MODELING AND ANALYSIS OF BIRD FLU OUTBREAK WITHIN A POULTRY FARM

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Abstract: Outbreak of avian influenza within a poultry farm is studied mathematically. A system of two nonlinear ordinary differential equations is introduced as a model. Unknown variables of these differential equations are populations of susceptible birds and infected birds. Analysis of the model shows that the most effective measure against outbreak of avian influenza within a poultry farm is a constant removal of infected birds, and that removal of infected birds can solely prevent an outbreak. The analysis also shows that vaccination is effective in conjunction with removal of infected birds, and that vaccination can not prevent an outbreak without the removal of infected birds.

1 INTRODUCTION

Since outbreaks of bird flu (avian influenza) spread widely in 2003, poultry farms have always been threatened by loss due to the disease characteristic of domestic birds. Source of the disease originates in the influenza virus H5N1 endogenous to wild birds. Unlike wild birds, infection of virus to domestic birds leads to serious symptoms that often result in death. Such loss due to infection increases the cost of production per individual. Not only the direct consequence of loss due to infection of bird flu, there are also secondary effects that can harm poultry production, one of which is decrease in demand due to biased view that the bird flu is a zoonosis infectious to human consuming product from domestic birds.

Transmission of bird flu involves three factors, existence of avian influenza virus as the source of the disease, poultry as host, and environment as medium. It is likely to provide opportunities for infection of the virus under inappropriate supervision in handling poultry products and sanitation of entry-exit, *etc*. Vaccination reduces the risk of infection both for humans and for domestic animals. Vaccinated chickens shed much fewer viruses when infected. However a downside of vaccination of chickens emerges in export trade (Breytenbach, 2005).

In this study, a mathematical model is analyzed to investigate effects of vaccination and removal of infected birds. In the following sections, a mathematical model is proposed to analyzed time evolution of susceptible birds and infected birds. Then dominant states of dynamics are determined. Analysis of the model shows that an intrusion by bird flu into a farm wipes out the entire population without removal of infected birds. It also shows that the state free of infection can be maintained with proper removal of infected birds.

2 MODELING INFECTION PROCESS

When a poultry farm is contaminated by bird flu, the population of domestic birds are divided into two

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classes, healthy but susceptible birds and infected birds. As time elapses, some of susceptible birds are infected to become infected birds, while some of infected birds are removed from the population. Suppose that x and y are the population of susceptible birds and infected birds, respectively. The following SI model is analyzed to study infection process as a part of avian-human influenza model (S. Iwami, 2007).

$$\frac{dx}{dt} = c - bx - \omega xy, \tag{1}$$

$$\frac{dy}{dt} = \omega xy - (b+m)y, \tag{2}$$

Parameter c is the rate at which new birds are born, parameter b is the death rate for susceptible birds and infected birds, and m is the additional death rate for infected birds. The term ωxy denotes the number of susceptible birds infected per unit time.

The model (1), (2) is not suitable as far as infection process within a poultry farm is concerned. In a poultry production process, the population is kept constant by shipping of healthy birds to be products when the entire population exceeds the capacity of the farm, or by supply of new birds when vacancies are created by shipping of healthy birds or death of healthy birds or infected birds. Then the first two terms in the right hand side of the equation (1) is replaced with $a\{c-(x+y)\}$. The parameter c represents the capacity of the farm. The parameter a represents the time rate of supply. Some of infected birds stay alive and others die of the disease. However, regardless of being alive or dead, infected birds remain as a source of infection unless they are removed from the population. Suppose that the time rate of removal of infected bird is proportional to the population of infected birds. Then the second term in the right hand side of the equation (2) is replaced with $-my$ where m is a constant representing the removal rate. Under the circumstances, the time evolution of susceptible birds and infected birds are governed by the following system of differential equations.

$$\frac{dx}{dt} = a\{c-(x+y)\} - \omega xy, \tag{3}$$

$$\frac{dy}{dt} = \omega xy - my. \tag{4}$$

3 NULL CLINES AND STATIONARY POINTS

Null clines of the system (3), (4) are curves in the xy plane obtained by setting the right-hand sides equal to zero. The curve defined by

$$y = \frac{a(c-x)}{a+\omega x}. \tag{5}$$

is an x null cline. Let $x(t)$ and $y(t)$ be the x component and the y component of the solution of the system, respectively. Then $x(t)$ is an increasing function of t when it lies below the curve, and it is a decreasing function of t when it is lies above the curve.

The curves defined by

$$y = 0, \tag{6}$$

and

$$x = \frac{m}{\omega} \tag{7}$$

are y null clines. The y null clines (6) and (7) divide the xy plane into four parts determined by the conditions $x < m/\omega$ and $y < 0$, $x > m/\omega$ and $y < 0$, $x < m/\omega$ and $y > 0$, and $x > m/\omega$ and $y > 0$. Then $y(t)$ is an increasing function if it lies in the region defined by $x < m/\omega$ and $y < 0$, or $x > m/\omega$ and $y > 0$. It is a decreasing function if it lies in the region defined by $x > m/\omega$ and $y < 0$, or $x < m/\omega$ and $y > 0$. For $c\omega - m > 0$ or $c\omega - m < 0$, the null clines (5), (6), and (7) divide the xy plane into seven parts (Figures 1, 2).

Stationary points of the system (3), (4) are constant solutions of the system. One stationary point is an intersection of x null cline (5) and y null cline (6), which is

$$(x, y) = (c, 0). \tag{8}$$

Another stationary point is an intersection of the x null cline (5) and y null cline (7), which is

$$(x, y) = \left(\frac{m}{\omega}, \frac{a(c\omega - m)}{\omega(a+m)}\right). \tag{9}$$

In particular, the stationary point (9) becomes

$$(x, y) = (0, c) \tag{10}$$

for $m = 0$. The y component of the stationary point (9) is positive if and only if

$$c\omega - m > 0. \tag{11}$$

It is negative if and only if

$$c\omega - m < 0. \tag{12}$$

Under the condition (11), the stationary point (9) is practically significant, whereas it is unrealistic under the condition (12).

Let $(x, y) = (\xi, \eta)$ be a stationary point of the system (3), (4). Suppose that there is a neighborhood of the point (ξ, η) with the following property. Any solution $(x(t), y(t))$ that starts from a point in the neighborhood at $t = 0$ is contained in a neighborhood

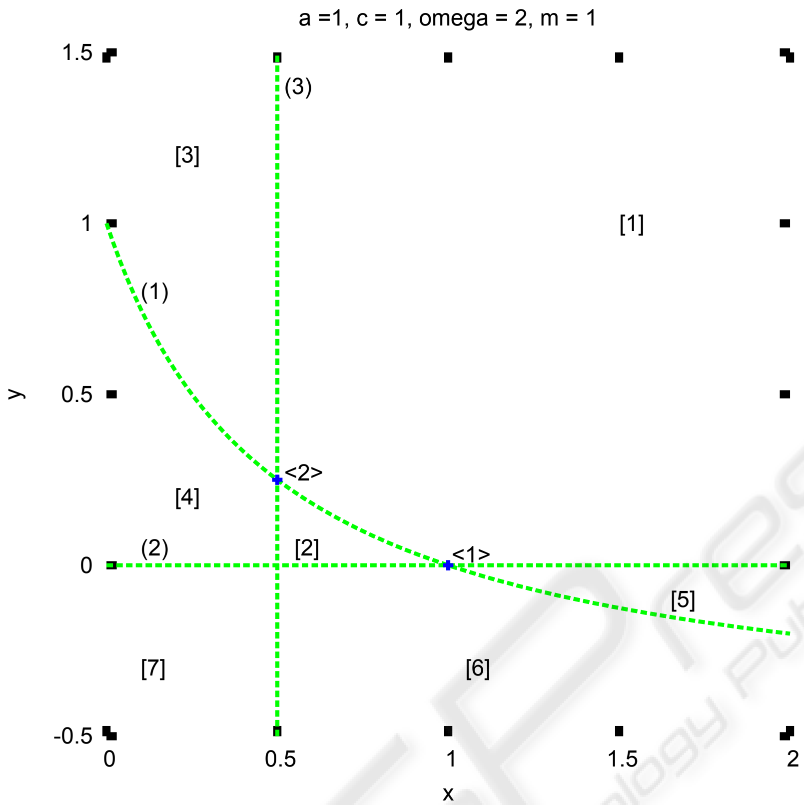

Figure 1: Null clines and stationary points for $c\omega - m > 0$. $a = 1$, $c = 1$, $\omega = 2$, $m = 1$. (1): $y = \frac{a(c-x)}{a+\omega x}$, (2): $y = 0$, (3): $x = \frac{m}{\omega}$, [1]: $\frac{dx}{dt} < 0$, $\frac{dy}{dt} > 0$, [2]: $\frac{dx}{dt} > 0$, $\frac{dy}{dt} > 0$, [3]: $\frac{dx}{dt} < 0$, $\frac{dy}{dt} < 0$, [4]: $\frac{dx}{dt} > 0$, $\frac{dy}{dt} < 0$, [5]: $\frac{dx}{dt} < 0$, $\frac{dy}{dt} < 0$, [6]: $\frac{dx}{dt} > 0$, $\frac{dy}{dt} < 0$, [7]: $\frac{dx}{dt} > 0$, $\frac{dy}{dt} > 0$, $<1>$: $(c, 0)$, $<2>$: $\left(\frac{m}{\omega}, \frac{a(c\omega-m)}{\omega(a+m)}\right)$.

of the stationary point for all $t \geq 0$. Then the stationary point is said to be stable. A stationary point is said to be unstable unless it is stable. In addition to being stable, suppose that there is a neighborhood of the point (ξ, η) with the following property. Any solution $(x(t), y(t))$ that starts from a point in the neighborhood at $t = 0$,

$$\lim_{t \longrightarrow \infty} (x(t), y(t)) = (\xi, \eta).$$

Then the stationary point is said to be asymptotically stable.

The stability of a stationary point $(x, y) = (\xi, \eta)$ depends on the eigenvalues of the Jacobian matrix, which we call A. It is asymptotically stable when all the eigenvalues of A have negative real parts, and it is unstable when at least one eigenvalue has a positive real part (E. A. Coddington, 1984). Let λ_- and λ_+ be the eigenvalues of A. Then

$$\lambda_{\pm} = \frac{\mathrm{tr}A}{2} \pm \frac{\sqrt{(\mathrm{tr}A)^2 - 4\det A}}{2} \tag{13}$$

where

$$\mathrm{tr}A = -(a + \omega\eta) + \omega\xi - m, \tag{14}$$

and

$$\det A = -(a + \omega\eta)(\omega\xi - m) + (a + \omega\xi)\omega\eta. \tag{15}$$

It follows that the steady state solution is asymptotically stable if and only if $\mathrm{tr}\,A < 0$ and $\det A > 0$.

4 DYNAMICS OF INFECTION

For the Stationary point (8), equations (13) - (15) lead to

$$\lambda_- = -a, \qquad \lambda_+ = \omega c - m.$$

Under the condition (11), the stationary point (8) is unstable. Under the condition (12), the stationary point is asymptotically stable.

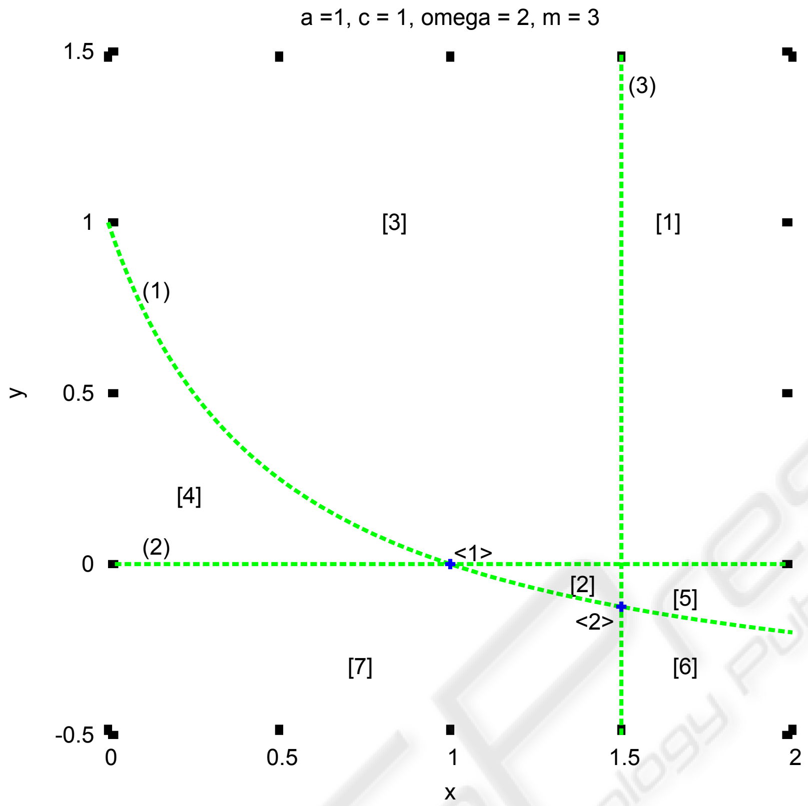

Figure 2: Null clines and stationary points for $c\omega - m < 0$. $a = 1$, $c = 1$, $\omega = 2$, $m = 3$. (1): $y = \dfrac{a(c-x)}{a+\omega x}$, (2): $y = 0$, (3): $x = \dfrac{m}{\omega}$. [1]: $\dfrac{dx}{dt} < 0$, $\dfrac{dy}{dt} > 0$, [2]: $\dfrac{dx}{dt} < 0$, $\dfrac{dy}{dt} > 0$, [3]: $\dfrac{dx}{dt} < 0$, $\dfrac{dy}{dt} < 0$, [4]: $\dfrac{dx}{dt} > 0$, $\dfrac{dy}{dt} < 0$, [5]: $\dfrac{dx}{dt} < 0$, $\dfrac{dy}{dt} < 0$, [6]: $\dfrac{dx}{dt} > 0$, $\dfrac{dy}{dt} < 0$, [7]: $\dfrac{dx}{dt} > 0$, $\dfrac{dy}{dt} > 0$, $< 1 >$: $(c, 0)$, $< 2 >$: $\left(\frac{m}{\omega}, \frac{a(c\omega - m)}{\omega(a+m)}\right)$.

For the stationary point (9), equations (14) and (15) become

$$\mathrm{tr}A = -\frac{a(a+c\omega)}{a+m} < 0, \qquad \det A = a(c\omega - m).$$

Under the condition (11), $\det A > 0$, and the stationary point (9) is asymptotically stable. It is unstable when the inequality (12) holds.

When the inequality (11) holds, the stationary point (8) is unstable, and the stationary point (9) is asymptotically stable. In particular, small perturbation of the stationary point (8) leads to convergence to the stationary point (9). When the inequality (12) holds, the stationary point (8) is asymptotically stable, and the stationary point (9) is unstable. In this case, small perturbation of the stationary point (8) does not affect the state. Figures 3 and 4 show the dynamics of the system (3), (4) under the conditions (11) and (12), respectively. Vectors defined by the right hand sides of the system (3), (4) are plotted. The figures also show some trajectories generated numerically together with the null clines and the stationary points. Those trajectories are generated numerically using the fourth-order Adams-Bashforth-Moulton Predictor-Corrector in PECE mode in conjunction with Runge-Kutta Method to generate values of approximate solution at the first three steps (Lambert, 1973).

5 DISCUSSION

It has been shown in Section 4 that perturbation of the stationary point (8) of the system (3), (4) leads to convergence to the steady state (10) when $m = 0$. Note that the stationary point (8) corresponds to the state free of infection. Note also that the stationary point (10) corresponds to complete infection where all the birds are infected. This result leads to the conclusion that intrusion of bird flu leads to infection of the en-

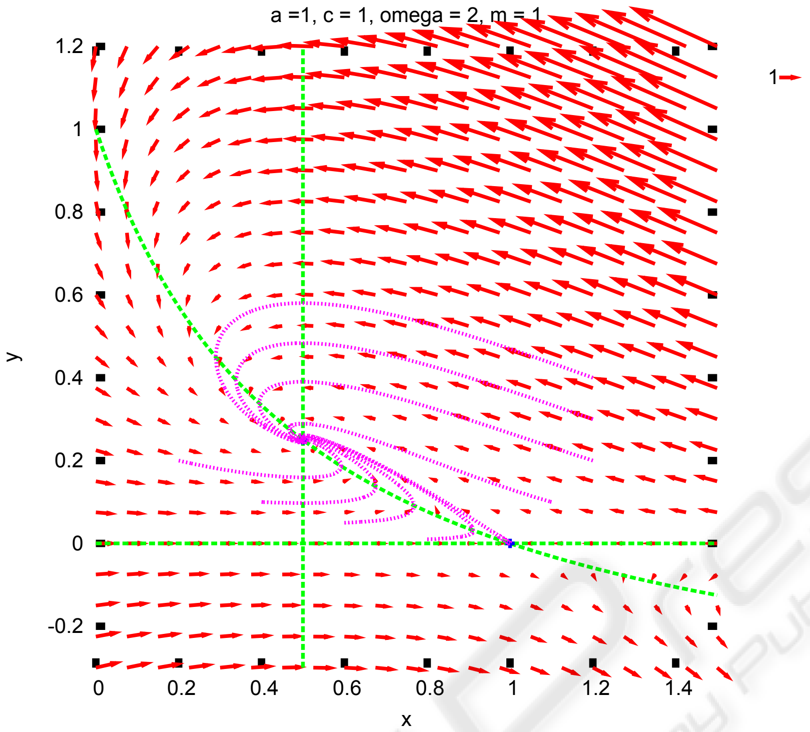

Figure 3: Null clines, stationary points, and vector field for $c\omega - m > 0$. $a = 1$, $c = 1$, $\omega = 2$, $m = 1$.

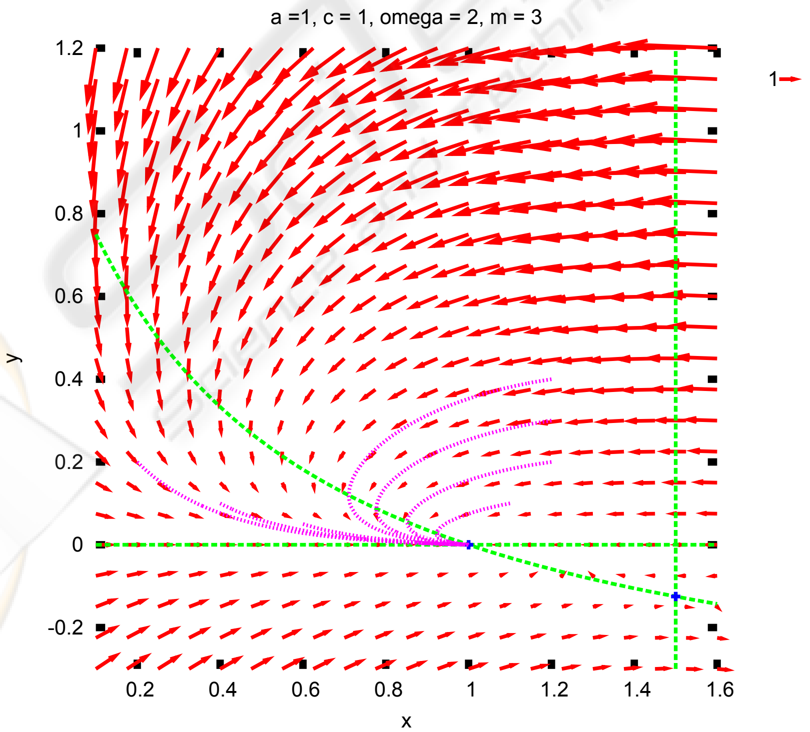

Figure 4: Null clines, stationary points, and vector field for $c\omega - m < 0$. $a = 1$, $c = 1$, $\omega = 2$, $m = 3$.

tire population without removal of infected birds. In this case, vaccination only slows down the infection process, but all the bird become infected eventually.

When $m > 0$, stationary point (8) can be made asymptotically stable by making the value of ωc small or the value of m large. Recall that ω denotes the infection rate, and that m denotes the removal rate of infected birds. The infection rate can be made small by vaccination. The removal rate can be made by secure management. Our analysis based on the model (3), (4) shows that the population can be made secure against infection by proper vaccination and proper removal of infected birds. The results also show that the population cannot be made secure by vaccination alone. But it can be achieved by removal of infected birds alone without vaccination. In conclusion, removal of infected birds is essential for prevention of outbreak of bird flu within a poultry farm.

In practice, so-called rapid test is conducted to detect infection. It is a spot-check in which some birds are taken randomly from a poultry farm. If one bird is found positive for infection, all the birds in the farm are disposed of. In a rapid test, blood or serum samples are collected from cloacae or anuses by swabs, and kept in glycerol to be taken to a laboratory for analysis. Analysis of serum takes approximately forty five minutes, while analysis of dirt takes approximately two hours. In order to make our results practicable, it is necessary to develop a detection system to cover the entire population of a farm in an appropriate time span. Then it will only be necessary to dispose of infected birds, not all the birds in the farm.

We propose the system of ordinary differential equations (3), (4) to analyze infection processes of bird flu with in a poultry farm. In modeling of infection processes of bird flu, there are other approaches including statistical transmission models to study transmission of bird flu from region to region (R. K. Upadhyay, 2008). Infection of bird flu to humans is limited so far, only high pathogenic viruses are contagious from bird to humans, and an infected human is hardly contagious to other humans so far. We focus on infection processes of bird flu within a poultry farm because an economic impact of bird flu on a farm is a significant issue, and because it is essential to understand mechanism of infection from a source to an outbreak within a farm.

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REFERENCES

Breytenbach, J. (2005). Vaccination and biosecurity is the key. *Poultry World; ProQuest Agriculture Journals*, 159(4):33.

E. A. Coddington, N. L. (1984). *Theory of Ordinary Differential Equations*. Robert E. Krieger Publishing Company, Malabar, Florida. Reprint. Originally published: McGraw-Hill, New York, 1955.

Lambert, J. D. (1973). *Computational Methods in Ordinary Differential Equations*. John Wiley & Sons, New York.

R. K. Upadhyay, N. Kumari, V. S. H. R. (2008). Modeling the spreade of bird flu and predicting outbreak diversity. *Nonlinear Analysis*, 9:1638–1648.

S. Iwami, Y. Takeuchi, X. L. (2007). Avian-human influenza epidemic model. *Mathematical Biosciences*, 207:1–25.