CULTURAL SWARMS

Knowledge-driven Framework for Solving Nonlinearly Constrained Global Optimization Problems

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Abstract: In this paper we investigate how diverse knowledge sources interact to direct individuals in a swarm population influenced by a social fabric approach to efficiently solve nonlinearly constrained global minimization problems. We identify how knowledge sources used by Cultural Algorithms are combined to direct the decisions of the individual agents in solving optimization problems using an influence function family based upon a Social Fabric metaphor. The interaction of these knowledge sources with the population swarms produced emergent phases of problem solving. This reflected an algorithmic process that emerged from the interaction of the knowledge sources under the influence of a social fabric using different configurations. This suggests that the social interaction of individuals coupled with their interaction with a culture within which they are embedded provides a powerful vehicle for the solution of nonlinearly constrained optimization problems. The algorithm can escape from the previously converged local minimizers, and can converge to an approximate global minimizer of the problem asymptotically. Numerical experiments show that it is better than many other well-known recent methods for constrained global optimization.

1 INTRODUCTION

The Cultural Algorithm (CA) is a class of computational models derived from observing the cultural evolution process in nature. It is a dual inheritance system that characterizes evolution in human culture at both the macro-evolutionary level, which takes place within the belief space, and at the micro-evolutionary level, which occurs at the population space. Knowledge produced in the population space at the micro-evolutionary level is selectively accepted or passed to the belief space and used to adjust the symbolic structures there. This knowledge can then be used to influence the changes made by the population in the next generation. The basic framework is shown in Figure 1.

Previous work by Reynolds (Reynolds and Saleem, 2003) identified five basic categories of knowledge that were useful in decision making. They were normative knowledge (ranges of acceptable behaviours), situational knowledge (exemplars of successful and unsuccessful solutions), domain knowledge (knowledge of domain objects, their relationships, and interactions), history knowledge (temporal patterns of behaviour), and topographical knowledge (spatial patterns of behaviour). This set of categories is viewed as being complete for a given domain in the sense that all available knowledge can be expressed in terms of one of these classifications.

Reynolds (Reynolds and Saleem, 2003) looked at the roles and contribution of these five generic knowledge classes (normative, topographical, domain, situational, and history knowledge) to the optimization problem-solving process using Evolutionary Programming (EP) as the population model. They observed the emergence of certain

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problem solving phases in terms of the relative performance of different knowledge sources over time. They labelled these phases as the coarse grained, fine grained, and backtracking phases. Each phase is characterized by the dominance of a suite or subset of the knowledge sources that are most successful in generating new solutions in that phase. In fact, the dominant subset of knowledge sources is often applied in a specific sequence within each phase. It appears that one knowledge source produces new solutions that are consequently exploited in by another knowledge source. Transitions between phases occur when the solutions produced by one phase can be better exploited by knowledge sources associated with the next phase. These phases emerged in static, dynamic, and deceptive problem environments.

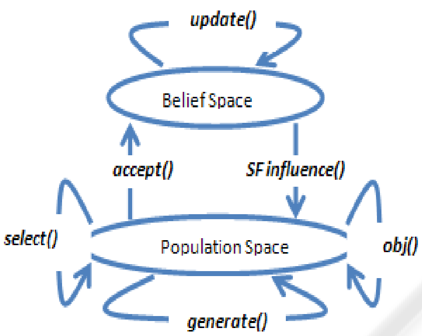


Figure 1: The framework of cultural algorithm.

The coarse grained phase often dominates at the beginning of the search process or when the problem solving landscape changes dynamically, and a search for a new solution must begin anew. In the coarse grained phase topographical knowledge dominates, producing the best new solution over 50% of the time. Situational knowledge is the second most successful, producing the best new solution over 25% of the time. In the fine-grained phase situational knowledge is the most successful at generating the best new individual, while Normative and Domain knowledge are a distant second best. In the backtracking phase all of the knowledge sources are equally successful at generating new solution. Static problems are the exception, in which cases the history component has little effect. Likewise, in non-deceptive environments backtracking occurred less frequently than the other two phases.

Cultural Algorithms can provide a flexible framework in which to study the emergence of complexity in a multi-agent system (MAS) (Reynolds, 1986). In this scenario the Cultural Algorithms framework has been embedded with the recursive porous agent simulation tool (Repast) (North, Collier, and Vos, 2006), producing a toolkit that is called Cultural Algorithms Toolkit (CAT). This tool is used to view the power Cultural Algorithms in solving many Engineering problems and other type of problems (Reynolds and Ali, 2007).

While many successful real-world applications of Cultural Algorithms have been produced, we are interested in studying the fundamental computational processes involved the use of Cultural Systems as problem solvers. In previous work the influence of the knowledge sources have been on individuals in the population only. The goal of this paper is to examine how Cultural Algorithms solve nonlinearly constrained global optimization problems. In our investigation here, we employ a set of standard test problems with differentiable objective function. These test problems are considered diverse enough to cover many kinds of difficulties that constrained global optimization faces. Agents then interact socially via the various knowledge sources to find the optimum after weaving the social fabric to motivate interaction. We then investigate the emergence of social patterns in both the population space and the belief space when the problem is successfully solved.

In this new approach, the Social Fabric influence function is the gear to find the optimal for a certain minimization problem. The agents are connected through a topology that determines connectivity type between agents, through which the fabric is weaved after the initial signal is sent from the Knowledge Sources.

2 PREVIOUS WORK

Several researchers have used different types of Algorithms for solving constrained optimization problems. A quick overview is as follows:

Coello and Mezura (Coello and Mezura-Montes, 2002) implemented a version of the Niched-Pareto Genetic Algorithm (NPGA) (Horn, Nafpliotis, and Goldberg, 1994) to handle constraints in single-objective optimization problems. The NPGA is a multiobjective optimization approach in which individuals are selected through a tournament based on Pareto dominance. However, unlike the first NPGA, Coello and Mezura's approach does not require niches (or fitness sharing (Deb and Goldberg, 1989)) to maintain diversity in the population. The NPGA is a more efficient technique than traditional multiobjective optimization algorithms, because it only uses a sample of the population to estimate Pareto dominance.

Deb (Deb and Goyal, 1996) proposed a Genetic Adaptive Search (GeneAS) to solve engineering optimization problems. He proposed to use both, binary and real encoding for each solution. This approach was tested on three engineering problems (Deb and Goyal, 1996), making emphasis in problems that have discrete and continuous variables. The obvious drawback of the approach is the need of implementing combined operators for the special encoding adopted. Mezura-Montes (Coello and Mezura-Montes, 2002) presented an enhanced Evolutionary Algorithm that doesn't require the definition of extra parameters other than those used by the Evolutionary strategy. The implemented mechanism allows the algorithm to maintain diversity during the process. Reynolds (Reynolds and Peng, 2005) implemented an algorithm that uses the Marginal Value Theorem (MVT) to influence the individuals in the population and drive the process of obtaining better solutions. The algorithm was a more efficient one than the one presented in (Coello and Mezura-Montes, 2002; Coello, 2002) and (Coelho, Souza, and Mariani, 2009).

3 THE SOCIAL FABRIC INFLUENCE FUNCTION

3.1 Concept

Knowledge sources are allowed to influence individuals through a network. From a theoretical perspective we view individuals in the real world as participating in a variety of different networks. Several layers of such networks can be supported within a population. The interplay of these various network computations is designated as the "social fabric". This notion of social fabric has appeared metaphorically in various ways within Computer Science. For example, IBM among others developed tools to reinforce the "social fabric" whereby designers and programmers interact to solve complex problems (Cheng, Patterson, Rohall, Hupfer, and Ross, 2005).

We adapt the Brock-Durlauf model of interactive discrete choice (Brock and Durlauf ;2001) to arbitrary interaction topologies represented by an arbitrary adjacency matrix Γ*:* All individuals faces the binary choice set $S = \{-1, 1\}$*:* Let agent *i* choose ω_i, $\omega_i \in S$, so as to maximize her utility, which depends on the actions of her neighbours: $Ui = U(\omega_i, \tilde{\omega}_{v(i)})$, where $\tilde{\omega}_{v(i)}$ denotes the vector of dimension d_i containing as elements the decisions made by each of agent *i*'s neighbours, $j \in v(i)$. The *I*-vector of all agents' decisions, $\tilde{\omega} = (\omega_1, \dots, \omega_i)$*;* is also known as a *configuration,* and $\tilde{\omega}_{v(i)}$ is known as agent *i*'s *environment.* We assume that an agent's utility function *Ui* is additively separable in a private utility component, which without loss of generality (due to the binary nature of the decision) may be written as $h\omega_i$, $h > 0$*;* in a social interactions component, which is written in terms of quadratic interactions between her own decision and of the expectation of the decisions of her neighbours, $\tilde{\omega}_{v(i)}$, $A\omega_i \varepsilon_i\{\frac{1}{|v(i)|} \sum_{j \in v(i)} J_{ij}\omega_j\}$*;* and a random utility component, $\epsilon(\omega_i)$*;* which is observable only by the individual *i.*

The social fabric is viewed as a computational tool that influences the action and interaction of the various knowledge sources. Informally, we have N networks and M individuals. An individual can be associated with one or more networks. For a given network only certain information is allowed to flow along that network between nodes. Each network can be viewed as being produced by a single thread that links up the participating nodes.

3.2 Weaving the Social Fabric into the CAT System

The networks that comprise the social fabric can emanate from either the Belief Space or the Population Space. In terms of the population, the network could reflect a kinship network or an economic network for example. In terms of the Belief Space, the network could be the Internet, or a local area network, or some other network directly accessible to the knowledge sources. It may be that the Knowledge Sources know something about the networks that they can access but are not sure how those networks are linked up to the low level social networks of the population. In other words, they may be aware of the outer layer of the social fabric, but can only infer about what is in the interior lining.

The experimental framework for the social fabric component is illustrated in Figure 2. The figure shows the initialization step, where each individual first will be affected by one knowledge source (as a special case) that will represent the initial signal to be passed to other individuals. The signal is passed to adjacent individuals in the topology. The individual is represented as a node in the landscape, where the number of connections or hops over which it can transmit this information to its neighbours will correspond to its influence, by a maximum hop distance and will be limited. The number of hops can be either 0 or d meaning either no connections or d connections at a time. The current system is using 0 hops as the individuals don't have any connections with each other.

From the standpoint of the Knowledge Sources they can seed or influence a subset of the population, and that subset may have population level affects but they can only guess what they might be. The key is to "seed" a subset of the population how represent "the weave" to these other networks, assuming that those that represent the weave between the networks have certain properties.

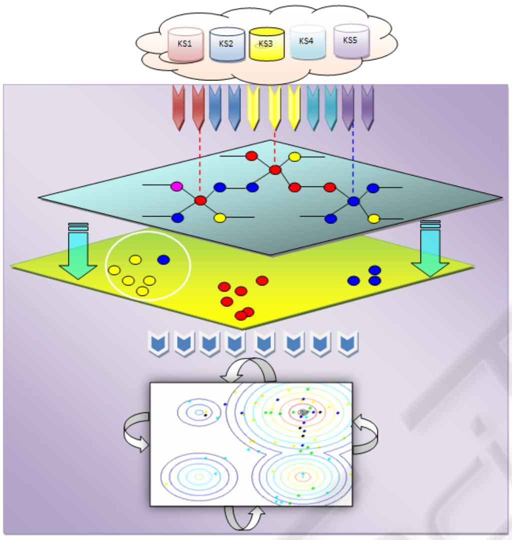


Figure 2: Embedded Social Fabric component in CAT with activated dynamics in the environment.

As a simple configuration in CAT we can simply specify just one network, one that is accessible to the Knowledge Sources in the Belief Space. What we wish to investigate is whether just having access to the Social Fabric is sufficient for the Knowledge Sources to improve the performance of the influence function as opposed to not having a network to distribute their influence at all.

The process starts where each individual first will be affected by one knowledge source (as a special case) that will represent the initial signal to be passed to other individuals. The signal is passed to adjacent individuals in the topology based on the network connectivity. The individual is represented as a node in the landscape, where the number of connections or hops over which it can transmit this information to its neighbours corresponds to its influence. The maximum number of hops can be either 0 or d meaning either no connections or d connections at a time. The simplest case is configured by assuming that each individual is connected to a fixed number of other individuals using a constant topology. The topologies that we used here were taken from work in Particle Swarm Optimization where the impact of various topologies on the communication of local information among particles has been studied.

Several frequently used topologies taken from the Particle Swarm Optimization literature are supported in CAT. For example, the *lBest* model is the simplest form of a local topology is known as the ring the ring model. The *lBest* ring model connects each individual to only two other individuals in the landscape and is shown in figure 3(a). Another frequently used topology is the gbest topology. In this topology each individual in the network is connected to all the individuals in the network as shown in figure 3(b). The advantage of the lbest model may lie in its lower convergence rate relative to the gbest model which may reduce the change of premature convergence to a false peak.

Another topology supported in CAT is the square topology in which each individual has four connections in addition to other individuals in the population.

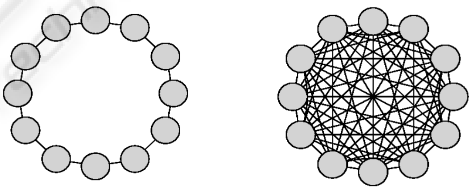


Figure 3: Topologies used in the Social Fabric model for connection between individuals. (a) *lBest* ring topology. (b) *gBest* topology.

At each time step, every individual is influenced by one of the knowledge sources. In this simplest version, Knowledge Sources do not know anything about the network and the selected individuals' position in it. The individual then transmits the name of the influencing Knowledge Source to its neighbours through as many hops as specified. Next, each node counts up the number of Knowledge source bids that it collects. It will have the direct influence from the Knowledge Source that selected it plus the names of the Knowledge Sources transmitted to it by its neighbours. The Knowledge Source that has the most votes is the winner and will direct the individual for that time step. In case of a tie, there are several tie breaking rules implemented in CAT. They include, select the "most frequently used KS, “the least frequently used Knowledge Source”, and “the Knowledge Source that selected the individual this time”, among others. In later sections, we will compare the performance of the *lBest* and square topologies when solving an example Engineering problem, used a benchmark.

The topographical knowledge in the belief space is used to generate a search direction d at a given solution x, and then use it to generate new trial solutions in a neighbourhood of x. The topographical knowledge structure is initialized by sampling a solution in every cell in the grid and creating a list of best cells. The update occurs when a cell is divided into sub-cells when an accepted individual’s fitness value is better than the best solution in that cell, or if the fitness value of the cell’s best solution has increased after a change event is detected.

4 EXPERIMENTAL FRAMEWORK & RESULTS ANALYSIS

4.1 Experimental Framework

The number of individuals is fixed to 100, and the total number of generations is 9000. If a tie is found when the social fabric is weaved the resolution approach used is to use the Knowledge Source that directly affected the individual at that step.

The algorithm will be tested on a set of standard test problems G1-G13 (Hedar and Fukushima, 2006; Hock and Schittkowski, 1981; Koziel and Michalewicz, 1999; Michalewicz and Schoenauer, 1996) except G2, since the objective function of problem G2 is not differentiable. These test problems are considered diverse enough to cover many kinds of difficulties that constrained global optimization faces (Hedar and Fukushima, 2006; Wenxing Z., Ali, M., 2009), and have been used to test performances of algorithms for constrained global optimization.

The algorithm was used to solve each problem 30 times with 100 individuals for the population space and a varying number of generations for each problem depending on its complexity with a maximum of 15000 generations for problems G1 and G15. We experimented with different kinds of topologies through which we found that the best was the lBest topology.

Throughout the next subsection, we will use problem G4 for explaining how our algorithm is used to solve such constrained optimization problems efficiently.

4.2 Analysis of Results

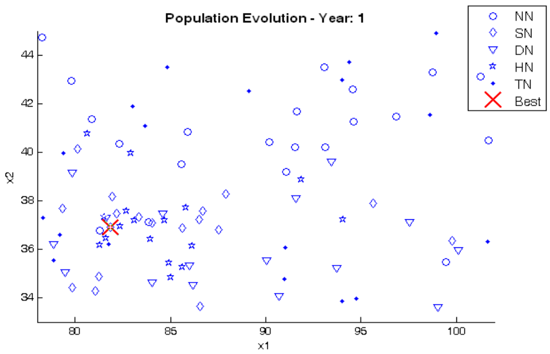
In this section we report the performance of our technique on 13 well-known test problems G1-G13. We put in Table 1 the best known objective function value in the second column. We report in Table 1 the best and the worst optimal values obtained from 30 runs for each test problem. To understand quality of the obtained solutions, we report in Table 1 for each problem the average optimal value and the standard deviation of the obtained objective function values for all 30 runs. Moreover, the success rate, the maximum number of generations before we stop each run, used to obtain these results in 30 runs, are reported in the third and the last columns of Table 1 for each problem respectively.

The approach used by Reynolds in (Reynolds and Peng, 2005) did not assume that there is any kind of connection between the individuals in the population space. Knowledge sources will pass their signals to the individuals at each time step. Our approach uses different topologies to pass abstract information obtained from the Knowledge Sources and then weave the social fabric to allow the individuals to pass the received info through the assumed used topology. The amount of interaction appears to affect the way the system solves the presented problem of a certain complexity. Not only the individual follows the successful Knowledge Source but also tries to adapt through neighbours in the built network to find a better value in the landscape. The results in table 2 show a statistical comparison between our new approach and some other known approaches from literature. When plotting the population swarms, individuals are plotted in different shapes to indicate which knowledge source is in control.

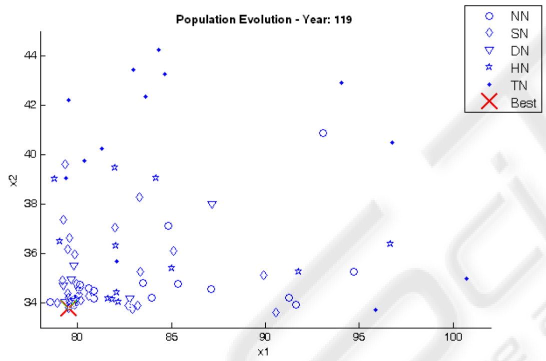
The population swarm plots in Figures 4-a and 4-b show the population (individuals) moving within the problem’s constructed landscape using the lBest topology used by our Social Fabric (SF) approach. Each individual is shape coded to reflect the knowledge source that has influences it in that generation. The best individual of a generation is stressed using a big cross ‘X’. Since the results of the dimensions of problems can be explained similarly we discuss only dimensions x1 and x2. Figure 5 shows a sample of the constructed Social Fabric-lBest topology for problem G4.

Figures 4-a and 4-b show the initial generation and generation 119 when running the system using the Social Fabric-lBest to illustrate how the different knowledge sources work under the influence of the social fabric technique to control individuals. The Topographic Knowledge followers draw the fine-tuning knowledge followers: Situational, Normative, and most of the Domain Knowledge followers.

By generation 119 most of the individuals are swarming around the best. Topographic knowledge individuals are still exploring the space hoping to find a better solution to report it later to the fine-tuning knowledge followers.



(a)



(b)

Figure 4: Population swarm of dimension x1+x2 using the lBest topology. (a) Plotted at generation 1. (b) Plotted at generation 119.

The power behind the algorithm lies in using the bounding boxes that the system calculates at each time step for each of the Knowledge Sources as illustrated in figure 6.

A bounding box represents the standard deviation of each "dot" produced during that generation for the mutation process. It is considered to be the focus of the generation process by each knowledge source. The main idea is how these bounding boxes of the Knowledge Sources interact (overlap area), and how focused these bounding boxes are at each time step. The branching phase of the algorithmic process is shown in Figures 6-a and 6-b, where initially the bounding boxes associated with the Topographic and Normative Knowledge Sources cover most of the space. The exploitation process takes place with time and the bounding boxes for the fine-grained search process have separated from those for the coarse-grained phase (focused search vs. wider search) and have surrounded the optimal value for this pair of dimensions. These bounding boxes are effectively channelling new individuals into this area as can be seen in figure 6-b.

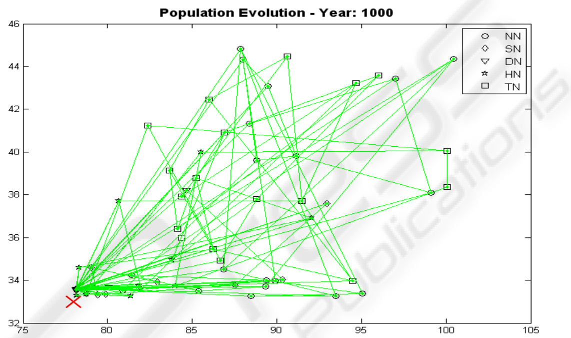
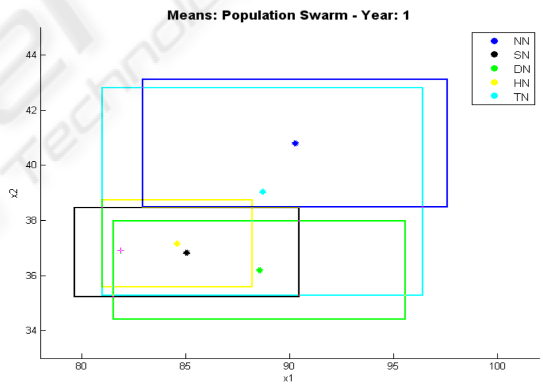
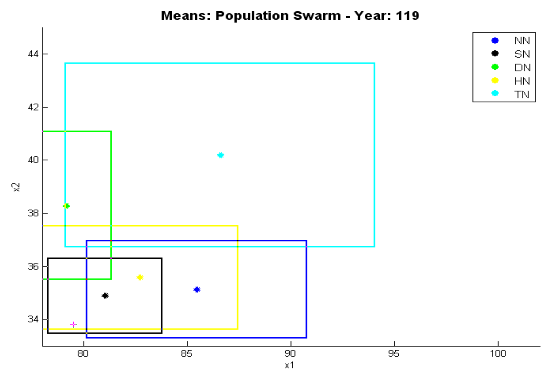


Figure 5: A sample Social Fabric swarm plot for problem G4 using lBest topology.



(a)



(b)

Figure 6: Knowledge Swarm Plot of dimension x1+x2. (a) Plotted at generation 1. (b) Plotted at generation 119.

Table 1: Test results for problems G1-G13.

Prob.	Opt.	Succ. (%)	Best	Av. opt.	Worst	S.D.	# generations
G1	-15	100	-14.99993	-14.99986	-14.99984	0.000140	15000
G3	1	100	0.999987	0.999977	0.999971	0.000032	9000
G4	-30665.539	100	-30665.52	-30665.47	-30665.40	0.055110	9000
G5	5126.4981	100	5126.499	5126.501	5126.520	0.098000	1000
G6	6961.81388	100	-6961.81	-6961.779	-6961.550	0.088575	1000
G7	24.3062091	100	24.30590	24.30595	24.306122	0.000400	90000
G8	0.095825	100	0.095825	0.095825	0.095825	0.000000	1000
G9	680.630057	100	680.6300	680.6310	680.6315	0.015212	10000
G10	7049.250	100	7049.244	7049.247	7049.253	0.050000	1000
G11	0.75	100	0.750000	0.700001	0.750004	0.000002	1000
G12	1	100	1.000000	0.999999	0.999989	0.000018	1000
G13	0.0539498	100	0.053950	0.053953	0.053959	0.000139	15000

Table 2: Comparison of test results for problems G1-G13.

Prob.: opt.		PSO	SR	ASCHEA	FSA	Our alg.
	Best	-15.0001	-15	-15	-14.999105	-14.99993
G1: -15	Av.	-13.2734	-15	-14.84	-14.993316	-14.99986
	Worst	-9.7012	-15	N.A.	-14.979977	-14.99984
	Best	1.0004	1.000	1	1.0000015	0.999987
G3: 1	Av.	0.9936	1.000	0.99989	0.9991874	0.999977
	Worst	0.6674	1.000	N.A.	0.9915186	0.999971
	Best	-30665.5398	-30665.539	-30665.5	-30665.5380	-30665.52
G4: -30665.539	Av.	-30665.5397	-30665.539	-30665.5	-30665.4665	-30665.47
	Worst	-30665.5338	-30665.539	N.A.	-30664.6880	-30665.40
	Best	5126.6467	5126.497	5126.5	5126.4981	5126.499
G5: 5126.4981	Av.	5495.2389	5128.881	5141.65	5126.4981	5126.501
	Worst	6272.7423	5142.472	N.A.	5126.4981	5126.520
	Best	-6961.8371	-6961.814	-6961.81	-6961.81388	-6961.81
G6: 6961.81388	Av.	-6961.8370	-6875.940	-6961.81	-6961.81388	-6961.779
	Worst	-6961.8355	-6350.262	N.A.	-6961.81388	-6961.550
	Best	24.3278	24.307	24.3323	24.310571	24.30590
G7: 24.3062091	Av.	24.6996	24.374	24.6636	24.3795271	24.30595
	Worst	25.2962	24.642	N.A.	24.644397	24.306122
	Best	0.095825	0.095825	0.09582	0.095825	0.095825
G8: 0.095825	Av.	0.095825	0.095825	0.09582	0.095825	0.095825
	Worst	0.095825	0.095825	N.A.	0.095825	0.095825
	Best	680.6307	680.630	680.630	680.63008	680.6300
G9: 680.630057	Av.	680.6391	680.656	680.641	680.63642	680.6310
	Worst	680.6671	680.763	N.A.	680.69832	680.6315
	Best	7090.4524	7054.316	7061.13	7059.86350	7049.244
G10: 7049.250	Av.	7747.6298	7559.192	7497.434	7509.32104	7049.247
	Worst	10533.6658	8835.655	N.A.	9398.64920	7049.253
	Best	0.7499	0.750	0.75	0.7499990	0.750000
G11: 0.75	Av.	0.7673	0.750	0.75	0.7499990	0.700001
	Worst	0.9925	0.750	N.A.	0.7499990	0.750004
	Best	1.0000	1.000000	N.A.	1.000000	1.000000
G12: 1	Av.	1.0000	1.000000	N.A.	1.000000	0.999999
	Worst	1.0000	1.000000	N.A.	1.000000	0.999989
	Best	0.05941	0.053957	N.A.	0.0539498	0.053950
G13: 0.0539498	Av.	0.81335	0.057006	N.A.	0.2977204	0.053953
	Worst	2.44415	0.216915	N.A.	0.4388511	0.053959

5 CONCLUSIONS

The Cultural Algorithm is a stochastic optimization method that uses evolutionary algorithmic mechanisms to model cultural evolution and social behaviors. Just as cultural evolution contributes to the adaptability of human society, CA provides an additional degree of adaptability to evolutionary computation. In this paper we have introduced the social fabric influence function in the Cultural Algorithms framework. This influence function is used to produce population and knowledge swarms that are used to optimally solve nonlinearly constrained optimization problems. The SF metaphor allows the knowledge sources to distribute their influence through a social network. We apply this approach to a set of well-known nonlinearly constrained optimization problems. It turns out that the used topology, frequency of distribution of influence, and conflict resolution play an important role in how efficiently the system produces knowledge and population swarms that represent structural patterns to solve problems.

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