EXTENDING REVISED AFFINE ARITHMETIC FOR FAST RELIABLE RAY-TRACING OF PROCEDURALLY DEFINED IMPLICIT SURFACES

Oleg Fryazinov, Alexander Pasko and Peter Comninos
The National Centre for Computer Animation, Bournemouth University, U.K.

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Abstract: Fast and reliable rendering of implicit surfaces is an important research area in the field of implicit modelling. Direct rendering, namely ray-tracing, is shown to be a suitable technique for obtaining good-quality visualisations of implicit surfaces. We present a technique for reliable ray-tracing of arbitrary procedurally defined implicit surfaces by using a modification of Affine Arithmetic called Revised Affine Arithmetic. A wide range of procedurally defined implicit objects can be rendered using this technique including polynomial surfaces, constructive solids, pseudo-random objects, procedurally defined microstructures, and others. We compare our technique with other reliable techniques based on Interval and Affine Arithmetic to show that our technique provides the fastest, while still reliable, ray-surface intersections and ray-tracing. We also suggest possible modifications for the GPU implementation of this technique for real-time rendering of relatively simple implicit models and for near real-time rendering of complex implicit models.

1 INTRODUCTION

In recent years, implicit surfaces (isosurfaces of trivariate real functions) have proved to be a powerful and simple solution to some complex problems in the area of modelling and animation. For example, implicit surfaces provide solutions for surface reconstruction from scattered points and for fluid simulation. Several operations, such as sweeping, metamorphosis and offsetting can be implemented quite easily with implicit models unlike traditional boundary-representation models. However, modelling with the whole range of implicit surfaces is still a complicated task because interactive rendering of arbitrary implicit surfaces is still an open problem. Currently, there are two ways to render an implicit model: generation of a polygonal mesh and direct rendering using ray-tracing. Polygonization is a widely used technique, but in many cases, when the model has sharp or thin features, large numbers of small-sized disjoint elements or internal microstructures, the generation of an appropriate polygonal mesh takes a long time and requires a large amount of memory. A more promising technique is that of interactive direct rendering of implicit surfaces using ray-casting and ray-tracing. Traditionally the main disadvantage of direct rendering was their slow speed due to the large number of ray-surface intersection calculations. With recent developments of hardware this problem becomes less critical, but not insignificant all together.

Many techniques of ray-tracing implicit surfaces have been developed. However, the majority of these techniques have disadvantages, because they either work with a small range of implicit surfaces (for instance those defined only by polynomials), or not reliable. For example, classical approximate techniques, such as ray marching, are fast, but can easily miss sharp features and small components of these models. Classical numerical techniques, such as the Newton search require different signs of the defining function at the ends of the ray interval, which is inappropriate for arbitrary rays. Sphere-tracing based techniques require a distance property of the defining function, which can not be provided for general models. Techniques based on interval analysis and other reliable numerical computations have also been applied to the ray-tracing of implicit surfaces. However, classical Interval Arithmetic is slow because of the interval

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overestimation.

The problem considered in this paper is that of finding a technique for ray-tracing general implicit surfaces, that has the following properties: a) Its ray-surface intersection procedure should be reliable, i.e. no roots should be missed; b) A wide range of implicit models should be supported – meaning that the algorithm should be able to work with procedurally defined models as well as with algebraic ones; c) The procedure should be fast and suitable for a GPU implementation for interactive rendering.

In this paper we propose to use Revised Affine Arithmetic (RevAA in the text) as a fast and reliable technique for calculating the range of a function for a given interval and hence for the core for the ray-surface intersection procedure. Also we extend affine arithmetic by affine forms related to the procedural definition of the model to decrease the number of computations while evaluating the interval for the function and hence decrease the rendering time. The main contributions in this paper are: 1) widening the scope of the reliable ray-tracing from algebraic surfaces defined by polynomials to general implicit surfaces defined by function evaluation procedures involving both affine and non-affine operations based on Revised Affine Arithmetic; 2) a technique to represent the procedural model by using special affine forms with a case study of affine forms for set-theoretic operations in form of R-functions; 3) a technique for optimising the proposed ray-tracing procedure by using argument pruning applied to Revised Affine Arithmetic.

2 RELATED WORK

Ray-tracing of implicit surfaces is a well-researched area. Most of the techniques described in a survey (Hart, 1993) are approximate and can miss small surface features, but on the other hand, they are suitable for all types of implicit surfaces. Later, several techniques were presented for particular types of implicit surfaces that provided not only an increase in speed but also reliability. Thus, in (Hart, 1994) a distance property is needed for the ray-tracing procedure, in (Sherstyuk, 1999) blobs, metaballs and convolution surfaces are the types of implicit surface that can be rendered fast.

Other ways to increase speed of ray-tracing are by reducing the number of processed rays intersecting the implicit surface and by using specialized hardware, for example graphical processors (GPUs). In (Hasan, 2003), the number of processed rays is reduced by using image-space subdivision, while (Gamito and Maddock, 2007a) uses progressive refinement. GPU-based ray-tracing of implicit surfaces was introduced only for particular types of the objects, such as radial-basis functions (Corrigan and Dinh, 2005). Ray-tracing of general implicit surfaces on the GPU was performed in (Fryazinov and Pasko, 2008) and in (Singh and Narayanan, 2009) by using approximate methods.

Reliable computational techniques based of Interval Arithmetic have been known for a long time. However, most of the literature relates to fields such as global optimisation rather than computer graphics. The works of (Mitchell, 1991) and (Snyder, 1992) discussed applications of Interval Arithmetic for computer graphics purposes, and Affine Arithmetic was used for ray-tracing of implicit surfaces in (de Cusatis Jr. et al., 1999) . A good comparison of different interval techniques can be found in (Martin et al., 2001), however the list of the implicit models used in this paper is limited to those given in the polynomial form. In (Gamito and Maddock, 2007b) the Reduced Affine Arithmetic was introduced for the purposes of stochastic implicit model rendering. However, the Reduced Affine Arithmetic could not be used for general implicit models, as only affine operations and multiplication were supported. Interval Arithmetic and Reduced Affine Arithmetic are applied for fast rendering of implicit surfaces by using the GPU in (Knoll et al., 2009). A more detailed comparison of these techniques with the one proposed here can be found in the "Results" section below. In this paper, we use Revised Affine Arithmetic (Vu et al., 2009), which was introduced recently for the purposes of constraint propagation and has not yet been used in computer graphics.

3 BACKGROUND

3.1 Procedurally Defined Implicit Surfaces

A zero level set or an isosurface of a trivariate real function f of a point with coordinates (x,y,z) is traditionally called an implicit surface and is defined as $f(x,y,z) = 0$. It can also be considered as the boundary of a solid (three-dimensional manifold) defined by the inequality $f(x,y,z) \geq 0$. There are many different ways to specify the function $f(x,y,z)$. The simplest form is that of an algebraic implicit surface defined by a polynomial function. Most of the extant work on reliable ray-tracing concentrates solely on algebraic surfaces. More complex forms involve exponential,

square root, trigonometric and other non-linear functions. We deal with the most general form of procedurally defined implicit surfaces, where the function f is evaluated by some procedure involving all kinds of non-linear functions as well as loops and conditional operations. This allows us to cover skeleton-based implicit surfaces, Constructive Solid Geometry (CSG) objects defined by nested R-functions, solid noise, fractals and other complex objects.

3.2 Affine Arithmetic

Affine Arithmetic (AA) is a technique for performing computations on uncertain numerical values. The main idea of AA is the calculation of an uncertain value (function) based on other uncertain values (arguments). Initially this model was introduced for self-validated numerical computations as an alternative to Interval Analysis and currently it is used in many different areas of computer science (de Figueiredo and Stolfi, 1997). By keeping track of the errors for each computed quantity, AA provides much tighter bounds for computed quantities compared to classical Interval Arithmetic. Uncertain values in AA are represented by affine forms, i.e. polynomials of the form
$\hat{x} = x_0 + x_1\varepsilon_1 + x_2\varepsilon_2 + \ldots + x_n\varepsilon_n$
where x_n are known real coefficients and ε_n are noise symbols, i.e. symbolic variables whose values are assumed to lie in the interval $\varepsilon_n \in [-1,1]$.

All operations on affine forms can be divided into affine (exact) and non-affine (approximate) operations. An affine operation is a function that can be represented by the linear combination of the noise symbols of its arguments. For example, a multiplication by a constant is an affine operation:

$$\alpha\hat{x} = \alpha x_0 + \alpha x_1\varepsilon_1 + \alpha x_2\varepsilon_2 + \ldots + \alpha x_n\varepsilon_n$$

Non-affine operations can not be performed over the linear combination of the noise symbols. In this case an approximate affine function is used and a new noise symbol is added to the affine form to represent the difference between the non-affine function and its approximation.

In the general case any non-affine operation can be represented in an affine form:

$$\hat{x} \otimes \hat{y} = \alpha\hat{x} + \beta\hat{y} + \zeta \pm \delta$$

where the value of the new noise symbol is represented by δ. In (de Figueiredo and Stolfi, 1997), different approximation techniques are discussed for the affine form of several functions: Optimal (Chebyshev), Min-range and Interval approximation. While optimal and min-range approximation require the function to be bounded, twice differentiable and with the same sign of the second derivative on the given argument range, the interval approximation requires only the function to be bounded, however the range of the function for interval approximation is wider than for other approximations.

3.3 Revised Affine Arithmetic

Pure AA is both computationally and memory expensive and can not be used in algorithms where the reduction of computational complexity is equally important as the quality of the computational result. In (Messine, 2002), several reduced affine forms were introduced to reduce the number of computations in Affine Arithmetic by accumulating errors. The Affine Form 1 (AF1) is the simplest one and represents the uncertain quantity as:

$$\hat{x} = x_0 + \sum_{i=1}^{n} x_i\varepsilon_i + x_{n+1}\varepsilon_{n+1}$$

The noise symbols $\varepsilon_1, \ldots, \varepsilon_n$ represent the errors of the initial arguments. The last noise symbol represents all the errors after the non-affine operations.

Revised Affine Arithmetic is an extension of AF1 and was introduced by Vu et al. (Vu et al., 2009) for the purposes of numerical constraint propagation. The revised affine form is similar to AF1:

$$\hat{x} = x_0 + \sum_{i=1}^{n} x_i\varepsilon_i + e_x[-1,1], e_x \geq 0$$

The general binary affine operation is defined as:

$$f(\hat{x},\hat{y}) = (\alpha x_0 + \beta y_0 + \zeta) + \sum_{i=1}^{n} (\alpha x_i + \beta y_i) + (\delta + |\alpha|e_x + |\beta|e_y)[-1,1]$$

where α, β and ζ can be taken from the affine approximation of the function f. The affine operation with one operand and more than two operands can be defined in the same way.

RevAA uses a special tight form for the multiplication operation:

$$\hat{x} * \hat{y} = (x_0y_0 + \tfrac{1}{2}\sum_{i=1}^{n} x_iy_i) + \sum_{i=1}^{n} (x_0y_i + x_iy_0)\varepsilon_i + e_{xy}[-1,1],$$

where $e_{xy} = e_xe_y + e_y(|x_0| + u) + e_x(|y_0| + v) + uv - \frac{1}{2}\sum_{i=1}^{n} |x_iy_i|$, $u = \sum_{i=1}^{n} |x_i|$, $v = \sum_{i=1}^{n} |y_i|$.

As for standard AA, RevAA has an inclusion property. In our technique we use the shortest possible revised affine form, i.e. $n = 1$ as it is most convenient for the purposes of ray-tracing and requires fewer computations than with a greater n.

4 RAY-TRACING WITH REVISED AFFINE ARITHMETIC

The main part of any ray-tracing procedure for implicit surfaces is the calculation of the zero roots of the defining function in the ray-surface intersection procedure. In this section we show how RevAA can be used for the intersection point calculation and we present several techniques for speeding up this calculation.

4.1 Ray-Surface Intersection

Our algorithm is based on a ray-surface intersection technique for implicit surfaces that uses interval techniques, which originally appeared in (Mitchell, 1991). We present the ray-surface intersection procedure in Algorithm 1.

Algorithm 1: Ray-surface intersection.

Procedure: bool intersect(t_{min}, t_{max})
  Calculate the affine form F for the function on the interval $[t_{min}, t_{max}]$
  Get the range of the function from the affine form
  if the range of the function does not include a 0 value **then**
    return FALSE (no roots in this interval);
  end if
  Calculate the argument estimation from the affine form: t'_{min}, t'_{max}
  Find the pruned argument range:
  $t_{min} = max(t_{min}, t'_{min})$;
  $t_{max} = min(t_{max}, t'_{max})$;
  if the length of the argument interval is less than some predefined accuracy **then**
    Store the midpoint of the interval as the root;
    return TRUE;
  end if
  Calculate the midpoint of the argument range:
  $t_{mid} = (t_{min} + t_{max})/2$;
  Repeat the procedure for the two subintervals:
  bool b1 = intersect(t_{min}, t_{mid});
  if b1 is TRUE and only the first root is needed **then**
    return TRUE;
  end if
  bool b2 = intersect(t_{mid}, t_{max});
  if b2 is TRUE **then**
    return TRUE;
  end if
  return FALSE;

The basic idea of the algorithm is quite simple: we calculate the range of the function for the given argument interval using RevAA, we reject the interval if the range does not include the zero value, otherwise we subdivide the interval into two intervals by using dichotomy and we repeat the procedure for both subintervals. An example of the affine form for the function after the dichotomy is shown in the figure 1. Note that in the case when only the first root is needed (for example, for primary rays), we can exit from the procedure earlier if we have found a root in the first subinterval after the recursive procedure.

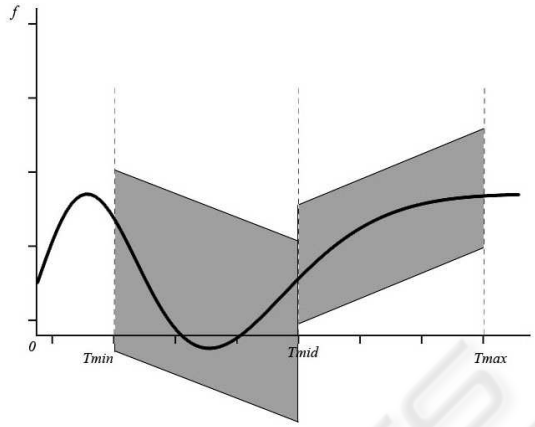


Figure 1: The revised affine form for the function on the two subintervals after the dichotomy on the interval $[t_{min}, t_{max}]$.

4.1.1 Affine Form Calculation and Interval Range for the Function in RevAA

The basic revised affine form for the function is obtained from the procedural definition of the function by replacing all the operations on real numbers by operations on the revised affine forms of coordinate variables: $\hat{x} = x_0 + \hat{t} * d_x, \hat{y} = y_0 + \hat{t} * d_y, \hat{z} = z_0 + \hat{t} * d_z$, where x_0, y_0, z_0 are the coordinates for the ray origin and d_x, d_y, d_z are the components for the ray direction vector and are constant for each ray, and $\hat{t} = \frac{t_{min}+t_{max}}{2} + \frac{t_{max}-t_{min}}{2}\varepsilon_1$ is an affine form for the argument interval.

Non-arithmetic operations, such as trigonometric, logarithmic and reciprocal, can be calculated by using the general affine form applied to RevAA. The formulation for the coefficients of the general affine form and most of the basic non-arithmetic operations can be found in (de Figueiredo and Stolfi, 1997).

We propose also to extend the number of operations by calculating affine forms for functions used in the procedural definition of the implicit model. As a case study we present the formulation for set-theoretic (CSG) operations using R-functions as follows:

$$R_{uni}(f_1, f_2) = f_1 + f_2 + \sqrt{f_1^2 + f_2^2}$$

$$R_{int}(f_1, f_2) = f_1 + f_2 - \sqrt{f_1^2 + f_2^2}$$

where R_{uni} is the function for set-theoretic union and R_{int} is the function for set-theoretic intersection (Shapiro, 2007). The optimal and min-range approximations can be obtained for these functions, because the requirements are satisfied, however the optimal approximation is computationally expensive for these particular functions comparing to the natural extension of these functions in RevAA. We propose to use the min-range approximation for these functions. By analogy with the construction of an approximation for the function of one variable presented in affine arithmetic-related literature, our approximation is constructed by using these rules:

- $\alpha = \frac{\partial R}{\partial x}$, $\beta = \frac{\partial R}{\partial y}$, where partial derivatives are taken at the base point, i.e. minimal point ($x = x_{min}, y = y_{min}$) for the union operation and maximal point ($x = x_{max}, y = y_{max}$) for the intersection operation because of the second derivative sign. Note that the base point should not be [0, 0] because of the discontinuity of the first derivative and an opposite point should be selected instead.
- We calculate the distance to the other corner points of the surface patch $z = R(x,y), x \in [x_{min}, x_{max}], y \in [y_{min}, y_{max}]$ by using the equation of the plane $R(x,y) = \alpha x + \beta y + d$. Note that for the base point $d = 0$, a maximal distance d_{max} between the three points is taken.
- ζ and δ can be obtained from d_{max}. For the union operation $\zeta = \frac{d}{2}$ and for the intersection operation $\zeta = -\frac{d}{2}$. $\delta = \frac{d}{2}$.

In the same way affine forms for other functions can be constructed. It is obvious that the affine form can not be obtained for an arbitrary function, as the requirements for the approximations are quite strict and moreover the construction for the affine form can be more computationally expensive than the natural affine extension of the function. However, the correct derivation of the affine form for the used functions can dramatically improve the speed of computations.

After the affine form computations, we obtain the range of the function from the affine form $\hat{f} = f_0 + f_1\varepsilon_1 \pm e_f$:

$$f_{min} = f_0 - |f_1| - e_f$$
$$f_{max} = f_0 + |f_1| + e_f$$

4.1.2 Argument Pruning

One of the useful properties of the reduced affine forms, including RevAA, is that of argument pruning (a term taken from the literature of interval slope methods), which means narrowing the argument range in case the root is contained in the interval. In (Gamito and Maddock, 2007b), the argument pruning formulation (the term interval optimisation is used in the paper) was suggested for Reduced Affine Arithmetic. As the geometric meaning of the revised affine form is similar to that of the reduced affine form, an analogous formulation can be used as follows. Given the revised affine form for the function $\hat{f} = f_0 + f_1\varepsilon_1 \pm e_f$ for the interval $\hat{t} = t_0 + t_1\varepsilon_1$, providing that $t_1 \neq 0$, $f_1 \neq 0$ and $e_f \neq 0$, the interval can be pruned by the points $t' = t_0 - \frac{t_1 f_0}{f_1} \pm e_f \frac{t_1}{f_1}$ if these points lie inside the interval $[t_{min}, t_{max}]$ (see Fig. 2).

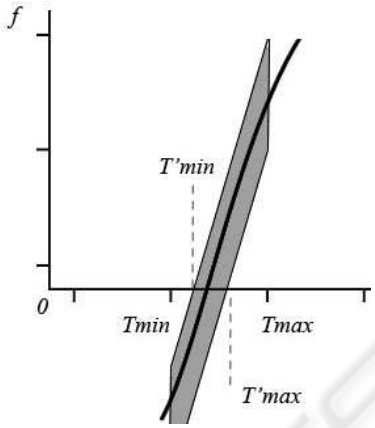


Figure 2: Pruning of the interval $[t_{min}, t_{max}]$ to the interval $[t'_{min}, t'_{max}]$ after the evaluation of the revised affine form for the function.

4.2 Implementation

In this section we present several details of the implementation of ray-tracing of procedurally defined implicit surfaces on the CPU and the GPU. The implementation can be subdivided into three parts: the RevAA representation, the function representation in the revised affine form and the ray-tracing procedure.

4.2.1 Affine Form Representation

The value in RevAA is represented by a polynomial with two terms and one interval. Thus, the value in the software implementation can be represented as a three-component vector, where the first component represents x_0, the second represents the noise symbol for the error along the ray and the third represents the half-length of the accumulating interval. The calculations in RevAA can be performed on these vectors. Almost all of the arithmetic operations have to be overridden as only summation in the RevAA matches the standard vector summation. For example, subtrac-

tion can be implemented as follows:

```
vec3 ra_subtraction(vec3 x, vec3 y){
vec3 ret;
ret[0] = x[0] - y[0];
ret[1] = x[1] - y[1];
ret[2] = x[2] + y[2];
return ret;
}
```

Similarly, other affine and non-affine operations can be implemented as operations on three-component vectors. Note that for non-affine operations we are most likely to use the affine constructor described above. In fact, any non-affine operation derived for pure Affine Arithmetic with known *alpha*, *dzeta* and *delta* can be adapted for RevAA.

4.2.2 Representation of the Function

The ray-tracing algorithm works with objects defined by a real-valued functions of real-valued arguments. In the same way this function can be rewritten by using the following rules:
- Each variable depending on the input arguments is replaced by a variable of the revised affine type, while each variable not depending on the input arguments and constants is left in the real form.
- If we have affine forms for functions or composition of functions for the given procedural model, replace the code over the affine variables by these forms
- Ensure that the implementation of the remaining operations are overridden in RevAA.

The returned value of the rewritten function is the range of the function in the affine form, which is used in the ray-surface intersection procedure described earlier.

4.2.3 Implementation of the Ray-tracing Procedure

The ray-tracing procedure includes the ray-surface intersection for the primary and secondary rays and shading. The ray-surface intersection procedure described earlier is suitable for CPU implementation, however it can not be used in a straightforward way for the GPU, as it is recursive. For the GPU implementation we used the version with a stack where the information of the current argument interval is stored. The detailed implementation of this is outside the scope of this paper. Another implementation without a stack and recursion can be found in (Knoll et al., 2009) for Interval Arithmetic, however this algorithm can be easily extended for RevAA. Note that in this implementation argument pruning can not be supported and hence the speed of rendering is most likely to be slower.

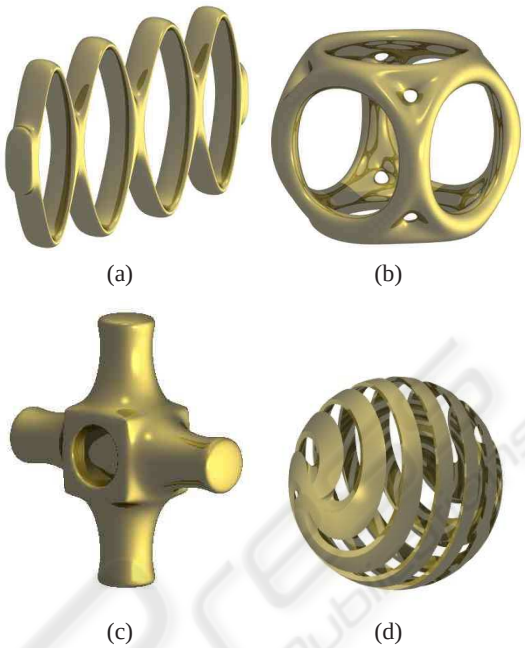
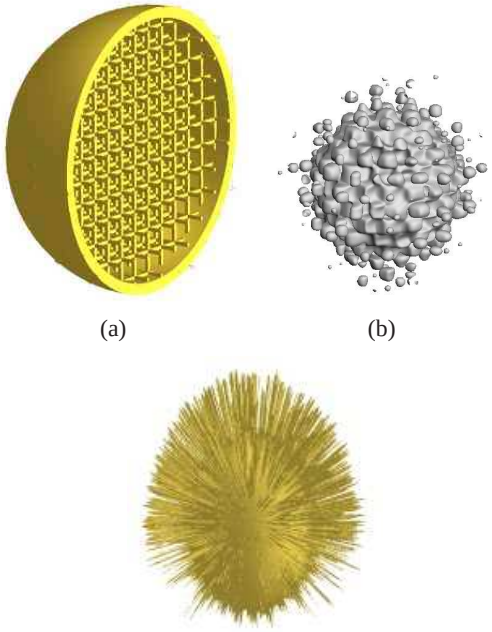


Figure 3: Ray-tracing of algebraic surfaces: a) Bretzel b) Decocube; and non-algebraic implicit surfaces: c) CSG with using blending union and blending intersection d) Sphere with trimming.

Similarly to other ray-tracing techniques for implicit surfaces, we use finite differences to obtain the normal vector for the shading and the secondary rays calculation. Therefore the real-valued defining function of real-valued arguments should be presented as well as the function in the revised affine form.

5 RESULTS

In our tests we used a modified version of the POV-Ray ray-tracing software for the CPU and a stand-alone renderer based on the GLSL language for the GPU. The results for the CPU as well as for the GPU implementations were generated on a PC with an Intel Pentium 4 3.20GHz processor and an NVidia 9600 graphics card.



(a) (b)

(c)

Figure 4: Ray-tracing of procedural implicit surfaces with thin elements or small disjoint components: a) Sphere with microstructure b) Sphere with procedural noise c) Procedural hair.

5.1 Offline Ray-tracing of Procedurally Defined Implicit Surfaces

We tested our ray-tracing algorithm on a wide range of procedurally defined implicit models (see Figs. 3, 4). First, we compare our procedure with other reliable techniques based on uncertain computations, the technique based on Interval Arithmetic described in (Knoll et al., 2009), the technique based on pure Affine Arithmetic described in (de Cusatis Jr. et al., 1999) and the technique based on Reduced Affine Arithmetic described in (Gamito and Maddock, 2007b). The results can be found in the table 1.

The results show that other rendering algorithms based on standard Interval and Affine Arithmetic are significantly slower than our algorithm which is based on Revised Affine Arithmetic. The reason for this is that with Interval Arithmetic algorithms there is an overestimation and with Affine Arithmetic algorithms there is an overestimation as well as a large number of terms in the polynomial form and thus there is a large number of arithmetic operations in the affine operation calculations. Reduced Affine Arithmetic can be used only for algebraic models and it proves to be faster than Interval and standard Affine Arithmetic for algebraic models, however the overestimation of the function in Reduced Affine Arithmetic is wider than the overestimation range for the Revised Affine Arithmetic. Therefore the range of a function based on Revised Affine Arithmetic is tighter than the range in all other techniques and hence the speed of the calculation of the ray-surface intersections is significantly better, especially for models with a large number of non-affine operations. Also, we used special affine forms for set-theoretic operations used in procedurally defined models. The results show that the speed increases drastically in this case.

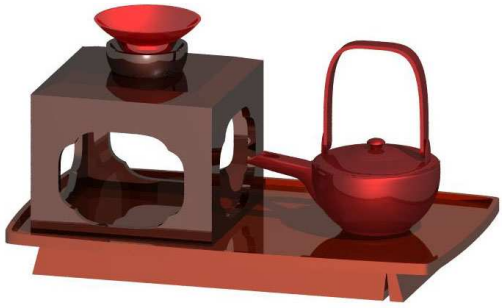


Figure 5: Ray tracing of procedural scenes: Virtual Shikki.

The reliability of the Revised Affine Arithmetic allows us to test our technique on several procedurally defined implicit models with small features or thin surfaces. For example, by using the proposed ray-surface intersection calculation we can reliably render models with internal structure (see Fig. 4a), stochastic procedural models with disjointed components (see Fig. 4b) and even procedurally-defined hair (see Fig. 4c).

Our ray-tracing technique can be applied to complex scenes with a number of procedurally defined implicit surfaces. For example, we show how a functionally defined scene ”Virtual Shikki” (Vilbrandt et al., 2004) can be rendered using our technique(see Fig. 5). Note that because of the thin elements in the models approximate techniques and polygonization do not work well for this scene.

5.2 Real-time Rendering

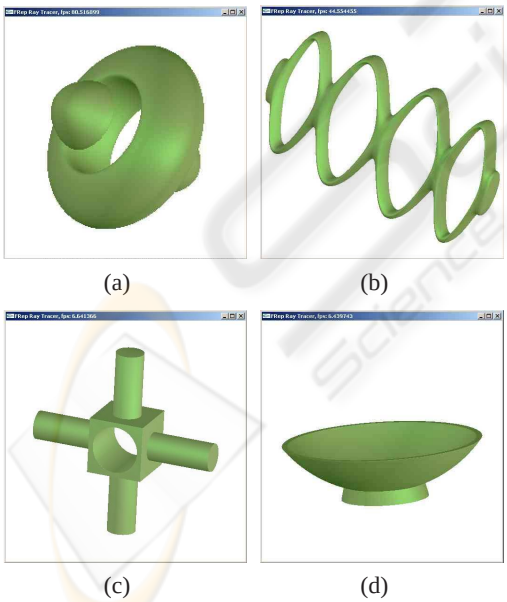
We tested the GPU implementation with several procedurally defined models (see Fig. 6). We compared our technique with the technique presented in (Knoll et al., 2009) for ray-tracing with Interval Arithmetic and Reduced Affine Arithmetic. The results are presented in the table 2. As can be seen from the table, RevAA gives faster ray-surface intersections – hence the higher speed if compared with Interval Arithmetic and near real-time frame rates for the models where RAA is not applicable.

Table 1: Comparison of the ray-tracing procedures for different computational models. IA stands for Interval Arithmetic, AA for Affine Arithmetic, RAA for Reduced Affine Arithmetic, RevAA for Revised Affine Arithmetic and RevAA* for Revised Affine Arithmetic extended by special non-affine operations. The timings for ray-tracing all the rays are shown in seconds.

	Resolution (pixels)	Number of operations All / Non-affine / Multiplications	IA	AA	RAA	RevAA	RevAA*
Mitchell	1280*1024	19 / 6 / 6	38	33	7	6	6
Bretzel	1280*1024	16 / 9 / 9	25	86	22	18	18
Decocube	1280*1024	30 / 17 / 17	17	226	19	13	13
CSG	640*480	96 / 40 / 32	126	129	n/a	18	11
Sphere with trimming	1024*768	142 / 54 / 37	837	2566	n/a	285	83
Sphere with noise	800*600	36 / 11 / 5	17	51	n/a	9	9
CSG with blending	640*480	105 / 42 / 32	266	72	n/a	31	16
Hair	640*480	88 / 34 / 22	4004	1935	n/a	658	23
Sphere with microstructure	640*480	65 / 33 / 22	1006	1079	n/a	293	12
Virtual Shikki	320*240	822 / 306 / 213	29244	50000+	n/a	390	32

Table 2: Comparison of ray-casting procedures on the GPU. IA stands for Interval Arithmetic, AA for Affine Arithmetic, RAA for Reduced Affine Arithmetic and RevAA for Revised Affine Arithmetic. All models were rendered by using only primary rays at a resolution of 512*512 pixels. Timings are shown in FPS (frames per second) and all models were rendered using the same camera parameters.

	IA	RAA	RevAA
Mitchell	28.1	90.1	92.2
Bretzel	83.8	90.2	90.2
Cup	0.56	n/a	5.95
CSG	4.3	n/a	15.6



(a) (b) (c) (d)

Figure 6: Example of real-time ray-casting using the GPU: a) The Mitchell surface b) The Bretzel surface c) The CSG model d) The cup model from the Virtual Shikki.

6 CONCLUSIONS

We presented a technique for ray-tracing of general procedurally defined implicit models based on RevAA. By using the inclusion property of RevAA we were able to obtain reliable ray-tracing of models and at the same time RevAA proved to be the fastest compared to other interval techniques. We also used argument pruning to further accelerate the ray-tracing procedure.

Currently the set of procedurally defined implicit models does not include models with conditional operators. Some research was done using Interval Arithmetic (Diaz, 2008), however further research using Affine Arithmetic and especially RevAA has to be done in this area. Also in this paper we present the affine form for set-theoretic operations based on R-functions and show that the speed of rendering can be dramatically improved. However, not all the functions can be rewritten in the affine form and currently there is no general criterion for the derivation of these forms for arbitrary implicit surface models. This is also an area that merits further research.

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