TRAINING FOURIER SERIES NEURAL NETWORKS TO MAP CLOSED CURVES

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Abstract: The paper presents the closed curve mapping method using several Fourier series neural networks having one input and one output only. The proposed method is also excellently fitted for a lossy compression of closed curves. The method does not require a large number of operations and may be used for multi-dimensional curves. Fourier series neural networks are especially well fitted for described purposes.

1 INTRODUCTION

Fourier Series Neural Networks (FSNNs) belong to the class of orthogonal neural networks which have been outlined, among other publications, in (Zhu, 2002), (Sher, 2001), (Tseng, 2004), (Rafajlowicz, 1994), (Halawa, 2008). These are feedforward networks. The output value of SISO FSNNs (Single-Inputs Single-Output FSNNs) is given by the following formula

$$\hat{f}(u) = \sum_{n=0}^{N} w_n \cos(a_n u) + \sum_{m=1}^{M} w_{N+m} \sin(b_m u), \qquad (1)$$

where u is the network input, N and M are sufficiently large numbers, $w_1,w_2,\ldots,w_{N+M}$ are the network weights, which values subject to changes during training process, $a_0,a_1,\ldots,a_N$, $b_1,b_2,\ldots,b_M$ are some natural numbers which meet the conditions $a_0 \neq a_1 \neq \ldots \neq a_N$ and $b_1 \neq b_2 \neq \ldots \neq b_M$. If $N=M$ and $a_0=0,a_1=1,\ldots,a_N=N$ and also $b_1=1,b_2=2,\ldots,b_N=N$, then (1) is the Fourier series.

FSNNs have numerous essential advantages of which the following are worth mentioning:

- the output is in linear relation to the weights,
- large speed of training caused by the lack of local minima of some popular cost functions (such situation is present, for instance, for the sum of error squares),
- the relationship between the number of neurons and the number of inputs and outputs is known,
- values of weights may be easily interpreted,
- there are no problems with arrangement of centres which exist in networks with radial basic functions (RBFs).

The functions $\cos(a_1u),\ldots,\cos(a_Nu)$ and $\sin(b_1u),\ldots,\sin(b_Mu)$ are orthogonal on the interval $[0,2\pi)$ and $[-\pi,\pi)$. The scale of u shall be selected so as the value of SISO FSNN input falls within one of these intervals.

FSNN have the property of periodicity, i.e.

$$\hat{f}(u) = \hat{f}(u + \gamma 2\pi), \qquad (2)$$

where γ is integer. Because of this property, FSNNs are especially well fitted for mapping closed curves.

Further in the text, the closed curve mapping method is illustrated for several SISO FSNNs. The outcomes presented refer to networks of various sizes. In Section 3, there is the procedure to be used when images has not all black pixels belonging to the closed curve. Section 4 is dedicated to the procedure applicable to disturbed images. Section 5 includes short comparison with some other methods for closed curves mapping.

2 METHOD OF TRAINING FSNNS TO MAP CLOSED CURVES

Figure 1 illustrates an example of monochromatic free-of-distortion image, resolution of 174x94 pixels, with a two-dimensional closed curve of a tank-like shape.

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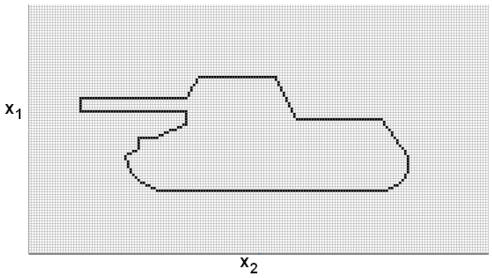

Figure 1: The closed curve of a tank-like shape.

We are denoting the number of black pixels in Figure 1 as P. Let's assume an arbitrary black pixel as the starting point (in the paper, the extreme left-hand upper end of the barrel was used). Moving along the closed curve in selected direction, let's number all successive black pixels from 0 to P-1. It is recommended to make the numbering in such a way that the neighbouring pixels have assigned numbers differing by no more than 2.

Let $x_1(k)$ and $x_2(k)$ denote the coordinates x_1 and x_2 of k-th pixel, respectively, where k=1,2,...,P-1. Let's create two SISO FSNNs. One of them shall be trained using the training set composed of the pairs $\{u_k, x_1(k)\}_{k=0}^{P-1}$, where $x_1(k)$ is the desired output value, when the network input equals to $u_k=2\pi k/P$. The other network is trained using the training set $\{u_k, x_2(k)\}_{k=0}^{P-1}$. We can notice that the condition $0 \le u_k < 2\pi$ is met. As the cost function, we may select, for example, the mean square function

$$J = \frac{1}{P}\sum_{k=0}^{P-1}\left(\hat{f}(u_k) - r(u_k)\right)^2, \quad (3)$$

where $r(u_k)$ is the desired output value of the trained network (for the first FSNN, $r(u_k)=x_1(k)$, while for the second one, $r(u_k)=x_2(k)$). If the minimization is made for the function (3), the least squares method is the most convenient to determine the values of weights (Groß, 2003). Upon completing the training process for the network, the closed curve may be approximated by plotting the pixels with the coordinates $\left(\hat{f}_{r1}(u_k), \hat{f}_{r2}(u_k)\right)$,where $\hat{f}_{r1}(u_k), \hat{f}_{r2}(u_k)$ denotes output values, rounded to the closest integer values, of the first and the second networks, when their inputs are equal to u_k. Since the points plotted in this way need not contact each other, it is additionally recommended to run interpolation of the curve under consideration by interconnecting with straight line the pixels of the coordinates $\left(\hat{f}_{r1}(u_k), \hat{f}_{r2}(u_k)\right)$ and $\left(\hat{f}_{r1}(u_{k+1}), \hat{f}_{r2}(u_{k+1})\right)$, which are not neighbouring each other.

The method presented may be also used, in an analogous way, for d-dimensional closed curves, where d is natural number greater than or equal to 2. The proposed algorithm can by concisely outlined by the following items:

a) find the starting point and number all the black pixels in succession.

b) create d-number of FSNNs.

c) train all FSNNs. For the i-th network, the training set $\{u_k, x_i(k)\}_{k=0}^{P-1}$, where i=1,2,...,d is used.

d) blacken appropriate pixels and run interpolation, if applicable.

Thanks to numbering the pixels as proposed, networks are taught the functions which include no violent changes. The property of periodicity of FSNNs makes this type of networks to be very well suitable for the purpose considered. The other advantages of FSNNs, as listed in the introduction, are also of great importance. It is worth to mention that due to application of the method proposed, the lossy compression of the closed curve is attained because the number of FSNN's weights is most often much less than the number of pixels of the curve under consideration. By changing the number of FSNN's neurons, we can modify the degree and the quality of the compression.

Below, there are results from training FSNNs to reconstruct the curve shown in Figure 1. The pixel in the left-hand upper corner of this figure has the coordinates (0,0) while the pixel in the right-hand bottom corner has the coordinates (174,94). Figures 2a and 2b show reproduced shape of the curve from Figure 1 for N=10 and N=30, respectively. During computer simulations, it was assumed that $a_0=0, a_1=1, \ldots, a_N=N$ and $b_1=1, b_2=2, \ldots, b_N=N$. The results shown are those without interpolation. The cost function (3) was minimized.

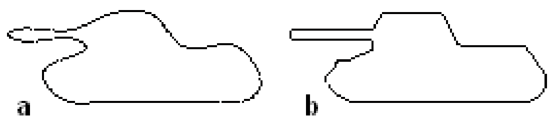

Figure 2: Reconstructed shape of the curve shown in Figure 1 a) for N=10, b) for N=30.

The method presented may be also applied to train FSNNs the shape of a closed curve with added line, e.g. the tank with an appended antenna as shown in Figure 3. However, the reconstructed image would be deformed by Gibbs effect, which will occur due to stepwise change of at least one co-ordinate value for pixels with successive numbers.

Figs. 5 and 6 provide results attained for the proposed method used to map the closed curves illustrated in Fig. 4. These curves were selected to get clear indication of distinctive features of the resulting mapping. For the sake of limited volume of the paper no further experimental results are presented.

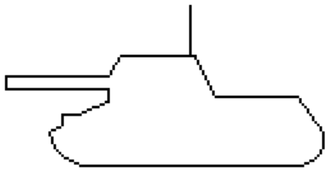

Figure 3: The tank from Figure 1 with an appended antenna.

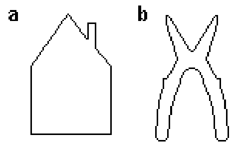

Figure 4: The closed curves a) house b) pliers.

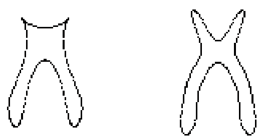

Figure 5: Reconstructed shape of the closed curve shown in Figure 4b for N=5 and for N=10.

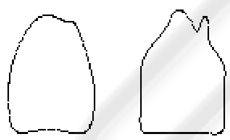

Figure 6: Reconstructed shape of the closed curve shown in Figure 4a for N=5 and for N=10.

3 TRAINING FSNNS TO MAP CLOSED CURVES FROM IMAGES WHERE SOME PIXELS WERE NOT MARKED OUT

It is enough to introduce minor modifications in the situation when we have the closed curve picture where not all pixels were marked out of this curve. An example of such image is shown in Figure 7.

Further in this paper, an assumption is made that the distance between two points is counted by means of norm L_1 (i.e. the so called Manhattan distance,

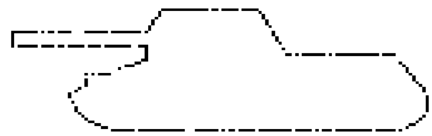

Figure 7: Picture of the closed curve where not all pixels of this curve are marked.

called also the taxicab distance). For instance, let's assume that the points w and q have the coordinates $w_1, w_2, \ldots, w_d$ and $q_1, q_2, \ldots, q_d$, respectively. Then, the distance between them equals to $\sum_{i=1}^{d} |v_i - q_i|$. In the situation under discussion, at first all the pixels shall be interconnected so as the sum of lengths of all connections was the shortest and so that these connections do not cross each other. Modification of the method consists in a change of pixel numbering way and assuming that the value P equals to the sum of lengths of all created connections. Natural numbers are assigned to the pixels and these numbers are the distances from the starting pixel counted along the route created by determined connections. Let's note that these would not always be the successive natural numbers.

4 TRAINING FSNNS TO MAP CLOSED CURVES FROM DISTURBED PICTURES

Figure 8 illustrates an example of a picture with closed curve disturbances where not all pixels related to this curve are marked out.

Figure 8: Picture with disturbances of the closed curve where not all pixels of this curve are marked out.

In situations similar to the case shown in Fig. 8, the picture may be first divided into smaller parts of identical dimensions.

Then, for each portion including the number of marked out pixels higher than r, where r is sufficiently large natural number selected according to a priori knowledge about disturbances, the average coordinates for all points belonging to the specific portion are calculated. These averages are then treated as the coordinates of pixels creating the image of closed curve with not all pixels of this curve being marked out. Then, the procedure outlined in Section 3 is used.

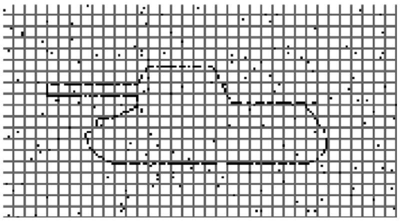

Figure 9: Figure 8 divided into smaller portions.

5 COMPARISON WITH SOME OTHER METHODS

A mapping method for closed curves using Fourier series is presented in (Ünsalan, 1998). This method makes use of polar/spherical coordinates. However, it is inapplicable when the radius is not a function, i.e. the same value of the turning angle may correspond to several different radius values. This constitutes its essential drawback which drastically reduces the number of closed curves which can be mapped. The method outlined in (Ünsalan, 1998) is applicable to map the curve shown in Fig. 4a but it may not be directly applied to map the curves shown in Figs 1 and 4b. The method presented in this paper is free of that drawback. The method proposed by the author has several similar advantages as that given in (Ünsalan, 1998), e.g. it requires less calculations than the 3L fit method (3LF) (Lei 1996). A valuable virtue of the proposed method is the fact that upon teaching FSNNs, we always attain the shape of closed curve which is not always a case when implicit polynomials are used with the methods of least squares fit (LSF), bounded least squares fit (BLSF) and 3L fit (Lei, 1996). The LSF, BLSF and 3LF methods are suitable for situation described in Section 3. If it is a priori known that the closed curve is given by equation of geometrical figure or shape, better results could be reached by specific-shape-dedicated methods, e.g. the method presented in (Pilu, 1996).

6 CONCLUSIONS

The presented method is well-suited for approximating closed curves. These curves may be presented on a monochromatic picture. The FSNNs, thanks to the property (2) and essential advantages listed in Section 1, are especially suitable for described purpose. As the problem size rises, it is enough to increase the number of FSNNs used. The FSNNs are taught the functions which include no rapid changes. The presented method may be used for the lossy compression of closed curves. It may be also used to find the shape of closed curves out of disturbed or incomplete pictures.

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REFERENCES

Zhu C, Shukla D, Paul F.W., 2002, Orthogonal Functions for System Identification and Control, *In Control and Dynamic Systems: Neural Network Systems Techniques and Applications*, Vol. 7, pp. 1-73 edited by: Leondes C.T., Academic Press, San Diego

Sher C.F, Tseng C.S. and Chen C.S., 2001, Properties and Performance of Orthogonal Neural Network in Function Approximation, *International Journal of Intelligent Systems,* Vol. 16, No. 12, pp. 1377-1392

Tseng C.S., Chen C.S., 2004, Performance Comparison between the Training Method and the Numerical Method of the Orthogonal Neural Network in Function Approximation, *International Journal of Intelligent Systems*, Vol. 19, No.12, pp. 1257-1275

Rafajłowicz E., Pawlak M., 1997, On Function Recovery by Neural Networks Based on Orthogonal Expansions, *Nonlinear Analysis, Theory and Applications*, Vol. 30, No. 3, pp. 1343-1354, Proc. 2nd World Congress of Nonlinear Analysis, Pergamon Press

Groß J., 2003, *Linear Regression*, Springer-Verlag, Berlin

Halawa K., 2008, Determining the Wegihts of a Fourier Series Neural Network on the Basis of Multidimensional Discrete Fourier Transform, *International Journal of Applied Mathematics and Computer Science*, Vol. 18, No. 3, pp. 369–375

Ünsalan C., Erçil A., 1998, Fourier Series Representation For Implicit Polynomial Fitting, *Bogazici Universitesi Research Report*, FBE-IE-08/98-09

Lei, Z., Blane, M.M., Cooper, D.B., 1996, 3L Fitting of Higher Degree Implicit Polynomials, *Proceedings of the 3rd IEEE Workshop on Applications of Computer Vision* (WACV '96), pp. 148-153

Pilu M., Fitzgibbon A.W., Fisher R.B., 1996, *Proceedings of IEEE International Conference on Image Processing,* Vol 3, pp. 599-602