# DEFORMABLE IMAGE REGISTRATION
## *Improved Fast Free Form Deformation*

Bartłomiej W. Papież, Tomasz P. Zieliński
*Department of Telecommunications, AGH University of Science and Technology, Krakow, Poland*

Bogdan J. Matuszewski
*ADSIP Research Centre, University of Central Lancashire, Preston, U.K.*

Keywords: Medical image registration, Free-form deformation, Numerical methods, Prostate cancer.

Abstract: In this paper, we describe a class of deformable registration techniques with application to radiotherapy of prostate cancer. To solve registration problem we introduced Jacobi and successive over-relaxation methods and compared them with the Gauss-Seidel used in the variational framework previously proposed in literature. A multi-resolution scheme was used to improve speed of computation, robustness and ability to recover bigger image deformations. To investigate the properties of these algorithms they were tested using simulated data with known displacement filed and real CT images . The results show that it is possible to improve currently widely used algorithms by introducing simple modifications in the numerical solving scheme.

## 1 INTRODUCTION

Prostate cancer is a common cause of cancer death among men in the world. In Poland in 2006 there were more than 7 thousand new cases estimated, whereas in the United Kingdom more than 35 thousand with respectively 3.5 and 10 thousand deaths due to prostate cancer. The accurate and fast tools for diagnosis, surgical planning and treatment are required. The image registration and segmentation are the fundamental tools, which are instrumental in achieving effective image-guided radiation therapy.

The image registration can be described as a process of finding optimal geometric transformation between images which have similar contents in some sense. The images can be taken from different scanners, at different time and from different positions. Moreover, in medical imaging there is no guarantee that there is one-to-one correspondence between images (e.g. due to missing data).

The image registration methods can be broadly divided into two main categories, feature-based and intensity-based methods. The feature-based registration methods require a pre-processing step to extract corresponding image features such as points, lines and curves. By matching the corresponding image features, deformation of the whole image can be calculated using one of "smooth" interpolation methods (Little et al (1997, Rohr et al (2001)). The intensity-based methods operate directly on image intensity values. One of the most popular methods is to calculate the transformation using a set of equally spaced sparse control points, which are not linked to any specific image features, by finding the optimum of the cost function defined in the neighbourhood of the control points. The image deformations are calculated from displacement of sparse control points using one of interpolation methods mentioned above (MacCraken et al (1996)). As the number of control points might be significant, additional regularisation measures are necessary to avoid excessive variation of the deformation field. Rueckert et al (1999) applied the global affine transformation first, and subsequently used the B-spline interpolation and a penalty function which is a 3D counterpart of the 2D bending energy of the Thin-Plate Spline. Schnabel et al (2001) extended and generalised the work described in (Rueckert et al (1999)) by introducing multi-resolution optimisation and allowing non-uniform distribution of control points. (Matuszewski et al (2003), Shen et


W. Papież B., P. Zieliński T. and J. Matuszewski B. (2010).
DEFORMABLE IMAGE REGISTRATION - Improved Fast Free Form Deformation.
In *Proceedings of the International Conference on Computer Vision Theory and Applications*, pages 530-535
DOI: 10.5220/0002920105300535

al (2005), Shen et al (2006)) proposed further extension by describing interaction between control points using physical analogies.

Another often used intensity-based method is to model the displacement field by using physical analogies. The first such methods were schemes using the Navier-Lamé Partial Differential Equations (*PDEs*) to model elastic behaviour of the registered data (Bajcy and Kovacic (1989)), and scheme using the Navier-Stokes *PDE*s to model fluid deformations (Christensen et al (1996)). The systematic overview of these methods can be found in (Modersitzki (2004))

This paper describes modifications in the numerical solving scheme of a previously proposed method (Lu et al (2004)) based on variational formulation of the registration problem. It is shown that relatively simple modifications improve the performance of the original method.

The remainder of this paper is organized as follows: Section 2 describes shortly method of image registration used in this paper: registration using variational formulation, method of solving resulting partial differential equations system (PDE) and similarity measure which was used for comparison. Section 3 describes and visualizes results of our experiment on simulated data and on real CT images. In section 4 we draw conclusions of those tests.

## 2 THEORY

In this section methods of image registration used for test are shortly described.

### 2.1 Deformable Registration

In general, deformable registration is problem of minimization distance $\varepsilon$ between reference image A($x$) and moving image B($x$) with respect to deformation $u$.

$$\hat{u} = arg\min_{u} \varepsilon(A(x), B(x+u)) \tag{1}$$

A minimization of distance measure is the ill-posed problem (e.g. the solution can be not unique). To solve this problem we add additional term S. There is no general way of choosing regularizing term and there is many different approaches provided depend on desired final results.

$$\hat{u} = arg\min_{u}(\varepsilon\big(A(x), B(x+u)\big) + \alpha S(u)) \tag{2}$$

#### 2.1.1 Free-form Deformation

W. Lu described the free-form deformable registration as process of computing a displacement $\hat{u}$ which minimizes energy of functional $\varepsilon(u)$:

$$\hat{u} = \arg\min \varepsilon(u) \tag{3}$$

where

$$\varepsilon(u) = \int_{x\in R^3} (R^2\,(x,u) + \lambda \sum_{i=1}^{3}\sum_{j=1}^{3} ({v_j}^i)^2)\,dx \tag{4}$$

Here $R(x,u) = B(x+u) - A(x)$ is residual, $A(x)$ is reference image, $B(x+u)$ is moving image and ${v_j}^i = \frac{\partial u^i}{\partial x^j}$. To find displacement field u, the calculus of variations is used and the problem of deformable registration becomes the problem of solving the non-linear elliptic partial differential equations.

$$\lambda\nabla^2 u - \mathrm{R}(x,u)\frac{\partial \mathrm{R}(x,u)}{\partial u} = 0 \tag{5}$$

To solve an equation (5) a finite difference scheme previously proposed in literature is used. After discretizing equation (5), we have:

$$L_{m,n} = \lambda\nabla^2 u_{m,n} - (B_m^* - A_m)\mathrm{g}_{m,n}^* \tag{6}$$

Where $m$=1,…,$N$ ($N$ denoting the total number of volume voxels) is a voxel index using lexicographical ordering; $n$=1,2,3 is an index corresponding x, y, and z dimensions, $B_m^* = B(x_m + u_m)$; $A_m = A(x_m)$; $\mathrm{g}_{m,n}^* = \mathrm{g}_n(x_m + u_m)$ with $\mathrm{g}(x) = \nabla B\,(x)$;

The displacement field is estimated using one step of Newton iterations:

$$u_{m,n}^{new} = u_{m,n}^{old} + L_{m,n}^{old}/(\lambda + \big(g_{m,n}^{*old}\big)^2) \tag{7}$$

### 2.2 Criterion of Registration Quality

To assess the quality of registration in tested data we calculated the correlation coefficient (CC).

$$CC = \frac{\sum\big(A(x) - \bar{A}(x)\big)\big(B(x+u) - \bar{B}(x+u)\big)}{\sqrt{\sum(A(x) - \bar{A}(x))^2 \sum(B(x+u) - \bar{B}(x+u))^2}} \tag{8}$$

Here $\bar{A}(x)$ and $\bar{B}(x+u)$ are the mean intensity of the reference and moving image. Better registration means than value of CC is closer 1.

For simulated data the sum of squared differences (SSD) is calculated between known deformation field $u(x)$ deformation field and estimated field $\hat{u}(x)$:

$$SSD = \frac{1}{N * M} \sum (u(x) - \hat{u}(x))^2 \quad (9)$$

For perfect registration the value of SSD is 0.

### 2.3 Solution Scheme

To solve system of nonlinear elliptic partial differential equation, the finite difference scheme was used. After that we can use iterative method to compute $u$. Update scheme for each iteration is as follows:

- Calculate $\nabla^2 u$ using central difference scheme
- Interpolate using tri-linear interpolation $B(x+u)$ and gradient $g$
- Calculate $L$ according to equation (6)
- Update deformation field $u$ using equation (7)

Previously in literature Gauss-Seidel scheme was proposed to calculate $L$ and $g$. In this paper Jacobi and SOR scheme is introduced to calculate $L$ and $g$.

#### 2.3.1 Jacobi Method

The Jacobi method solves element $x^k$ using previously computed value $x^{k-1}$ for each iteration. This may be written as:

$$x_i{}^k = \frac{d_i - \sum_{i \neq j} l_{ij} x_j{}^{k-1}}{l_{ii}} \quad (10)$$

#### 2.3.2 Gauss-Seidel method

The Gauss-Seidel method solves element $x^k$ using already computed values of $x^k$ and previously computed value $x^{k-1}$. This may be written as:

$$x_i{}^k = \frac{d_i - \sum_{i>j} l_{ij} x_j{}^k - \sum_{i<j} l_{ij} x^{k-1}}{l_{ii}} \quad (11)$$

#### 2.3.3 Successive Over-relaxation Method

If we make an overcorrection to $x^k$ at the $k$-th iteration of Gauss-Seidel method by introducing over-relaxation parameter ω we get method called successive over-relaxation (SOR). This may be written as:

$$u_i{}^k = (1-\omega) x_i{}^{k-1} + \omega \left( \frac{d_i - \sum_{i \neq j} l_{ij} x_j{}^{k-1}}{l_{ii}} \right) \quad (12)$$

There is many various automated method for choosing parameter ω but there are no general rules.

## 3 EVALUATION AND RESULTS

Evaluation of registration quality is done in two ways. At the first stage, we prepared simulated data with generated ground truth displacement field and then we used previously described algorithms to estimated this displacement field. At the second stage we used real CT data from radiotherapy of prostate cancer.

Free-form deformable registration with each scheme was implemented in Matlab. All three methods are implemented in the multiresolution manner to reduce the computation time, improve accuracy by avoiding local extremes and improve ability to recover large displacement.

### 3.1 Simulated Data

Simulated data used in our tests are shown on Figure 1. First image is the reference image, second image is the warped version of the first image by applying the known displacement field. Figure 2 visualizes deformation field introduced into reference image, arrows shows the direction of displacement from reference image to moving image. The arrows are calculated as gradient of function used to deform image. For those experiments, the objective is to recover known deformation field using three previously described methods.

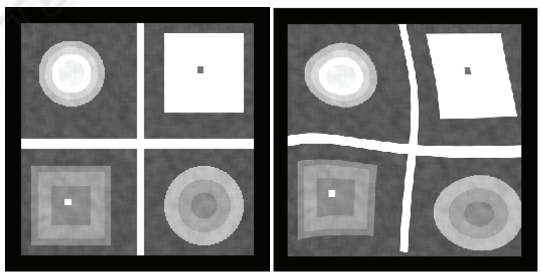

Figure 1: Simulated reference and moving image.

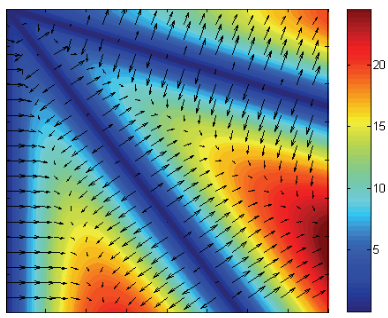


Figure 2: Simulated deformation field introduced into reference image.

Figure 3 shows image differences between reference image and moving image and the error magnitude between known and estimated deformation after registration. For the same number of iteration the FFD method using SOR scheme achieved slightly better results than Gauss-Seidel and Jacobi scheme. It is due to the fact that SOR and Gauss-Seidel scheme provide faster convergence.

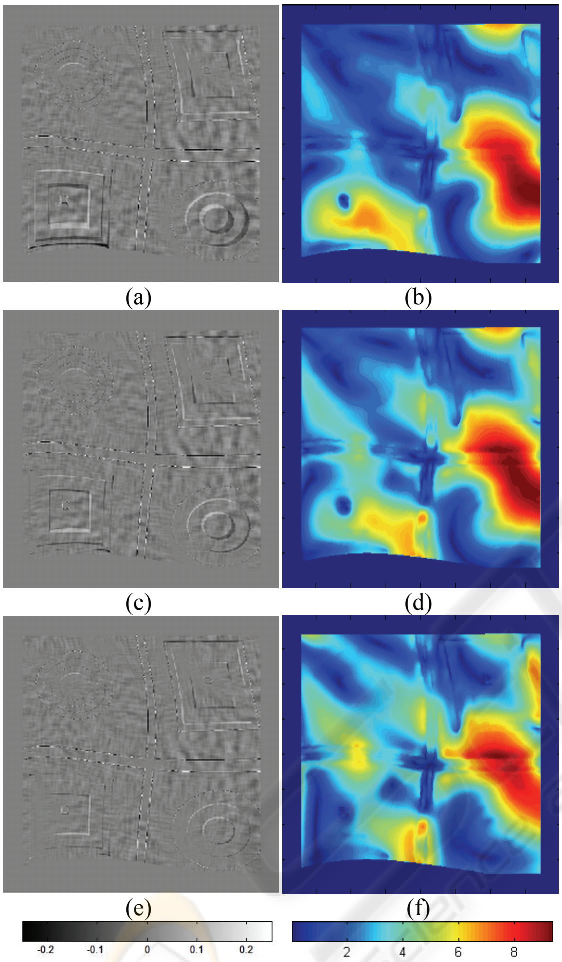


Figure 3: Image difference between references image and moving image after registration and error magnitude between known and estimated deformation field for Jacobi method (a)-(b), Gauss-Seidel method (c)-(d) and SOR method (e)-(f) .

## 3.2 Real CT Data

The evaluation of registration methods was done on real medical data from radiotherapy of prostate cancer. For each patient, we had got two CT data sets, first from radiotherapy planning used as reference image and second taken during treatment process used as moving image. We provide tests for 2D images (taken as slice form 3D data set) and for 3D data. Figure 4 shows slices from CT images. The quality of registration was measured for the same number of iterations for each method in two ways: first as value of correlation coefficient and the second as image differences between images. The relaxation parameter and weight of Laplacian were selected empirically.

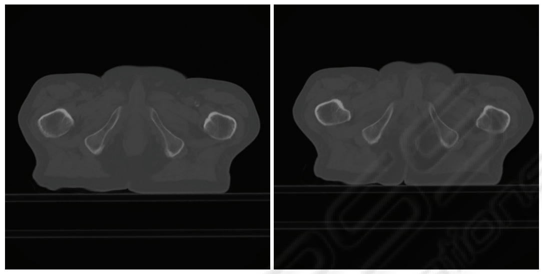

Figure 4: CT images: reference and moving image.

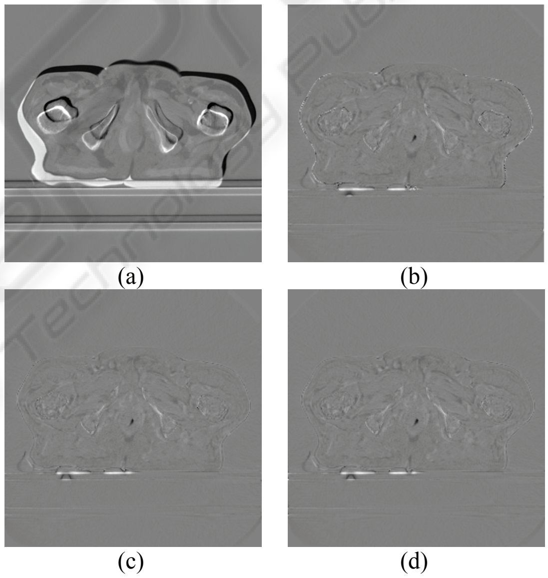


Figure 5: Image differences: (a) between reference image and moving image before registration, and after registration using Jacobi scheme (b), Gauss-Seidel scheme (c) and SOR scheme (d).

Table 1: Intensity CC and values of SSD calculated for corresponding displacement fields after 2D registration using tested methods. (the same number if iterations was applied to each method).

| | After 20 iterations | | After 80 iterations | |
|---|---|---|---|---|
| | CC | SSD | CC | SSD |
| Jacobi | 0,9979 | 25,55 | 0,9985 | 9,91 |
| G-S | 0,9983 | 16,34 | 0,9987 | 7,73 |
| SOR | 0,9986 | 9,49 | 0,9988 | 6,17 |

Figure 5 visualises results images after registration using different scheme for fast free-form registration.

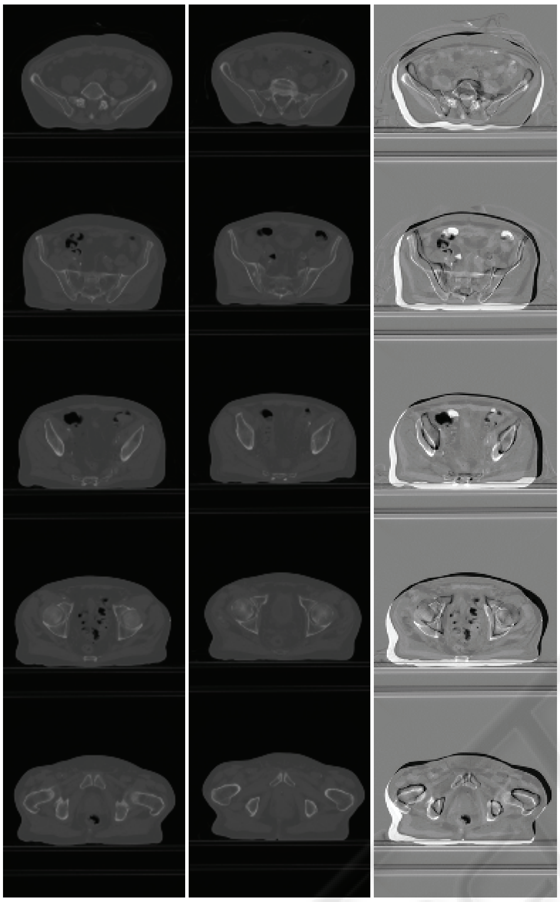

Figure 6: 3D reference and moving image used in tests and image difference between them.

Figure 6 shows reference image taken from planning and moving image taken from treatment process and difference between them.
Figure 7 shows difference after registration. All methods were able to recover large displacement of patient and for each the similarity measures was significantly improved. Figure 8 shows value of CC for different number of executed iterations for each method. SOR scheme is dependent on value of relaxation parameter. In some cases we can achieve better accuracy of registration using Gauss-Seidel scheme than SOR with non-optimal relaxation parameter. For Jacobi scheme it is necessary to calculate more iterations to achieve the similar accuracy of registration.

Figure 7: Image difference between (from left to right) Jacobi, Gauss-Seidel and SOR scheme.

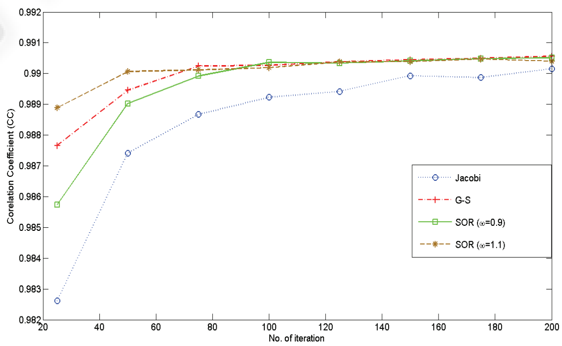


Figure 8: Correlation coefficient for each method after the same number of executed iterations.

## 4 CONCLUSIONS

The paper has been focused on evaluation of currently known methods of registration. We show that it is possible to achieve better quality of registration for these methods by introducing simply numerical improvements.

For simulated data we have achieved slightly better results using SOR scheme, this is due to fact that SOR scheme get convergence faster than Gauss-Seidel and Jacobi scheme. Each method is able to recover large deformation field introduced into moving image.

For CT data, all methods achieve similar results. The main differences between tested methods were the number of executed iterations to achieve similar value of correlation coefficient and the sum of squared differences. In every case the Jacobi method required twice the number of iteration compared to Gauss-Seidel. It is due fact that Gauss-Seidel and SOR scheme has faster convergence.

The main difficulty with SOR scheme is an optimal selection of the over-relaxation parameter. The optimal value of this parameter is data dependent. In our experiments this values was chosen empirically. In some cases, using non-optimal values of over-relaxation parameter provides smaller accuracy and quality of registration than Gauss-Seidel scheme.

In general the quality of registration depends on data and there is no possibility to show the most accurate method. Fast Free-Form Deformation algorithm is suitable to recover large displacement. Also it is possible to use this algorithm during the radiotherapy of prostate cancer because of short computation time of deformation field.

## ACKNOWLEDGEMENTS

The work presented in this paper has been supported from the MEGURATH project (EPSRC project No. EP/D077540/1). The authors would like to thank Dr Paweł Kukołowicz from the Holy Cross Hospital for supplying CT images.

## REFERENCES

W. Lu, M. L. Chen, G. H. Olivera, K. J. Ruchala, T. R. Mackie, 2004, Fast free-form deformable registration via calculus of variations. In *Physics in Medicine and Biology*, 49:3067-3087

B. J. Matuszewski, J.-K. Shen and L.-K. Shark, 2003, Elastic Image Matching with Embedded Rigid Structures Using Spring-Mass System. In *Proceedings of IEEE International Conference on Image Processing*, ICIP-2003. Vol. 10, pp.937-940

J. Modersitzki, 2004, *Numerical Methods for Image Registration*, Oxford University Press,

D. Rueckert, L. I. Sonoda, C. Hayes, D. L. G. Hill, M. O. Leach, D. J. Hawkes, 1999, Nonrigid Registration Using Free-Form Deformation: Applications to Breast MR Images. In *IEEE Transactions on Medical Imaging*. Vol. 18 No.8. pp. 712-721

J.-K. Shen, B.J. Matuszewski and L.-K. Shark, 2003, Deformable Image Registration. In *Proceedings of IEEE International Conference on Image Processing*, ICIP'2005. Vol. 3, pp. 1112-1115.

J.-K. Shen, B.J. Matuszewski, L.-K. Shark and C.J. Moore, 2006, Deformable image registration using spring mass system. In *British Machine Vision Conference, BMVC*06, Vol. 3, pp 1199-1208.

J.-P. Thirion, 1998, Image matching as a diffusion process: an analogy with Maxwell's demons. In *Medical Image Analysis*, Vol. 2, No. 3, pp-243-260

W. H. Press, S. A. Teukolsky, W. T. Vetterling, B. P. Flannery, 1992. *Numerical recipes in C:The art of scientific computing*, Cambridge University Press

T. S. Yoo, 2004. *Insight Into Images. Principles and Practice for Segmentation, Registration and Image Analyses.* National Library of Medicine.

J. A. Little, D.L.G. Hill and D.J. Hawkes, 1997. Deformations Incorporating Rigid Structures. In *Computer Vision and Image Understanding*. Vol. 66, No. 2, pp. 223-232.

K. Rohr, H.S. Stiehl, R. Sprengel and et al., 2001. "Landmark-based elastic registration using approximating thin-plate spline". In *IEEE Transactions on Medical Imaging*, Vol. 20, pp. 526-534.

R. MacCraken, K. Joy, 1996. Free-form deformations with lattices of arbitrary topology. In *Computer Graphics Proceedings, Annual Conference Series, Proceedings of SIGGRAPH 96*. pp 181-188. ACM SIGGRAPH.

J. A. Schnabel, D. Rueckert and et al., 2001. A Generic Framework for Non-rigid Registration Based on Non-uniform Multi-level Free-Form Deformations. In *Proc. MICCAI 2001, Lecture Notes in Computer Science*. Vol.2208, pp.512-721.

R. Bajcsy and S. Kovacic, 1989. Multiresolution elastic matching. In *Computer Vision, Graphics and Image Processing*. Vol. 46, pp. 1-21.

G. E. Christensen, R.D. Rabbitt and M.I. Miller, 1996. Deformable Templates Using Large Deformation Kinematics. In *IEEE Transactions on Image Processing*. Vol. 5, pp. 1435-1447.