISCUSSIONS

COGA-II was compared against NSGA-II and SPEA-II by the benchmark problems DTLZ1-7 with 3-6 objectives. The parameter setting is shown in Table 2. The average (Avg.) and standard deviation (SD) of M_1 and *CI* are shown in Table 3 to Table 10. From Table 3 to Table 10, COGA-II outperforms NSGA-II and SPEA-II in terms of M_1. This performance superiority is clearer with larger numbers of objectives. In addition, the M_1 performance of NSGA-II and SPEA-II is very close to one another in the three-objective DTLZ1, DTLZ2, DTLZ4 and DTLZ7 problems. However, NSGA-II is better than SPEA-II once the number of objectives increases. From the *CI* values, the performance of COGA-II and SPEA-II is quite similar. In contrast, the performance of NSGA-II is significantly worse than that of them.

Table 2: Parameter setting for NSGA-II, SPEA-II, and COGA-II.

Parameter	Setting and Values
Test Problems	DTLZ1-7 (Deb et al, 2005) with $\|x_m\| = 10$
Number of Objectives	3-6
Chromosome coding	Real-value chromosome
Crossover method	SBX crossover (Deb, 2001) with probability = 1.0
Mutation method	Variable-wise polynomial mutation (Deb, 2001) with probability = 1/number of decision variables.
Population size	100
Archive size (except NSGA-II)	100
Number of generations	800
Number of repeated runs	30

Table 3: Summary of M_1 of DTLZ1-7 with 3 objectives.

Problem		NSGA-II	SPEA-II	COGA-II
DTLZ1	Avg	0.0184	0.0197	**0.0099**
	SD	0.0494	0.0392	0.0300
DTLZ2	Avg	0.0090	0.0089	**0.0033**
	SD	0.0018	0.0014	0.0006
DTLZ3	Avg	0.0102	0.0324	**0.0079**
	SD	0.0133	0.0521	0.0115
DTLZ4	Avg	0.0088	0.0094	**0.0026**
	SD	0.0015	0.0018	0.0008
DTLZ5	Avg	0.0012	0.0010	**0.0004**
	SD	0.0003	0.0003	0.0001
DTLZ6	Avg	0.0641	0.1269	**0.0369**
	SD	0.0168	0.0331	0.0097
DTLZ7	Avg	0.0163	0.0162	**0.0069**
	SD	0.0034	0.0026	0.0014

The number displayed in boldface is the best result while the underlined number is the second best result.

Table 4: Summary of M_1 of DTLZ1-7 with 4 objectives.

Problem		NSGA-II	SPEA-II	COGA-II
DTLZ1	Avg	228.41	343.97	**3.3139**
	SD	105.49	56.344	4.8817
DTLZ2	Avg	0.0395	0.0687	**0.0054**
	SD	0.0120	0.0201	0.0015
DTLZ3	Avg	351.18	316.23	**15.454**
	SD	59.885	49.012	10.236
DTLZ4	Avg	0.0416	0.1200	**0.0043**
	SD	0.0204	0.0276	0.0018
DTLZ5	Avg	1.5054	1.5696	**1.4583**
	SD	0.0643	0.0627	0.0751
DTLZ6	Avg	10.166	7.2514	**4.6022**
	SD	0.6209	0.4098	0.4689
DTLZ7	Avg	0.1076	0.1082	**0.0306**
	SD	0.0093	0.0136	0.0047

Table 5: Summary of M_1 of DTLZ1-7 with 5 objectives.

Problem		NSGA-II	SPEA-II	COGA-II
DTLZ1	Avg	836.17	956.50	**28.118**
	SD	89.932	68.417	24.311
DTLZ2	Avg	0.4600	1.3523	**0.0108**
	SD	0.0987	0.1092	0.0034
DTLZ3	Avg	843.19	1024.7	**254.63**
	SD	96.118	88.152	68.619
DTLZ4	Avg	1.2253	1.5949	**0.0054**
	SD	0.2204	0.0742	0.0017
DTLZ5	Avg	2.2099	2.3259	**2.1381**
	SD	0.0863	0.1088	0.0593
DTLZ6	Avg	14.370	15.287	**6.5050**
	SD	0.2809	0.1743	0.3127
DTLZ7	Avg	0.2236	0.3917	**0.0652**
	SD	0.0328	0.0590	0.0078

Table 6: Summary of M_1 of DTLZ1-7 with 6 objectives.

Problem		NSGA-II	SPEA-II	COGA-II
DTLZ1	Avg	1114.0	1230.98	**161.84**
	SD	62.783	23.460	78.768
DTLZ2	Avg	1.6093	2.2346	**0.0237**
	SD	0.1630	0.0308	0.0079
DTLZ3	Avg	1223.3	1674.4	**482.55**
	SD	74.211	64.004	57.373
DTLZ4	Avg	1.9653	2.2703	**0.0088**
	SD	0.0834	0.0268	0.0045
DTLZ5	Avg	3.2738	4.5426	**2.5996**
	SD	0.2849	0.1072	0.0794
DTLZ6	Avg	19.451	20.362	**10.495**
	SD	0.3376	0.2201	0.4016
DTLZ7	Avg	0.4441	0.9042	**0.0836**
	SD	0.0664	0.1348	0.0127

Table 7: Summary of *CI* of DTLZ1-7 with 3 objectives.

Problem		NSGA-II	SPEA-II	COGA-II
DTLZ1	Avg	0.5467	**0.8420**	0.8080
	SD	0.0411	0.0816	0.0551
DTLZ2	Avg	0.5880	**0.8967**	0.8440
	SD	0.0322	0.0234	0.0304
DTLZ3	Avg	0.5573	0.7673	**0.7900**
	SD	0.0264	0.0571	0.0574
DTLZ4	Avg	0.6153	**0.8847**	0.8473
	SD	0.0229	0.0249	0.0277
DTLZ5	Avg	0.7940	**0.9200**	0.8900
	SD	0.0267	0.0174	0.0217
DTLZ6	Avg	0.6533	0.7833	**0.8360**
	SD	0.0416	0.1088	0.0251
DTLZ7	Avg	0.5633	**0.8340**	0.8180
	SD	0.0282	0.0243	0.0307

Table 8: Summary of *CI* of DTLZ1-7 with 4 objectives.

Problem		NSGA-II	SPEA-II	COGA-II
DTLZ1	Avg	0.4053	0.7253	**0.7847**
	SD	0.0588	0.0226	0.0646
DTLZ2	Avg	0.5213	**0.8253**	0.8220
	SD	0.0352	0.0326	0.0250
DTLZ3	Avg	0.4720	0.7427	**0.7800**
	SD	0.0509	0.0314	0.0638
DTLZ4	Avg	0.5513	0.7973	**0.8067**
	SD	0.0415	0.0289	0.0192
DTLZ5	Avg	0.5007	0.7693	**0.7913**
	SD	0.0284	0.0291	0.0252
DTLZ6	Avg	0.4980	**0.7880**	0.7587
	SD	0.0300	0.0307	0.0249
DTLZ7	Avg	0.5400	**0.8187**	0.7747
	SD	0.0270	0.0270	0.0344

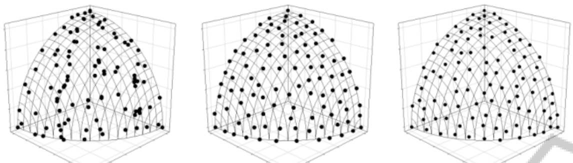
Table 9: Summary of *CI* of DTLZ1-7 with 5 objectives.

Problem		NSGA-II	SPEA-II	COGA-II
DTLZ1	Avg	0.4700	**0.7400**	0.7353
	SD	0.0358	0.0507	0.0557
DTLZ2	Avg	0.4367	0.7780	**0.8327**
	SD	0.0358	0.0351	0.0272
DTLZ3	Avg	0.4527	0.6813	**0.7033**
	SD	0.0427	0.0310	0.0976
DTLZ4	Avg	0.4827	0.7867	**0.8153**
	SD	0.0282	0.0270	0.0219
DTLZ5	Avg	0.4740	**0.7620**	0.7613
	SD	0.0321	0.0313	0.0292
DTLZ6	Avg	0.5053	**0.8880**	0.7607
	SD	0.0323	0.0175	0.0227
DTLZ7	Avg	0.5033	**0.7793**	0.7413
	SD	0.0209	0.0347	0.0373

Table 10: Summary of *CI* of DTLZ1-7 with 6 objectives.

Problem		NSGA-II	SPEA-II	COGA-II
DTLZ1	Avg	0.4740	**0.8353**	0.7627
	SD	0.0478	0.0358	0.0432
DTLZ2	Avg	0.4653	**0.8653**	0.8320
	SD	0.0426	0.0213	0.0316
DTLZ3	Avg	0.4413	**0.8047**	0.7420
	SD	0.0240	0.0371	0.0448
DTLZ4	Avg	0.5077	0.8120	**0.8147**
	SD	0.0233	0.0263	0.0273
DTLZ5	Avg	0.4340	**0.8420**	0.8113
	SD	0.0228	0.0167	0.0326
DTLZ6	Avg	0.4973	**0.8653**	0.7593
	SD	0.0221	0.0265	0.0353
DTLZ7	Avg	0.5157	0.7620	**0.7727**
	SD	0.0327	0.0291	0.0307

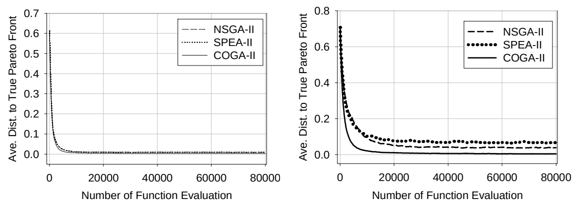
Figure 3 shows uniformity of solutions from a run of DTLZ4 with 3 objectives from all algorithms. Obviously, SPEA-II and COGA-II yeild more diversity of solutions than NSGA-II. Figure 3 also shows that solutions from COGA-II are quit well uniform distribution.



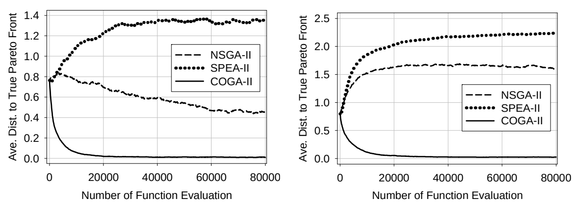
(a) NSGA-II, *CI* = 0.60 (a) SPEA-II, *CI* = 0.88 (c) COGA-II, *CI* = 0.87

Figure 3: Examples of solutions from a run of DTLZ4 with 3 objectives of (a) NSGA-II, (b) SPEA-II, and (c) COGA-II.

Graphs of average M_1 versus number of function evaluations from all 30 runs from two selected problems – DTLZ2 and DTLZ6 are shown in Figure 4 and Figure 5, respectively. All algorithms can search solutions that are close to the true Pareto front for the problems with 3 objectives with a close convergence rate. However for the problems with 4 objectives, COGA-II clearly can search for solutions with a better convergence rate. The performance of COGA-II very obviously improves the NSGA-II and SPEA-II when graphs of average M_1 versus number of function evaluations of the problems with 5-6 objectives are considered. Surprisingly, solutions searched by NSGA-II and SPEA-II diverged from the true Pareto front for the DTLZ2 with 6 objectives and DTLZ6 with 5 and 6 objectives if the number of function evaluations is increased.



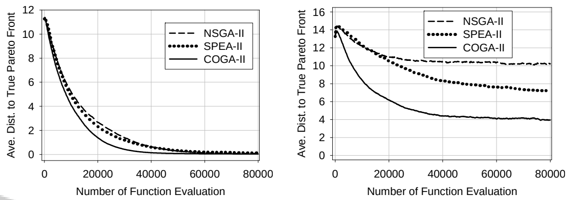
(a) DTLZ2 – 3 Objectives (b) DTLZ2 – 4 Objectives



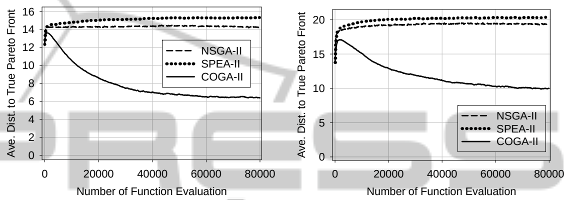
(c) DTLZ2 – 3 Objectives (d) DTLZ2 – 3 Objectives

Figure 4: Graphs of M_1 vs. Number of Function Evaluations of DTLZ2 with 3-6 objecitves.

Therefore solutions obtained by the algorithms are worse than solutions in the initial population. This shows that Pareto domination alone is not enough to solve problems with a large number of objectives.



(a) DTLZ6 – 3 Objectives (b) DTLZ6 – 4 Objectives



(c) DTLZ6 – 5 Objectives (d) DTLZ6 – 6 Objectives

Figure 5: Graphs of M_1 vs. Number of Function Evaluations of DTLZ2 with 3-6 objecitves.

5 CONCLUSIONS

This paper presents COGA-II which integrates preference notion into the process of non-dominated solution screening for Pareto front approximation in multi-objective problems with large number of objectives. A criterion for selecting a winner from the competition for survival between two non-dominated solutions is thus defined. The proposed ranking technique is referred to as a winning score based ranking mechanism. The effectiveness of COGA-II has been compared in benchmark trials against those of NSGA-II and SPEA-II with multi-objective DTLZ test problems. The performance evaluation criteria are an average distance between the non-dominated solutions and the true Pareto front (M_1) [0] and a new clustering index (*CI*) for solution distribution description. In overall, the results indicate COGA-II are superior to NSGA-II and SPEA-II in terms of the M_1 index while the solution distribution is comparable to that of SPEA-II. COGA-II can also prevent solutions diverge from true Pareto solution that occur on NSGA-II and SPEA-II for problems with 5-6 objectives. Thus it can conclude that COGA-II is suitable for solving an optimization problem with large number of objectives.

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