# SWEEPING BASED CONTROLLABLE SURFACE BLENDING

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Abstract: In this paper, we propose a novel sweeping surface based blending method. A generator defined by the solution of a vector-valued fourth order ordinary differential equation is swept along the two trimlines, which meets the boundary tangent constraints of the primary surfaces at the trimlines. The blending surface generated therefore satisfies both the positional and tangential continuity constraints at the interfaces between the primary surfaces and the blending surface. Since the vector-valued shape control parameters are embedded in the blending surface, its shape can be effectively controlled and manipulated by adjusting these vector-valued shape control parameters. Several surface blending examples are given to demonstrate the applications of the proposed method.

## 1 INTRODUCTION

In computer-aided design and geometric modeling, often it requires to smoothly connect two separate surfaces together. This operation is called surface blending. The surfaces to be connected are called the primary surfaces. The surface which forms a smooth transition between the primary surfaces is called a blending surface. The interfaces between the primary surfaces and the blending surface are called trimlins. The geometric properties at the trimlines form the boundary conditions, which need to be satisfied when a blending surface is generated.

Surface blending has been a research topic for decades especially in computer-aided design. Recently, it has found its way to character modeling in 3D animation. Several surface blending methods have been proposed in the existing literature.

The rolling-ball method is the most popular. It was pioneered by Rossignac and Requicha (1984). According to different surface representations, the rolling-ball blending method can be classified into those of implicit surfaces and parametric surfaces. Lukács (1998) discussed how to blend implicit surfaces using the rolling-ball method. Kós et al. (2000) investigated how to recover constant radius rolling ball blends used in reverse engineering. For the rolling-ball blending of parametric surfaces, two different blends can be identified depending on whether the radius of the rolling ball varies or not. One is the constant-radius rolling-ball blend method, and the other is variable-radius rolling-ball blend method. The constant-radius rolling-ball blend method is studied by Choi and Ju (1989), Harada et al. (1990), Sanglikar et al. (1990), Ying et al. (1991), Barnhill et al. (1993), and Farouki and Sverrisson (1996). The variable-radius rolling-ball blend method is addressed by Harada et al. (1991), Chuang et al. (1995), Chuang and Hwang (1997), Chuang and Lien (1998), and Hartmann (2000).

Cyclides are also useful in some simple blending tasks such as a cylinder obliquely meeting a plane. In general, implicit quartic equations or parametric representations in the form of trigonometrical parameterisation or rational biquadratic Bézier equations are used to describe cyclides. Cyclides were investigated by Allen and Dutta (1997a, 1997b), and Shene (1998).

Partial differential equations (PDEs) based surface blending was pioneered by Bloor and Wilson (1989). In the work, a biharmonic-like fourth order PDE with one vector-valued parameter was used to solve blending problems. The perturbation method developed by Bloor and Wilson (2000) is suitable for solving more complicated surface blending problems than their previously proposed analytical solution. In order to improve the capability of PDE based surface blending, numerical methods were introduced to solve partial differential equations and


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created blending surfaces. Following their analytical work, Bloor and Wilson (1990) developed a collocation method based on B-spline representation of blending surfaces. Finite difference method was proposed by Cheng et al. (1990) to solve a vector-valued fourth order partial differential equation for generation of blending surfaces between two cylinders and between a cone and a cylinder. At the same year, a B-spline finite element method was presented by Brown et al. (1990). Later on, a boundary penalty finite element method was investigated by Li (1998, 1999) and Li and Chang (1999). By studying an efficient semi-analytical and semi-numerical method, You et al. (2004a, 2004b) used a vector-valued fourth order partial differential equation to generate blending surfaces with tangential continuity and a vector-valued sixth order partial differential equation to create blending surfaces with curvature continuity.

In contrast to the applications of partial differential equations in geometric modelling, little work exists where ordinary differential equations were used for geometric modelling and computer animation. Surface creation and manipulation with time-independent ordinary differential equations was investigated by You et al. (2007). By introducing a time variable and considering the dynamic effect, time-dependent ordinary differential equations were applied in animating skin shapes in character animation (You et al. 2008).

Up to now, we have not found any publications investigating ordinary differential equation based surface blending. This paper will address this issue. It uses the solutions to a vector-valued fourth order ordinary differential equation together with the boundary conditions to create a blending surface; and to control the shape of the blending surface through the manipulation of the shape control parameters involved in the equation.

## 2 MATHEMATICAL MODEL AND SOLUTION

Surface blending with tangential continuity is most frequently met in computer-aided design and geometric modelling. In this paper, we concentrate on such surface blending tasks.

The boundary conditions for surface blending with tangential continuity consist of the positional and tangential information of the primary surfaces at the trimlines, i.e., boundary curves and boundary tangents. They can be represented with the equation below.

$$\begin{aligned} u=0 \qquad & \mathbf{X}=\mathbf{C}_0(v) \qquad \frac{\partial \mathbf{X}}{\partial u}=\overline{\mathbf{C}}_0(v) \\ u=1 \qquad & \mathbf{X}=\mathbf{C}_1(v) \qquad \frac{\partial \mathbf{X}}{\partial u}=\overline{\mathbf{C}}_1(v) \end{aligned} \tag{1}$$

where subscript 0 indicates the boundary $u=0$ and subscript 1 indicates the boundary $u=1$, those without an overbar denote boundary curves, and those with an overbar stand for boundary tangents.

In equation (1), all the vector-valued functions $\mathbf{C}_0(v)$, $\overline{\mathbf{C}}_0(v)$, $\mathbf{C}_1(v)$ and $\overline{\mathbf{C}}_1(v)$ have three components. Taking the vector-valued function $\mathbf{C}_0(v)$ to be an example, the three components can be written as $C_{0x}(v)$, $C_{0y}(v)$ and $C_{0z}(v)$ and $\mathbf{C}_0(v)=(C_{0x}(v), C_{0y}(v), C_{0z}(v))$.

A blending surface can be created by sweeping a generator along two trimlines and satisfying the tangential continuity at the trimlines. If the mathematical representation of a blending surface is $\mathbf{S}(u,v)$, the mathematical representation of the generator at the position $v_i$ is $\mathbf{G}(u)=\mathbf{S}(u,v_i)$.

In order to control the shape of a blending surface, we must deform the generator. Through the following fourth order ordinary differential equation, the generator is related to vector-valued shape control parameters which will be used to manipulate the generator.

$$\mathbf{b}\frac{d^4\mathbf{G}(u)}{du^4}+\mathbf{c}\frac{d^2\mathbf{G}(u)}{du^2}+\mathbf{d}\mathbf{G}(u)=0 \tag{2}$$

where $\mathbf{b}$, $\mathbf{c}$ and $\mathbf{d}$ are vector-valued shape control parameters, and $\mathbf{G}(u)$ has three components $G_x(u)$, $G_y(u)$ and $G_z(u)$.

The analytical solution of equation (2) can be taken to be

$$\mathbf{G}(u)=e^{ru} \tag{3}$$

Substituting equation (3) into (2), the ordinary differential equation is changed into an algebra equation below

$$\mathbf{b}r^4+\mathbf{c}r^2+\mathbf{d}=0 \tag{4}$$

Depending on combinations of the vector-valued shape control parameters, equation (4) has different solutions. Here, we only consider the situation of $\mathbf{c}^2=4\mathbf{bd}$ and $\mathbf{c}/\mathbf{b}<0$.

For this situation, the roots of equation (4) are

$$r_{1,2,3,4}=\pm q_1 \tag{5}$$

where

$$q_1 = \sqrt{-\mathbf{c}/(2\mathbf{b})} \qquad (6)$$

With the roots given in equation (5), the solution of equation (2) is

$$\mathbf{G}(u) = \mathbf{c}_1 e^{q_1 u} + \mathbf{c}_2 u e^{q_1 u} + \mathbf{c}_3 e^{-q_1 u} + \mathbf{c}_4 u e^{-q_1 u} \qquad (7)$$

where $\mathbf{c}_1, \mathbf{c}_2, \mathbf{c}_3, \mathbf{c}_4$ are unknown constants which will be determined below.

The unknown constants in equation (7) can be determined by substituting it into boundary conditions (1).

With the obtained solution, we can generate blending surfaces constrained by boundary conditions (1).

## 3 SHAPE CONTROL OF BLENDING SURFACES

In this section, we investigate how the vector-valued shape control parameters are used to control the shape of blending surfaces through a surface blending example below.

This blending task is to find a transition surface which smoothly connects an open surface and a cylinder together. The boundary conditions for this blending task can be written as

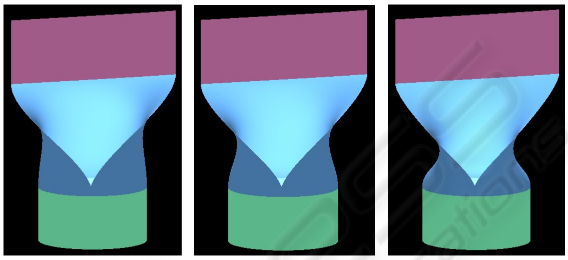
$$\begin{array}{lll}
u=0 & x = d_0 + d_1 v + d_2 e^v & \dfrac{\partial x}{\partial u} = 0 \\
 & y = d_3 + d_4 v & \dfrac{\partial y}{\partial u} = 0 \\
 & z = h_0 & \dfrac{\partial z}{\partial u} = h_1 \\
u=1 & x = -r\cos v & \dfrac{\partial x}{\partial u} = 0 \\
 & y = r\sin v & \dfrac{\partial y}{\partial u} = 0 \\
 & z = 0 & \dfrac{\partial z}{\partial u} = h_2
\end{array} \qquad (8)$$

where $d_i$ (i=0,1,2,3,4), $h_i$ (i=0,1,2), and $r$ are known constants.

For the blending surface given in Figure 1, $d_0 = -0.5001$, $d_1 = 0.1584$, $d_2 = 1.02\times10^{-4}$, $d_3 = 1.5$, $d_4 = -0.4775$, $h = 2.0$, $h_1 = h_2 = -2.0$, and $r = 1.0$.

By solving Eq. (2) subjected to boundary conditions (8), we obtained the analytical solution.

Initially, we set the vector-valued shape control parameters $\mathbf{b} = 1$, and $\mathbf{c} = -3$. The blending surface indicated in Figure 1a was created. Then, we changed the vector-valued shape control parameter $\mathbf{c}$ to -15, the blending surface depicted in Figure 1b was generated. Finally, we further changed the shape control parameter to -30, the blending surface in Figure 1b was changed into that in Figure 1c. Comparing these figures, we can conclude that the vector-valued shape control parameters can be used to change the shape of a blending surface but still maintain the original boundary conditions of the blending task.



a b c

Figure 1: Different shapes of the blending surface created by different vector-valued shape control parameters.

## 4 APPLICATION EXAMPLES

In this section, we give a number of examples to demonstrate the applications of the proposed method in surface blending.

The first example is to generate a blending surface between the frustum of an irregular conical surface and an elliptic cylinder. The boundary conditions for this blending task are

$$\begin{array}{lll}
u=0 & x = Ru_0\cos v & \dfrac{\partial x}{\partial u} = R'\cos v \\
 & y = Ru_0 v\sin v & \dfrac{\partial y}{\partial u} = R' v\sin v \\
 & z = h_0 u_0 & \dfrac{\partial z}{\partial u} = h_0' \\
u=1 & x = x_0 + a\cos\alpha\cos v + h_1 u_{01}\sin\alpha & \\
 & & \dfrac{\partial x}{\partial u} = h_1'\sin\alpha \\
 & y = b\sin v & \dfrac{\partial y}{\partial u} = 0 \\
 & z = z_0 - a\sin\alpha\cos v + h_1 u_{01}\cos\alpha & \\
 & & \dfrac{\partial z}{\partial u} = h_1'\cos\alpha
\end{array} \qquad (9)$$

where $R$, $R'$, $u_0$, $u_{01}$, $h_0$, $h_0'$, $h_1$, $h_1'$, $x_0$, $z_0$, $a$, $b$, and $\alpha$ are known constants.

Using the same method, we obtained the analytical solution of equation (2).

Taking the geometric parameters in the above equation to be: $R=-R'=1.2$, $h_0=-h_0'=2$, $u_0=0.3$, $x_0=u_{01}=1$, $a=0.8$, $b=0.6$, $\alpha=-40^o$, $h_1=1.5$, $h_1'=-2$, and $z_0=-1.9$, and setting the vector-valued parameters $\mathbf{a}$ and $\mathbf{c}$ to 1, and $\mathbf{b}$ to -5, the blending surface created from the analytical solution is shown in Figure 2.

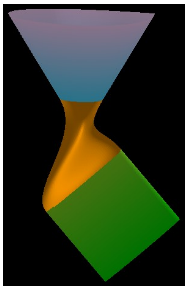


Figure 2: Blending between the frustum of an irregular conical surface and an elliptic cylinder.

The second example is to blend an ellipsoid to an elliptic paraboloid. The boundary conditions for this blending task are

$$\begin{array}{lll} u=0 & x_1=cu_0\cos v & \dfrac{\partial x_1}{\partial u}=-c\cos v \\ & x_2=du_0\sin v & \dfrac{\partial x_2}{\partial u}=d'\sin v \\ & x_3=h_0+h_1u_0^2 & \dfrac{\partial x_3}{\partial u}=h_1'u_0 \\ \mathrm{u}=1 & x_1=a\sin u_1 \cos v & \dfrac{\partial x_1}{\partial u}=a'\cos u_1\cos v \\ & x_2=b\sin u_1 sin v & \dfrac{\partial x_2}{\partial u}=b'\cos u_1\sin v \\ & x_3=h_2+h_3\cos u_1 & \dfrac{\partial x_3}{\partial u}=h_3'\sin u_1 \end{array} \tag{10}$$

where $c$, $d$, $d'$, $h_0$, $h_1$, $h_1'$, $a$, $a'$, $b$, $b'$, $h_2$, $h_3$, $h_3'$, $u_0$, and $u_1$ are known constants.

Using the same treatment, we obtained the analytical solution of this blending task which was used to produce the blending surface shown in Figure 3.

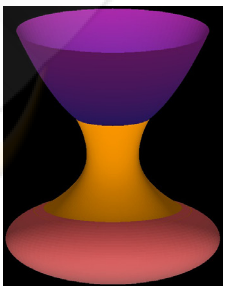


Figure 3: Blending between an ellipsoid and an elliptic paraboloid.

The third example is to investigate the blending between an elliptic paraboloid and a sphere. The boundary conditions for this blending task have the form of

$$\begin{array}{lll} u=0 & x=au_0 \cos v & \dfrac{\partial x}{\partial u}=a'\cos v \\ & y=bu_0\sin v & \dfrac{\partial y}{\partial u}=b'\sin v \\ & z=h_0+h_1u_0^2 & \dfrac{\partial z}{\partial u}=2h_1'u_0 \\ u=1 & x=R\sin u_{01}\cos v & \dfrac{\partial x}{\partial u}=R'\cos u_{01}\cos v \\ & y=R\sin u_{01}\sin v & \dfrac{\partial y}{\partial u}=R'\cos u_{01}\sin v \\ & z=R\cos u_{01} & \dfrac{\partial z}{\partial u}=-R'\sin u_{01} \end{array} \tag{11}$$

where $a$, $a'$, $b$, $b'$, $h_0$, $h_1$, $h_1'$, $R$, $R'$, $u_0$, and $u_{01}$ are known constants.

With the method discussed above, the blending surface was obtained from equation (2) and the above boundary conditions. It was depicted in Figure 4.

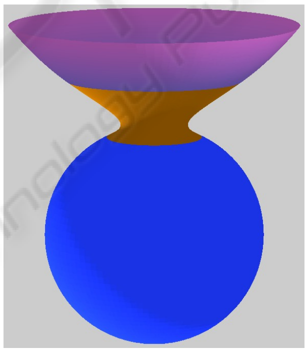


Figure 4: Blending between an elliptic paraboloid and a sphere.

The fourth example is to blend an open surface to a plane at a specified pedal-like curve. The boundary conditions for this surface blending take the form of

$$\begin{array}{lll} u=0 & x_1=a_1\sinh(a_2v+a_3)+a_4\sin v & \\ & \dfrac{\partial x_1}{\partial u}=a_5\sin v & \\ & x_2=a_6\cosh(a_7v)+a_4\cos v & \\ & \dfrac{\partial x_2}{\partial u}=a_5\cos v & \\ & x_3=h_0+\mathrm{e}^{0.2} & \dfrac{\partial x_3}{\partial u}=e^{0.2} \\ u=1 & x_1=a\sin v+b\sin(a_8v) & \dfrac{\partial x_1}{\partial u}=a'\sin v \\ & x_2=a\cos v+b\cos(a_8v) & \dfrac{\partial x_2}{\partial u}=a'\cos v \\ & x_3=h_1 & \dfrac{\partial x_3}{\partial u}=0 \end{array} \tag{12}$$

where $a_i$ ($i$=1,2,…, 8), $h_0$, and $h_1$ are known constants.

Using the analytical solution obtained from equation (2) and the above boundary conditions, the blending surface was created and indicated in Figure 5.

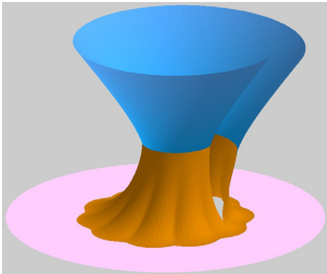


Figure 5: Blending between an open surface and a plane at a specified pedal-like curve.

The fifth example is to generate a blending surface between a circular cylinder and an elliptic hyperboloid of two sheets. The boundary conditions for this blending task are given below.

$$\begin{array}{ll}
u=0 & \\
x=R\cos v & \dfrac{\partial x}{\partial u}=0 \\
y=R\ \sin v & \dfrac{\partial y}{\partial u}=0 \\
z=h_0 & \dfrac{\partial z}{\partial u}=-h_0' \\
u=1 & \\
x=a\sinh 1\cos v & \dfrac{\partial x}{\partial u}=a'\cosh 1\cos v \\
y=b\sinh 1\sin v & \dfrac{\partial y}{\partial u}=b'\cosh 1\sin v \\
z=-h_1\cosh 1 & \dfrac{\partial z}{\partial u}=-h_1'\sinh 1
\end{array} \tag{13}$$

where $R$, $a$, $a'$, $b$, $b'$, $h_0$, $h_0'$, $h_1$ and $h_1'$ are known constants.

The blending surface produced from the solution to equation (2) subjected to the above boundary conditions was given in Figure 6.

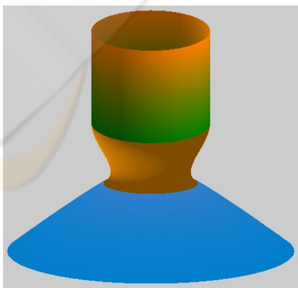


Figure 6: Blending between a circular cylinder and an elliptic hyperboloid of two sheets.

The last example is to smoothly connect two intersecting cylinders together. The boundary conditions for this blending task can be written as

$$\begin{array}{lll}
u=0 & x=s\cos v & \dfrac{\partial x}{\partial u}=0 \\
 & y=s\sin v & \dfrac{\partial y}{\partial u}=0 \\
 & z=h_0 & \dfrac{\partial z}{\partial u}=h_1 \\
u=1 & x=(s+l_1)\cos v & \dfrac{\partial x}{\partial u}=t\cos v \\
 & y=(s+l_1)\sin v & \dfrac{\partial y}{\partial u}=t\sin v \\
 & z=\sqrt{r^2-(s+l_1)^2\cos^2 v} & \dfrac{\partial z}{\partial u}=-\dfrac{t\cos^2 v}{\sqrt{r^2-(s+l_1)^2\cos^2 v}}
\end{array} \tag{14}$$

where $s$, $h_0$, $h_1$, $l_1$, $t$, and $r$ are known constants.

Using the same treatment, analytical solution of equation (2) under boundary conditions (14) was obtained which was used to generate the blending surface indicated in Figure 7.

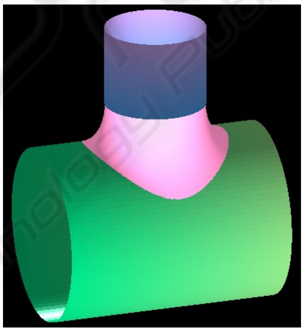


Figure 7: Blending between two intersecting cylinders.

## 5 CONCLUSIONS

With our new surface blending method, a sweeping surface is generated along two trimlines. The key task is to ensure that this sweeping surface satisfies the tangential continuity constraints at the trimlines. The shape of the generator is controlled by the vector-valued shape parameters associated with the fourth order ordinary differential equation. This makes the blending surfaces controllable and applicable for different conditions and applications. The validity of the proposed method is demonstrated with application examples given in this paper.

Since our proposed blending method is based on the closed form solution to a vector-valued fourth order ordinary differential equation, it is simple and efficient in creating blending surfaces. We intend to implement it into a user-friendly interface for interactive shape manipulation of blending surfaces and apply this method to tackle more surface blending problems in our future work.

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