# Data Improvement using Form Factors Reconstructed in Latent Dimensions

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**Abstract.** A new approach to the problem of enhanced description of cluster boundaries in the sample is developed. New density estimates calculated on the base of nonlinear reconstruction latent form factors in additional dimensions are used for resolving critical loci in empirical distributions

## 1 Introduction

The idea to use form factors from accompanying dimensions, with a view of the improved reconstruction of aprioristic density, is not new and is actively realized in the field of Image Processing and Image Mining. A typical example: aposteriori estimations of an arrangement of the boundary points, depending on contextual conditions of type of elasticity, entropy minimization, etc., used in functionals for Active Contours and Balloons [1] [2] [3].

The reconstructed contour or spatial border of object can be considered as an example of the substantial knowledge mined from the image. For a problem of tracing objects in structure of scene or video sequence when the time interrelations are the subject to be mined, adequate use of the contextual information is even more important. So, at elimination scratches on the image, at segmentation of video sequence, at construction of tools of type Time Machine for GIS [4], etc., aprioristic data on character of display of the factor of time in data are used. The same is valid for use of other dominating latent parameters proving in structure of the image. The choice both necessary types of procedures and their parameters directly depends on the correct account of such data. In an ideal, it is possible to look for systematization of aprioristic data, and for automatic adjustment of data processing procedures on this base. Researches in this direction are actual, first of all, in the field of Image Mining where the choice of a method directly depends on type of the image [5] [7], but also in many other areas of data analysis [4] [6]. In this paper a new approach to a problem of reconstruction of aprioristic density is proposed in which improvement of the geometrical form of empirical distributions is carried out on the basis of form factors determined by the relevant contextual information.

#### 2 Latent Dimensions

Methods of reconstruction and estimation of aprioristic distributions are most demanded in the problems related to research of cluster structures. As a rule, the incorrectness of the problem of separation of clusters arises in view of the fact that attributes in used space set a mediated data representation, and their numerical values in some combined form reflect interrelation of various (stochastic and determined) factors defining individuality of a class and localization of its representatives. From all variety of the statements concerning to a problem of reconstruction of latent parameters, we address to a situation when there is a uniformity of such mediation within the limits of a sample. This assumption yields to investigate opportunities of the resolving critical variations of empirical density on the basis of its reconstruction in latent dimensions. In case of one additional dimension, on this way a problem arises that's inverse to the problem of estimation of integrated density on subspaces of co-dimension 1. For a direct problem there is the fast algorithm based on the use of geometrical properties of clusters of representation [8]. In this work the specified inversion is formulated as a problem of a choice of the optimal discrete K-layer scaling which corresponds to weakly changing distributions of density in layers and itself satisfies to structural restrictions of type of smoothness.

## 3 Hidden Exact Dependencies and Model of Data

In case of images the accompanying dimensions are present in initial representation from the very beginning, but, in case of abstract feature spaces  $R^N$  the comprehensible kind of the latent parameters is not guaranteed in advance, and substantiations of a correctness of their reconstruction in additional coordinate subspace are necessary every time. We shall be limited to a case when ontological assumptions specify an acceptability of such treatment. In what follows, there is proposed a computational scheme where these requirements are consolidated, and a method is developed for aprioristic density estimation for that of involved in a solving rule in the form of projections on actual feature space some natural approximations of the distributions of classes reconstructed in extended space with use of these structural restrictions. It is shown, that thus the various difficulties related to overfitting and to adequate involvement of objects of type outliers can be overcome. More strictly, we shall use the following model of data:

a) For the some n there are the strict analytical dependencies setting domain X in space  $\mathbb{R}^{N+n}$  as admissible for representatives of a class.

b) These dependencies are smooth, and the geometrical form of immersing  $X \subset \mathbb{R}^{N+n}$  is simple.

c) Essential share of tangent spaces  $T_x, x \in X$ , contains vectors from initial space  $R^N$  (i.e., in arrangement of sample there are local correlations between the latent and observable dimensions).

d) Stochastic components bring their contribution to the mediated representation at the stage of realization of the class in actual feature space  $R^N$ .

e) Within the limits of area X stochastic components vary insignificantly.

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Let's explain in more detail the condition of item c). If there is more then one latent parameter, metric parities between them are coordinated with scales of actual representation, and degrees of their dependence on accessible dimensions can be compared among themselves. Hence, every two smooth curves in  $\mathbb{R}^{N+n}$  are compatible on degree of visibility in actual space (i.e., on the sum of nonzero local projections on  $\mathbb{R}^N$ ), that is, among them there is the best in this sense. We can not know both the value nand nature of latent parameters, but, if some curve as their combination dominates over others in the specified sense ('main hyper-surface' of the sample, in a considered case it's 'main curve'), then it will be automatically reconstructed on the proposed way.

### 4 Clusters with Longitude

We shall search at first stages of algorithm for some natural longitude of the cluster, considering variations of empirical density as displays of non-linearity in  $X \subset \mathbb{R}^{N+n}$ , which determine local relations between latent and actual coordinates.

#### 4.1 Algorithm in Case of One Additional Dimension

1. Choose model of a layer (for example, unique Gaussian kernel) that describe local features of realization of sample in  $\mathbb{R}^N$ , including both geometrical and stochastic components. It is important to provide transformability of the density function  $p_L$  on a layer in  $\mathbb{R}^N$  in a density  $p_{L'}$  of a prototype of this layer L' in  $\mathbb{R}^{N+1}$ .

2. Build approximation of the whole empirical distribution in the form of convolution of discrete set of points with the model of a layer (normal mixture with equal weights and fixed kernel is an example). Let  $F_a$  be the functional of the quality of approximation, C is a set of central points of the mixture  $C \otimes L = \sum \frac{1}{K} L(c_k)$ .

3. Build smooth ordering the centers of a mixture. It's the central item of the method. Let  $F_b$  be a functional of quality of approximation of the discrete sequence of the centers  $c_k$  by some smooth curve S in  $\mathbb{R}^N$ . Let  $s_k$  be the nearest to centers  $c_k$  points on S. Obligatory condition consists in that smooth should be either the evolution of distances on S between new centers of consecutive layers. These restrictive condition gives an opportunity of representation C in  $\mathbb{R}^N$  as a projection of some smooth uniform chain C' merged in space  $\mathbb{R}^{N+1}$ .

4. Build prototype  $S' \subset \mathbb{R}^{N+1}$  of the curve  $S \subset \mathbb{R}^N$  so that images  $s'_k \in S'$  of consecutive points from S settle down through the same intervals equal to the maximal distance between consecutive centers in C. Corresponding fragments become horizontal in  $\mathbb{R}^{N+1}$  (i.e., parallel to  $\mathbb{R}^N$  in immersion  $S' \subset \mathbb{R}^{N+1}$ ).

5. Fill extended space with models L' of the layer L transformed from dimension  $R^N$  to  $R^{N+1}$ . The goal is to construct smooth tube **S** as an approximation of the prototype distribution in  $R^{N+1}$ . The simplest way is to choose lot of equidistant points and place transformed layers uniformly with corresponding small weights.

6. Projecting S back in  $\mathbb{R}^N$ , we receive improved estimation of the empirical density.

It is above presented the simple structure of algorithm which basic elements are available in a ready form in MatLab environment. Furthermore, if back projections on

 $R^N$  all transformed models of layers L' restore these layers without changes then items 4)-6) correspond simply to convolution of curve S with model of layer L of a kind  $S \otimes L = \int p_L dp_S$ .

We recall once again that in advance it is not known, which latent parameter will get representation in additional dimension, but it will be one of the those better represented in actual space, according to the assumption of item c).

So, on Fig. 1 a modelling example of an arrangement of clusters for two deeply overlapped training classes describing two similar types of plants are represented.

Fig. 1. An example of two deeply overlapped classes. Some representatives merged in alien media seem to be outliers.

On Fig. 2 the result of modelling in additional dimension of the dominating latent parameter is shown. At substantial interpretation it is visible, that representation receives the time scale which in this case is presented in the form of seasonal changes of parameters of objects (color of leaves, and so on). Thus it is found out, that the main distinction between classes consists in duration of the green period of vegetation of these two types of plants while in all other respects they do not differ essentially from each other.

On Fig. 3 it is shown, that adequate use of conditions of smoothness can lead to separability of models of clusters reconstructed in extended feature space on the basis of form factors. In particular, certain elements in an initial mix which from the point of view of many criteria would be assigned to the category of outliers, in the extended space take natural positions in own classes and bring therefore positive contribution to the decision making.

## 5 Discussion

Introduction in structure of algorithm of more complex stages allows to involve the important new opportunities. So, items 2 and 3 are closely interconnected, and for them it is possible to construct joint functional of a kind  $F_{ab} = (F_a, F_b)$ . This junction determines a search of such approximation of cluster by a mix  $\sum \frac{1}{K}L(x_k)$ , in which the configuration of the centers of layers provides their comprehensible ranking along

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**Fig. 2.** Modelling the dominating latent parameter in additional dimension. It becomes visible that unique essential distinction between two classes of plants consists in duration of the green period of vegetation. Equi-moment surfaces cross the lower plane from top-left to bottom-right.



Fig. 3. Use of conditions of smoothness in the extended space. Some candidates to category of outliers are rehabilitated and take natural positions in own classes.

a smooth curve, and either smooth evolution of distances between the adjacent centers. In particular, for the sample originally represented by small symmetric kernels, this problem can be put as a problem of a choice one-dimensional scaling in which the model of a layer (item 1) serves as good approximation for all crossings of own layers

of the scaling with initial representation of the cluster. Thus, the constructed scale will define the necessary ordering by itself.

Far more complicated variant assumes association of items 1, 2 and 3, i.e., simultaneous optimization of a choice as well the model of layer.

By introduction of the given complications it would be possible to build better approximations of the sample by ordered mix  $\sum \frac{1}{K}L(x_k)$ , but even for  $F_{ab}$  the optimization space can appear too complex and inaccessible for gradient methods. Inclusion in analysis also a changing layer more aggravates the situation. One of possible ways here can be the direct search and use of parallel calculations. Let  $\{C\}$  be the set of initial configurations of the centers,  $\{L\}$  be the set of models of a layer. Product of their counters can serve as rough lower estimate for the factor of parallelizing.

Certainly, domination any one of the latent parameters takes place not always and even not often. Suitable approximation of the set of the centers C, for instance, on a plane in the form of a discrete grid which could serve as a projection of the uniform grid placed on a smooth surface in  $\mathbb{R}^{N+2}$ , in alternative cases can be more adequate. But, the latter is considerably more challenging task, and a development of the method at least for plane hardly will be immediate.

Therefore, the most natural niche for the method is its use in big systems for multimodal data analysis, where choice of procedures is coordinated with aprioristic information. Systems of this type accumulate multiple partial decisions, the individual potential of each of which can be limited, but the contribution in aggregated or collective decision can become essential at association with other partial decisions.

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