NONLINEAR CONSTRAINED PREDICTIVE CONTROL OF EXOTHERMIC REACTOR

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Abstract: Predictive method which allows applying constraints in the process of designing control system has wide practical significance. The method developed in the article consists of feedback linearization and linear quadratic control applied to obtained linear system. Employment of interpolation method introduces constraints of variables into control system design. The control algorithm was designed for a model of exothermic reactor, results illustrate its operation in comparison with PI control.

1 INTRODUCTION

The predictive algorithms have a wide industrial applications because of the simplicity of its operation and good features of regulation. One of important advantages of the predictive control is the possibility to impose the signal constraints in the process of designing the control law. In the practical applications it is convenient to use the linear models for the theory of them is well known.

First examples of the industrial use of the MPC applications had place in 1970's, but the idea was known earlier (Lee, Markus, 1967). One of the most important algorithms was the Dynamic Matrix Control (Cutler, Ramaker,1980) and Quadratic DMC (Garcia et al.,1989) with linear models. There appeared a number of articles with nonlinear models with the exact and suboptimal algorithms. The use of nonlinear models cause additional problems with finding global minimum and can have an effect on calculation time (Tatjewski, 2002). Adaptation of a controller with linearization around the working point may result in system instability (Dimitar et al., 1991), changes of variables have to be limited.

The aim of the work was to design an application used for control of an exothermic reactor with constraints, to propose use of feedback linearization for this nonlinear plant, present predictive control method solving problem of constraints(Poulsen et al., 2001) and its modification (Ziętkiewicz 2008) for changed reference signal.

2 EXOTHERMIC REACTOR

2.1 CSTR Model

The plant to be controlled is the Continuous Stirred Tank Reactor (CSTR). The structure of reactor is presented on figure 1. It contains tank, cooling jacket, inflow and outflow of both elements. It is assumed that, because of perfect mixing, there are no spatial gradients of parameters in the tank area.

The work of reactor is described by 3 differential equations. First equation (1) illustrates the mass balance,

$$V\frac{dC(t)}{dt} = \phi\left[C_i - C(t)\right] - VR(t), \qquad (1)$$

where $C(t)$ is the concentration of product measured in [kmol/m^3]. The second and the third equations (2,3) represent the balance of energy in the reactor,

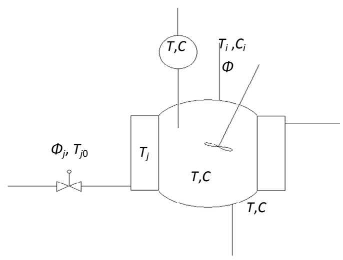

Figure 1: Model of exothermic reactor.

$$V\rho c_p\frac{dT(t)}{dt} = \phi\rho c_p\left[T_i - T(t)\right] - Q(t) + (-\Delta_i)VR(t), \qquad (2)$$

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and the balance of energy in the cooling jacket

$$v_j \rho_j c_{pj} \frac{dT_j(t)}{dt} - \varphi_j(t)\rho_j c_{pj}\left[T_{j0} - T_j(t)\right] + Q(t) \quad (3)$$

where T(t) is the temperature inside the reactor and $T_j(t)$ temperature in the cooling jacket, measured in Kelvin. $\phi_j(t)$ [m^3/h] represents cooling flow through the reactor jacket. Remaining equations represent

$$Q(t) = \alpha A_c \left[T(t) - T_j(t)\right] \quad (4)$$

- thermal energy in the process of cooling,

$$R(t) = C(t) k_0 e^{\frac{-E}{RT(t)}} \quad \text{- velocity of reaction.} \quad (5)$$

Constant values used in experiments are placed in the table 1.

Table 1: Constant values of CSTR model.

const.	value	const.	value
ϕ	1.13 [m^3/h]	T_{j0}	294.4 [K]
V	1.36 [m^3]	ρ_j	998 [kg/m^3]
C_i	8 [kmol/m^3]	c_{pj}	4186.8 [J/(kgK)]
ρ	801 [kg/m^3]	k_0	7.08*10^{10} [1/h]
c_p	3140.1 [J/(kgK)]	E	6.96*10^7 [J/kmol]
T_i	294.4 [K]	R	8314.3 [J/(kmolK)]
$(-\Delta_i)$	6.96*10^7 [J/kmol]	α_c	3.07*10^6 [J/(hKm^2)]
v_j	0.109 [m^3]	A_c	23.2 [m^2]

In the further parts of the paper the function of time will be omitted to simplify equations. The control signal will be denoted as $u = \phi_j(t)$ and the state variables $x_1=C(t)$, $x_2=T(t)$, $x_3=T_j(t)$. The system (1-3) can be describe by 3 equations:

$$\begin{aligned} \dot{x}_1 &= A_1 C_i - (A_1 + k_0 e^{-E/Rx_2}) x_1, \\ \dot{x}_2 &= A_1 T_i - (A_1 + B_1) x_2 + B_1 x_3 + C x_1 k_0 e^{-E/Rx_2}, \\ \dot{x}_3 &= B_2 (x_2 - x_3) + \frac{T_{j0} - x_3}{v_j} u, \end{aligned} \quad (6)$$

where $A_1 = \frac{\phi}{V}$, $B_1 = \frac{\alpha_c A_c}{V \rho c_p}$, $B_2 = \frac{\alpha_c A_c}{v_j \rho_j c_{pj}}$, $C = \frac{(-\Delta_i)}{\rho c_p}$.

2.2 Formulation of Control Problem

The objective of control is to make the temperature inside the reactor $T(t)$ track a desired trajectory $w(t)$ using the control signal u. The complete model with output signal can be described by (6) with defined output signal

$$y = x_2. \quad (7)$$

Furthermore the control signal is constrained

$$0 m^3/h \le \phi_j \le 2.5 m^3/h \quad (8)$$

3 FEEDBACK LINEARIZATION

The functions describing the considered system are smooth and have continuous derivatives of any required order in region $\Omega=\{(x_1, x_2, x_3)\epsilon R^2 | x_2>T_{j0}, x_3>T_{j0}\}$, which is the normal area of reactor operation. Since the relative degree is equal to 2 and the system order was equal to 3, the system has internal dynamic described by one equation. From (6) it takes form:

$$\dot{x}_1 = A_1 C_i - (A_1 + k_0 e^{-E/Ry}) x_1. \quad (9)$$

Parameters E and R are positive (tab.1). The output signal y is also positive. If we assume, that control law provides, that signal y is bounded ($y(t)=e(t)+w(t)$, where $e(t)$ is the tracking error), then the internal dynamic of the system is stable.

The system (6,7) can be described in a the form

$$\begin{aligned} \dot{\mathbf{x}} &= f(\mathbf{x}) + g(\mathbf{x}) u \\ y &= h(\mathbf{x}). \end{aligned} \quad (10)$$

There exists a diffeomorphism $\mathbf{z}=\varphi(\mathbf{x})$ in region Ω,

$$\begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \varphi(\mathbf{x}) = \begin{bmatrix} h(x) \\ L_f h(x) \\ \eta(x) \end{bmatrix} \quad (11)$$

which conditions normal form of transformed system. $L_f h(\mathbf{x})$ is the Lie derivative of $h(\mathbf{x})$ with respect to $f(\mathbf{x})$. All variables of vector z have to be independent, therefore $\eta(\mathbf{x})$ should satisfy $L_g \eta(\mathbf{x})=0$. One of solutions is $\eta(\mathbf{x})=x_1$. The feedback law is defined as

$$u = \psi(v, \mathbf{x}) = \frac{v - L_f^2 h(\mathbf{x})}{L_g L_f h(\mathbf{x})}, \quad (12)$$

where v is the new input signal. The feedback linearization method is illustrated in fig.3.

The system with new coordinates takes form

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \end{bmatrix} = \begin{bmatrix} z_2 \\ v \\ A_1 C_i - (A_1 + k_0 e^{-E/Rz_1}) z_3 \end{bmatrix}, \quad (13)$$

$$y = z_1, \quad (14)$$

for which the mapping $\mathbf{z}=\varphi(\mathbf{x})$:

$$\varphi(\mathbf{x}) = \begin{bmatrix} x_2 \\ A_1T_1 - (A_1 + B_1)x_2 + B_1x_3 + Cx_1k_0e^{-E/Rx_2} \\ x_1 \end{bmatrix},$$

and the inverse mapping $\mathbf{x}=\varphi^{-1}(\mathbf{z})$: (15)

$$\phi^{-1}(\mathbf{z}) = \begin{bmatrix} z_3 \\ z_1 \\ \dfrac{(A_1 + B_1)z_1 + z_2 - Cz_3k_0e^{-E/Rz_1} - A_1T_1}{B_1} \end{bmatrix}.$$

The transformed system is linearized partly, the third equation is nonlinear. However, the relation between input and output signal is linear, which will be used in control algorithm.

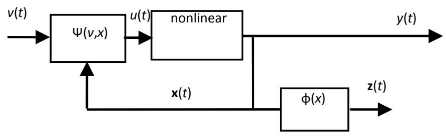

Figure 2: Feedback linearization.

4 PREDICTIVE CONTROL

To design the control algorithm we will use linear model obtained in previous section

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \begin{bmatrix} z_2 \\ v \end{bmatrix} \qquad (16)$$
$$y = z_1.$$

Third equation of (13) will be used only to calculate successive variables of vector **z**, and then from (15) vector **x**. After discretisation of the linear model with T_s=60s and adding reference signal w_k which is imposed by using an additional variable

$$p_{k+1} = p_k + w_k + y_k, \qquad (17)$$

we obtain a discrete model

$$\begin{bmatrix} z \\ p \end{bmatrix}_{k+1} = \begin{bmatrix} A_d & 0 \\ -C_d & 1 \end{bmatrix}\begin{bmatrix} z \\ p \end{bmatrix}_k + \begin{bmatrix} B_d \\ 0 \end{bmatrix} v_k + \begin{bmatrix} 0 \\ 1 \end{bmatrix} w_k, \qquad (18)$$
$$y_k = \begin{bmatrix} C_d & 0 \end{bmatrix}\begin{bmatrix} z \\ p \end{bmatrix}_k,$$

where A_d, B_d, and C_d denote matrices of discrete model.

4.1 Linear Quadratic Control

The predictive control algorithm for the system without constraints and infinite horizon can be designed by LQ control method (Maciejowski, 2002). The cost function which prevents too large deviation from equilibrium point is given by:

$$J_t = \sum_{k=t}^{\infty} \begin{bmatrix} z_k - z_k^0 \\ p_k - p_k^0 \end{bmatrix}^T Q \begin{bmatrix} z_k - z_k^0 \\ p_k - p_k^0 \end{bmatrix} + R(v_k - v_k^0)^2, \qquad (19)$$

with $Q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and R=0.1. The optimal gain L is obtained from LQ method. Then the control law describes

$$\hat{u}_{k|t} = Mw_t - L\begin{bmatrix} \hat{z} \\ \hat{p} \end{bmatrix}_{k|t}, \qquad (20)$$

where M is the first element of L, because the output is the first element of the state vector **z**. The index $k|t$ denotes the sample of variable predicted for the moment t and calculate in the instant k.

4.2 Constrained Predictive Control

In order to include the constraints to the control problem, there will be applied the interpolation technique (Poulsen et al., 2001). It consists in using the LQ method for a system with so changed required output trajectory $\widetilde{w}_{k|t}$ that the obtained variables fulfil the constraints. The changed trajectory is defined by

$$\widetilde{w}_{k|t} = w_t + \hat{s}_{k|t}, \qquad (21)$$

then the control law

$$\hat{v}_{k|t} = M\widetilde{w}_{k|t} - L\hat{z}_{k|t}. \qquad (22)$$

The so called perturbation trajectory $\hat{s}_{k|t}$ calculated in the instant k for successive steps $k \le t \le H$ is obtained from

$$\hat{s}_{k|t} = \alpha_k \hat{s}_{k-1|t}, \qquad (23)$$

where $0 \le \alpha_k \le 1$.

It can be seen from (21) and (23), that α_k=0 corresponds to the unconstrained LQ control. To find proper $\hat{s}_{k|t}$ assuring feasibility of $\widetilde{w}_{k|t}$ we use the initial perturbation trajectory $\hat{s}_{0|t}$, which ensures fulfilling the constraints. One of solution is to chose the $\hat{s}_{0|t}$ so it maintains trajectory $\widetilde{w}_{k|t}$ unchanged for future t, therefore every variable in model is unchanged (assuming that initial condition is stable and fulfil given constraints).

With above reasoning the objective of control is to minimize the parameter α_k with respect to constraints on assumed horizon H. Even though the model (18) is linear, the relation between constrained variable u and α is nonlinear, because it goes through the function $u = \psi(v, \mathbf{x})$. To solve this nonlinear problem it is possible to use simple numeric procedure as bisection.

The above procedure was designed for the instant change of the set point. When desired output trajectory w_k changes in another way the following method of calculation of $\hat{s}_{k|t}$ can be used:

$$\hat{s}_{k|t} = w_{k|t-1} - w_{k|t+}\alpha_k \hat{s}_{k-1|t} \,. \qquad (24)$$

Under assumption that initial conditions are stable and then the initial perturbations sequence is stable, because of the constraints values the control law designed on the interpolation algorithm is asymptotically stable.

5 RESULTS

Two experiments were performed in matlab environment. The PI controller tuned experimentally was used as comparison was. In the first experiment the trajectory w_t was suddenly changed from one value to another. In the second experiment w_t was changed along the linear function, which is a proper behaviour of desired temperature in the reactor. In every figures placed below first chart illustrate the desired trajectory w_t and the output y_t, whilst the second chart show the behaviour of constrained input of the reactor u_t.

The results of the first experiment are illustrated below. The desired trajectory was changed from 333 to 338K with jump in t=20min. Figure 4 illustrate the result obtained from use of PI method, figure 5 with predictive algorithm developed in the article.

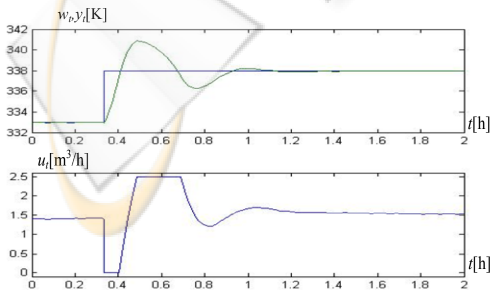

Figure 3: First experiment, PI control.

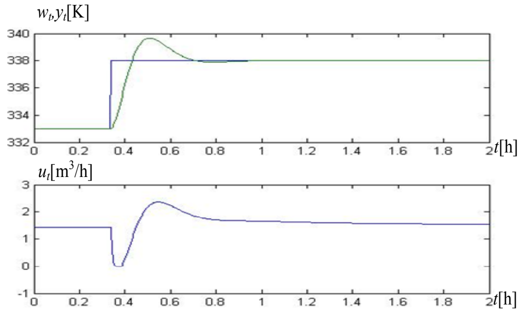

Figure 4: First experiment, predictive control.

In the second experiment trajectory was changed in linear function from 310 to 340K. Results are placed below in a way as in the first experiment.

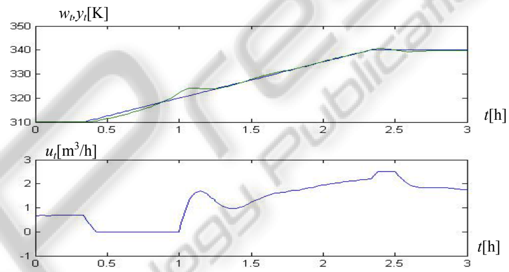

Figure 5: Second experiment, PI control.

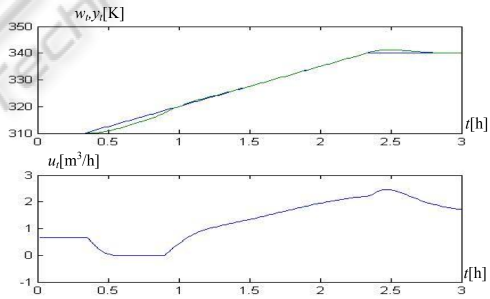

Figure 6: Second experiment, predictive control.

5.1 Conclusions

The operation of predictive method presented in the paper was correct, it fulfils the constraints. In both experiments the use of the algorithm improved the quality of control in comparison with PI control. However the disadvantage of the method is that it relies on feedback linearization, which can be use to limited class of objects.

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