RANDOM VARIATES GENERATING METHODS OF TIME-BETWEEN-FAILURES FOR THE REPAIRABLE SYSTEMS UNDER AGE-REDUCTION PREVENTIVE MAINTENANCE

Chun-Yuan Cheng
Dept. of Industrial Engineering and Management, Chaoyang University of Technology
168, Jifong E. Rd., Wufong, Taichung County, 41349, Taiwan

Renkuan Guo
Department of Statistical Sciences, University of Cape Town, Cape Town, South Africa

Mei-Ling Liu
Dept. of Industrial Engineering and Management, National Taipei University of Technology, Taipei, Taiwan
The Office of Academic Affairs, Chaoyang University of Technology, Taichung County, Taiwan

Keywords: Simulation method, Time-between-failure random variates, Preventive maintenance, Age reduction.

Abstract: Based on the theoretical model, a numerical method is usually necessary for obtaining the optimal preventive maintenance (PM) policy for a deteriorating system since the theoretical model becomes complicated when the system's hazard rate function is changed after each PM. It makes the application of the theoretical model not suitable for real cases. Moreover, the theoretical model assumes using infinite time span to obtain the long-term expected number of failures. Yet, in reality, the deteriorating systems always have a finite life time. Hence, an optimal solution might not be resulted as compared to the infinite time span. Therefore, we consider using the simulation method to obtain a range of the near-optimal PM policy. The critical step of the simulation method for obtaining a near-optimal PM policy is the generation of the random variates (RV). In this research, three methods are developed to generate the required RVs of the time-between-failures (TBF) for the finite-time-span preventive maintenance model with age reduction effect. It is found that there are no significant differences among three proposed RV generating methods when comparing the dispersion of the generated RV's. However, the rejection method is the simplest method for obtaining the near-optimal PM policies. Examples of the near-optimal PM policies are also presented in this paper.

1 INTRODUCTION

Based on the theoretical model, a numerical method is usually necessary for finding the optimal preventive maintenance (PM) policy for a deteriorating system since the theoretical model becomes complicated when the system's hazard rate function is changed after each PM. It makes the application of the theoretical model not suitable for real cases. Furthermore, by the theoretical model, the optimal policy is obtained based on the long-term failures occurrence under the assumption of the infinite time span. Yet, in reality, the life time of a system is always finite. Hence, the optimal solution from the theoretical model may not suitable for a single system with finite life time. In practical, a near-optimal PM policy might be good enough for the real applications. In order to obtain a near-optimal PM policy for the real situations, the simulation method is applied to generate random variates (RV) of the time between failures (TBF). However, recent literature survey has shown that

Cheng C., Guo R. and Liu M. (2009).
RANDOM VARIATES GENERATING METHODS OF TIME-BETWEEN-FAILURES FOR THE REPAIRABLE SYSTEMS UNDER AGE-REDUCTION PREVENTIVE MAINTENANCE.
In *Proceedings of the 6th International Conference on Informatics in Control, Automation and Robotics - Robotics and Automation*, pages 325-330
DOI: 10.5220/0002219903250330

little research has been done to obtain a near-optimal PM policy by using the simulation method.

The critical step of the simulation method for obtaining a near-optimal PM policy is the generation of the random variates (RV). In this research, three methods are developed to generate the required RVs of the time-between-failures (TBF) for the finite-time-span PM model with age reduction effect.

Based on the simulation method developed by Cheng (2005), the first proposed method applies the inverse transformation method to generate the random variates (RV) of the time between failures (TBF) for a PM model with age reduction effect. The algorithm assumes that the occurrence time of the last failure in the i^{th} PM cycle is irrelative to the occurrence time of the first failure in the i+1st PM cycle. This RV generating method for the TBF is called "the offset inverse transformation method" in this paper.

Intuitively, however, the occurrence time of the first failure in the i+1st PM cycle is affected by the occurrence time of the last failure in the i^{th} PM cycle since the failure occurrence of the system follows the non-homogenous Poisson process (NHPP) and the PM is imperfect (i.e., the PM will not renew the system to zero failure rate). Therefore, in this research, we have developed a modified inverse transformation method for generating the RVs of the TBF which is called "the trace-back inverse transformation method". The second proposed method assumes the occurrence time of the first failure in the i+1st PM cycle is affected by the occurrence time of the last failure in the i^{th} PM cycle.

Furthermore, since the rejection method is often applied to generating RVs of complicated distributions, we also present the third proposed method, the rejection method, for generating the RVs of the TBF under the age-reduced PM model. In this paper, the algorithms and the simulation results for the above three RV generating methods are presented and compared. An example of finding the near-optimal PM policy is provided by using the rejection method of RV generation.

2 THE BACKGROUND FOR THE THEORITICAL MODEL

2.1 Nomenclature

- L — the finite life time span for the system or equipment
- T — the time interval of each periodic PM
- N — the number of PM performed in the finite life time span (L)
- k_i — the generated number of failures in the i^{th} PM cycle, i = 0, 1, …, N
- $x_{i,j}$ — the generated time between the j-1st and the j^{th} failures in the i^{th} PM cycle, i = 0, 1, …, N; j = 1, 2, …, k_i
- $t_{i,j}$ — the generated occurrence time of the j^{th} failure in the i^{th} PM cycle where $t_{i,j} = t_{i,j-1} + x_{i,j}$
- x_{i,k_i+1} — the generated time between the last (k_i^{th}) and the k_i+1st failures (not existing) in the i^{th} PM cycle
- t_{i,k_i+1} — the generated occurrence time of k_i+1st failure (not existing) in the i^{th} PM cycle, i.e., t_{i,k_i+1} exceed the time of the i^{th} PM cycle
- γ — the reduced age after each PM
- $w_{i,j}$ — the generated effective occurrence time (age) of the j^{th} failure in the i^{th} PM cycle where $w_{i,j} = t_{i,j} - i\gamma$
- $U_{i,j}$ — the random number required for the generation of $x_{i,j}$
- $\lambda(t)$ — Original hazard rate function (before the 1st PM action)
- $\lambda_i(t)$ — Hazard rate function at time t where t is in the i^{th} PM cycle and $\lambda_0(t)=\lambda(t)$
- $F(t)$ — the cumulated distribution function (CDF) of the TBF at age t
- $R(t)$ — the reliability at age t
- C_{pm} — Cost of each PM
- C_{mr} — Minimal repair cost of each failure
- TC — The total maintenance cost function in the finite life time span

2.2 Assumptions

- The system has a finite useful life time L.
- The system is deteriorating and repairable over time where the failure process follows the non-homogenous Poisson Process (NHPP) with increasing failure rate (IFR). Weibull distribution with hazard rate function: $\lambda(t)=\left(\frac{\beta}{\theta}\right)\left(\frac{t}{\theta}\right)^{\beta-1}$ is used to illustrate the examples in this paper, where β is the shape parameter and θ is the scale parameter.
- The periodic PM actions with constant interval (T) are performed over the finite time span L.
- The system's age can be reduced γ units of time to result in a younger age (called the effective age) after each PM. Hence, the hazard rate function at time $t_{i,j}$ in the i^{th} PM cycle can be written as

$$\lambda_i(t_{i,j}) = \lambda(t_{i,j}-i\gamma) = \lambda(w_{i,j}). \quad (1)$$

- Minimal repair is performed when failure occurs between each PM.
- The time required for performing PM, minimal repair, or replacement is negligible.

2.3 The Theoretical Model

Based on the theoretical PM model with age reduction proposed by Cheng *et al.* (2004) and Yeh and Chen (2006), the optimal PM policy is obtained by the following steps. The first step is to find the expected cost rate function for the PM model as shown below.

$$C(T,N) = \frac{(N-1)C_{pm} + C_{pr} + C_{mr}\Lambda(T,N)}{NT}, \quad (2)$$

where $\Lambda(T, N)$ is the expected number of failures occurred in the finite time span and is defined as

$$\Lambda(T,N) = \sum_{i=0}^{N-1}\int_{iT}^{(i+1)T} \lambda_i(t)dt \quad (3)$$

with $\lambda_i(\cdot)$ being defined in Eq. (1). Second step is to obtain the time interval of PM (T) as a function of N by taking the partial derivative of T of the above expected cost rate function and letting it equal to zero, i.e., $\frac{\partial C(T,N)}{\partial T} = 0$

Third, the optimal value T^* and N^* of the theoretical model can be obtained by numerically searching $\min_N C(T,N)$, N = 1, 2, … since the cost rate function is a convex function. The hazard rate function of the PM model with age reduction is illustrated in Figure 1.

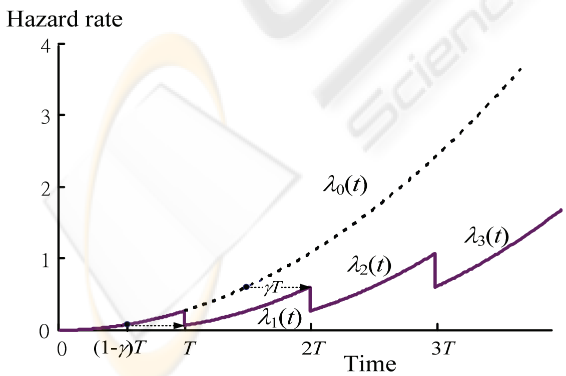

Figure 1: The hazard rate function of the PM Model with age reduction.

3 THE RV GENERATING METHODS OF THE TBF

3.1 The Offset Inverse Transformation Method

This RV generating method assumes that the occurrence time of the last failure in the i^{th} PM cycle is irrelative to the occurrence time of the first failure in the i+1st PM cycle. Thus, when the generated occurrence time of the k_i^{th} failure is within the i^{th} PM cycle but the occurrence time of the k_i+1st failure exceeds the i^{th} PM cycle, i.e., $t_{i,k_i+1} > iT$, we discard the k_i+1st failure and start to generate the occurrence time for the first failure of the i+1st PM cycle.

According to the concept of the inverse transformation method, if x is the time between failures, then, we have

$$U = F(x). \quad (4)$$

However, since the PM model assumes that the minimal repair is performed at each failure occurred between each PM. Therefore, we can re-write Eq.(4) as

$$U_{i,j} = F(x_{i,j}|t_{i,j-1}) = 1 - R(x_{i,j}|t_{i,j-1}). \quad (5)$$

Then, for the age-reduction PM model, we apply Eq.(1) to Eq. (5) and it can be resulted as the following equation.

$$1-U_{i,j} = R(x_{i,j}|t_{i,j-1}) = \exp\left\{-\int_{t_{i,j-1}}^{t_{i,j-1}+x_{i,j}} \lambda_i(t')dt'\right\} \quad \text{for } i = 0,1,\ldots,N;\ j = 1,2,\ldots,k_i \quad (6)$$

which, according to Eq. (1), can be expressed as function of effective age as follows.

$$1-U_{i,j} = R(x_{i,j}|t_{i,j-1}) = \exp\left\{-\int_{w_{i,j-1}}^{w_{i,j-1}+x_{i,j}} \lambda(t)dt\right\} \quad \text{for } i = 0,1,\ldots,N;\ j = 1,2,\ldots,k_i. \quad (7)$$

where $w_{0,j} = t_{0,j}$, $w_{i,0} = t_{i,0} - i\gamma = iT - i\gamma$; $w_{i,j} = t_{i,j} - i\gamma = t_{i,j-1} + x_{i,j} - i\gamma$. When the TBF of a system is a Weibull random variable, based in Eq. (7), we can generate the TBF random variates by the following equation.

$$x_{i,j} = \left\{-\theta^{\beta}\ln(1-U_{ij}) + \left[t_{i,j-1} - i\gamma\right]^{\beta}\right\}^{1/\beta} - t_{i,j-1} + i\gamma \quad \text{for } i = 0,1,\ldots,N;\ j = 1,2,\ldots,k_i. \quad (8)$$

The algorithm for the offset inverse transformation method is presented as follows.

(1) Specify the values of the following parameters: β, θ, γ, N, T, L and let i = 0.

(2) Let $t_{i,0} = iT$.
(3) Let $j = 1$.
(4) Generate random number $U_{i,j}$.
(5) Obtain the value of $x_{i,j}$ according to Eq.(8); let $t_{i,j} = t_{i,j-1}+x_{i,j}$.
(6) If $t_{i,j} < iT$, let $j = j + 1$ and go back to (4) else go to (7).
(7) If $t_{i,j} < L$, let $i = i + 1$ and go back to (2) else stop.

It can be seen that the occurrence time of the first failure in the i+1st PM cycle does not relate to the occurrence time of the last failure (t_{i,k_i}), i.e.,

$$t_{i+1,1} = t_{i+1,0} + x_{i+1,1} = (i+1)T + x_{i+1,1}.$$

3.2 The Trace-back Inverse Transformation Method

The proposed second method is modified from the offset inverse transformation method. For the following reasons: (1) the failure occurrence of the system follows the non-homogenous Poisson process (NHPP); (2) the PM is imperfect (i.e., the PM will not renew the system to zero failure rate), this generating method assumes that the occurrence time of the first failure in the i+1st PM cycle is affected by the occurrence time of the last failure in the ith PM cycle. Hence, the theoretical concept for the generation of $x_{i+1,1}$ is shown below.

$$R(x_{i+1,1}|t_{i,k_i}) = \Pr\{T' > t_{i,k_i} + x_{i+1,1}|T' > t_{i,k_i}\}$$
$$= \frac{\Pr\{T' > t_{i,k_i} + x_{i+1,1}\}}{\Pr\{T' > t_{i,k_i}\}} = \frac{R(t_{i,k_i} + x_{i+1,1})}{R(t_{i,k_i})},$$

where

$$R(t_{i,k_i} + x_{i+1,1}) = R(t_{i+1,1})$$
$$= \exp\left[-\sum_{l=0}^{i}\int_{lT}^{(l+1)T}\lambda_l(t')dt' - \int_{(i+1)T}^{t_{i,k_i}+x_{i+1,1}}\lambda_{i+1}(t')dt'\right]$$

and

$$R(t_{i,k_i}) = \exp\left[-\sum_{l=0}^{i-1}\int_{lT}^{(l+1)T}\lambda_l(t')dt' - \int_{iT}^{t_{i,k_i}}\lambda_i(t')dt'\right]$$

It turns out that

$$R(x_{i+1,1}|t_{i,k_i}) = \exp\left\{-\left[\int_{t_{i,k_i}}^{(i+1)T}\lambda_i(t')dt' + \int_{(i+1)T}^{t_{i+1,1}}\lambda_{i+1}(t')dt'\right]\right\}.$$

Then, let $U_{i+1,1} = U_{i,k_i+1} = 1 - R(x_{i+1,1}|t_{i,k_i})$. For the Weibull case, we can generate the first TBF random variate of the i+1st PM cycle by the following equation.

$$x_{i+1,1} = \left\{\begin{matrix}[(i+1)(T-\gamma)]^\beta + (t_{i,k_i} - i\gamma)^\beta \\ -[(i+1)T - i\gamma]^\beta - \theta^\beta \ln(1-U_{i+1,1})\end{matrix}\right\}^{1/\beta} - t_{i,k_i} + (i+1)\gamma \quad \text{for } i = 0,1,2,\ldots,N. \tag{9}$$

The algorithm for the trace-back inverse transformation method is provided below.

(1) Specify the values of the following parameters: β, θ, γ, N, T, L.
(2) Let $i = 0$, $t_{0,0}$=0.
(3) Let $j = 1$.
(4) Generate random number $U_{i,j}$.
(5) Obtain the value of $x_{i,j}$ according to Eq.(8); let $t_{i,j} = t_{i,j-1}+x_{i,j}$.
(6) If $t_{i,j} < iT$, let $j = j + 1$ and go back to (4) else go to (7).
(7) If $t_{i,j} < L$,
  obtain the value of $x_{i+1,1}$ according to Eq.(9);
  let $t_{i+1,1} = t_{i,k_i} + x_{i+1,1}$;
  let $i = i + 1$ and $j = 2$;
  go back to (4)
  else stop.

It can be seen that the occurrence time of the first failure in the i+1st PM cycle depends on the occurrence time of the last failure (t_{i,k_i}), i.e.,

$$t_{i+1,1} = t_{i,k_i} + x_{i+1,1}.$$

3.3 The Rejection Method

It can be seen from Eq.(4) or Eq.(5) that the hazard rate function is changed when performing a PM. This makes the formula for generating the TBF random variates shown in Eq.(6) and Eq.(7) very complicated. Therefore, the rejection method is applied in this research.

In the rejection method, two random numbers, say U_1 and U_2, are required for generating each RV. Suppose $\lambda_i(t)$ is the hazard rate function of the ith PM cycle. U_1 is used to generate a RV from a hazard rate function with a simple formula, say $\lambda(t)$ where $\lambda(t) \geq \lambda_i(t)$ for any $t \geq 0$. Then, the RV generated by using U_1 is accepted if $U_2 < \lambda_i(t)/\lambda(t)$.

In this research, we use the original hazard rate function $\lambda(t)$ (i.e., the hazard rate function before the first PM) to generate the RV of the TBF corresponding to U_1. For the Weibull case, we can obtain the TBF formula as the following equation.

$$x_m = \left[-\theta^\beta \ln(1-U_1) + (t_{m-1})^\beta\right]^{1/\beta} - t_{m-1}. \tag{10}$$

The algorithm of the rejection method is presented as follows.

(1) Specify the values of the following parameters: β, θ, γ, N, T, L.
(2) Let $t_{0,0}$=0, t_0=0.
(3) Let m = 0, i = 0, j = 1.
(4) Generate random number U_1.
(5) Obtain the value of x_m according to Eq. (10); let $t_m = t_{m\text{-}1}+x_{i,j}$.
(6) If $t_m < iT$, go to (7) else go to (10).
(7) Generate random number U_2
(8) Calculate $\lambda(t_m)$ and $\lambda_i(t_m)=\lambda(t_m\text{-}i\gamma)$.
(9) If $U_2 \le \lambda_i(t_m)/\lambda(t_m)$, let $t_{i,j} = t_m$; $j = j$+1; $m = m$+1; go back to (4) else $j = j$; $m = m$+1; go back to (4).
(10) If $t_m < L$, let $i = i$ + 1 and j = 1; go back to (7) else stop.

It can be seen that the rejection method is easy to use since it does not need to derive the formula of $R_i(t)$ for i = 1, 2, …, N.

4 EXAMPLES AND DISCUSSION

In the examples, let the finite life time period (L) be 6 time units and the PM interval (T) be 1 time unit. The values of parameters are set as: θ = 0.4; N = 5; C_{pm} = a+bi = 5+100i for the i^{th} PM; C_{mr} =3.1036. Then, we construct 25 experiments for each RV generating method, which consist of 5 different β values, each with 5 replicates. There are 30 runs for each experiment. We compare the differences between the mean number of failures obtained from Eq. (3) and the sample averages from the three RV generating methods. The analysis of variance (ANOVA) for the number of failures generated is also provided in Table 1. It can be seen that the three RV generating methods do not have significant different. Parameter β and the number of PM performed do significantly affect the number of failures generated, which demonstrates the validity of the simulation models.

4.1 The Near-Optimal Solution

Table 2 shows the parameter values used in the proposed simulation models as well as in the theoretical model of Yeh and Chen (2006). By using the rejection method, Table 3 presents the 30-run simulation results for N = 1 to 6. The smallest (best) total maintenance cost of each run is highlighted by shadow background.

Table 1: The ANOVA of the generated number of failures.

Source	SS	DF	MS	F	Sig.
Model	8070.401[a]	74	109.059	97.332	.000
Intercept	12773.386	1	12773.386	11399.784	.000
BETA	6358.646	4	1589.661	1418.715	.000
METHOD	3.799	2	1.899	1.695	.185
PM	782.269	4	195.567	174.537	.000
BETA * METHOD	4.838	8	.605	.540	.826
BETA * PM	898.965	16	56.185	50.143	.000
METHOD * PM	4.524	8	.565	.505	.853
BETA * METHOD * PM	17.360	32	.542	.484	.992
Error	336.148	300	1.120		
Total	21179.934	375			
Adj. Total	8406.549	374			

Table 2: Parameters applied in the PM model.

β	θ	L	h	a	b	C_m
3.2	0.4	2	0.19	5	100	3.1036

It can be seen from Table 3 that, for each N, the average value of TC from the 30-run simulation is very close to the value obtained by using the theoretical method based on Yeh and Chen (2006). Both methods (simulation and theoretical) provides the same optimal policy of N^*=3 and γ^*=0.4781. Again, it has demonstrated that the experiment results obtained by simulation methods are consistent with those obtained by the theoretical model when large sample runs are generated.

It should be noted that the best solution of N, γ, and TC (marked with shadow) resulted from each simulation run are different from those obtained by the theoretical model. It is because the optimal solution of the theoretical model is obtained by taking the expected cost rate over the infinite time interval or over the large number of systems in a finite time interval. However, the simulation method considers the situations of a single system in a finite time interval.

For a single system in a finite time span, according to Table 3, the best solutions of each run (with shadow) can be categorized into three near-optimal policies: (N=2, γ=0.6667), (N=3, γ=0.4781), and (N=4, γ=0.3655). Table 4 lists the simulation runs in each near-optimal policy and presents the average, the smallest, and the largest minimal TC of the near-optimal policy. Among these best solutions, the average of the minimal TC (184.1143) is significantly different from the theoretical minimal TC (189.7280). The results have demonstrated that the theoretical PM model might not be suitable for a single system over a finite time interval.

Hence, in practical, when considering a single system to be preventively maintained in a finite time period, especially for short time period, more than one single near-optimal policy is suggested. In this example, either (N=2, γ=0.6667) or (N=3, γ=0.4781) or (N=4, γ=0.3655) may be chosen as the best (near-optimal) PM policy.

Table 3: The results of the 30 Simulation runs.

Run#	N	1	2	**3**	4	5	6
	γ	1	0.6667	**0.4781**	0.3655	0.2957	0.2483
1		216.730	189.894	**180.155**	194.132	194.575	210.016
2		229.144	196.101	192.570	**187.925**	210.093	194.498
3		250.869	196.101	**189.466**	197.236	213.197	206.912
4		250.869	186.790	**180.155**	200.340	188.368	213.120
5		204.315	199.205	204.984	**187.925**	219.404	197.602
6		263.284	**189.894**	201.880	194.132	197.679	197.602
7		213.626	**165.065**	183.259	197.236	185.264	203.809
8		232.248	214.723	**192.570**	194.132	206.990	194.498
9		241.558	**177.480**	183.259	200.340	191.472	206.912
10		216.730	189.894	**180.155**	194.132	188.368	206.912
11		219.833	211.619	**204.984**	209.650	213.197	216.223
12		222.937	189.894	189.466	**181.718**	197.679	197.602
13		247.766	**177.480**	180.155	209.650	197.679	216.223
14		247.766	196.101	**180.155**	206.547	197.679	203.809
15		198.108	196.101	189.466	**181.718**	200.782	194.498
16		216.730	192.998	**183.259**	200.340	185.264	203.809
17		226.040	205.412	**189.466**	191.029	194.575	206.912
18		195.004	202.308	211.191	**191.029**	210.093	191.394
19		216.730	**183.687**	186.362	191.029	200.782	213.120
20		226.040	199.205	**183.259**	203.443	200.782	206.912
21		232.248	186.790	189.466	**184.822**	194.575	203.809
22		204.315	**196.101**	198.777	197.236	197.679	200.705
23		207.419	186.790	**180.155**	206.547	188.368	206.912
24		210.522	208.516	195.673	**187.925**	206.990	197.602
25		219.833	205.412	**195.673**	203.443	197.679	213.120
26		257.076	192.998	**173.948**	215.858	191.472	219.327
27		216.730	211.619	195.673	**172.407**	210.093	188.291
28		216.730	**189.894**	201.880	200.340	197.679	216.223
29		210.522	**168.169**	180.155	206.547	191.472	216.223
30		247.766	**186.790**	189.466	187.925	197.679	200.705
Avg.		225.316	193.101	**189.569**	195.891	198.92	204.843
Theo.		221.495	191.076	**189.728**	192.850	197.222	202.051

Table 4: The near-optimal Policies of the Simulation.

Policy 1 (N^*=2, γ^*=0.6667)		Policy 2 (N^*=3, γ^*=0.4781)		Policy 3 (N^*=4, γ^*=0.3655)	
Run#	Min. *TC*	Run#	Min. *TC*	Run#	Min. *TC*
6	189.8940	1	180.1552	2	187.9252
7	165.0652	3	189.4660	5	187.9252
9	177.4796	4	180.1552	12	181.7180
13	177.4796	8	192.5696	15	181.7180
19	183.6868	10	180.1552	18	191.0288
22	196.1012	11	204.9840	21	184.8216
28	189.8940	14	180.1552	24	187.9252
29	168.1688	16	183.2588	27	172.4072
30	186.7904	17	189.4660		
		20	183.2588		
		23	180.1552		
		25	195.6732		
		26	173.9480		
Runs	9	Runs	13	Runs	8
Avg.	181.6177	Avg.	185.6462	Avg.	184.4337
Max.	196.1012	Max.	204.9840	Max.	191.0288
Min.	165.0652	Min.	173.9480	Min.	172.4072
Overall average of min. *TC*: 184.1143					

5 CONCLUSIONS

The proposed three simulation methods are not significant different in generating the time-between-failure RVs .for the PM model with age reduction. The rejection method seems simple and easy to use in practical.

For the infinite time span, the results from the simulation method are very close to those obtained by the theoretical model. However, for a finite time span, more than one near-optimal policy can be obtained by the simulation method. Each of the near-optimal solution can be the best PM policy for any single system having a finite life time period. The simulation results have demonstrated that the theoretical PM model might not always suitable for a single system in a finite time span.

The simulation method can be applied in solving more complicated real world situation, such as the consideration of the random shock in a PM model, which is difficult to be solved by the theoretical model.

ACKNOWLEDGEMENTS

This research has been supported by the National Science Council of Taiwan under the project number NSC96-2221-E-324-010.

REFERENCES

Cheng, C.-Y. and Liaw, C.-F., 2005. Statistical estimation on imperfectly maintained system, *European Safety & Reliability Conference 2005 (ESREL 2005)*, Jun. 27-30, 2005, Tri-City, Poland, pp 351-356.

Cheng, C.-Y. Liaw, C.-F., and Wang, M., 2004. Periodic preventive maintenance models for deteriorating systems with considering failure limit, *4th International Conference on Mathematical Methods in Reliability—Methodology and Practices*, Jun. 21-25, 2004, Santa Fe, New Mexico.

Murthy, D. N. P. and Nguyen, D. G., 1981. Optimal Age-Policy with Imperfect Preventive Maintenance, *IEEE Transactions on Reliability* Vol.R-30, No.1, pp.80-81.

Pongpech, J. and Murthy, D. N. P., 2006. Optimal Periodic Preventive Maintenance Policy for Leased Equipment, *Reliability Engineering & System Safety*, Vol.91, pp.772-777.

Ross, S. M., 1997. *Simulation*, Academic Press, San Diego, pp.62-85.

Yeh, R. H. and Chen, C. K., 2006. Periodical Preventive-Maintenance Contract for a Leased Facility with Weibull Life-Time, *Quality & Quantity*, Vol.40, pp.303-313.