MEMORY-BASED SPECKLE REDUCING ANISOTROPIC DIFFUSION

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Abstract: Diffusion filters are usually modelled as *partial differential equations* (PDEs) and used to reduce image noise without affecting the image main features. However, they have a drawback of broadening object boundaries and dislocating edges. Such drawbacks limit the ability of diffusion techniques applied to image processing. Yu and Acton. introduced the *speckle reducing anisotropic diffusion* (SRAD) to reduce speckle noise from *ultrasound* (US) and *synthetic aperture radar* (SAR) images. Incorporating the *instantaneous coefficient of variation* (ICOV) as the diffusion coefficient and edge detector, SRAD gives significantly enhanced images where most of the speckle noise is reduced. Yet, SRAD still faces the same problem of ordinary diffusion filters where the boundary broadening and edge dislocation affect its overall performance. In this paper, we introduce a novel approach to the diffusion filtering process, where a memory term is introduced as a reaction-diffusion term. By applying our new memory-based diffusion to SRAD, we significantly reduced the boundary broadening and edge dislocation effect and enhanced the diffusion process itself. Experimental results showed that the performance of our proposed memory-based scheme surpass other diffusion filters like normal SRAD and Perona-Malik filter as well as various adaptive linear de-noising filters.

1 INTRODUCTION

Diffusion has been widely used in image processing for smoothing and reducing noise. Sharing the physical properties of the diffusion process and being modelled as *partial differential equation* (PDE), diffusion arises as a powerful tool in various fields of image enhancement. However, the usual drawbacks of the diffusion process (e.g., the broadening of objects boundaries and edges dislocation) are hindering its applications. Weickert gave an in-depth analysis of the diffusion process and its application in image processing(Weickert, 1997).

Perona-Malik introduced one of the earliest edge-sensitive diffusion filter for additive noise reduction (Perona and Malik, 1990). Using nonlinear anisotropic diffusion, the filter greatly reduced the additive noise, where weighted image gradient is used as the diffusion coefficient. . For correcting Perona-Malik feature distortion effect and preserving edges, a nonlinear edge enhanced anisotropic diffusion is introduced (Fu et al., 2005). Yu et. al. introduced the *speckle reducing anisotropic diffusion* (SRAD). SRAD (Yu and Acton, 2002) combined both, the ordinary nonlinear anisotropic diffusion process proposed by Perona-Malik, as well as the adaptive speckle multiplicative noise filters of Lee (Lee, 1980) and Frost (Frost et. al., 1982). SRAD alleviates the reliance of adaptive filters of Lee (Lee, 1980) and Frost (Frost et. al., 1982) on the window size (i.e. mask size) of the filter.

On contrary to Perona-Malik filter, SRAD uses *instantaneous coefficient of variation* (ICOV) (Yu and Acton, 2004) of the image as the diffusion coefficient instead of the image gradient. ICOV has superior edge maps compared to ordinary edge detectors due to its incorporation as the diffusion coefficient into SRAD. SRAD enhances the reduction of speckle noise while ICOV extracts edges.

However, SRAD suffers from the drawbacks of ordinray diffusion (boundary broadening and edges migration). It produces a set of a coarse to fine images. The features identified at the finer scale are

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distorted and having dislocated edges. Meanwhile, features identified at coarse scale are noisy.

Trying to limit SRAD boundary broadening effect, a more robust diffusion coefficients tensor is introduced to further stop diffusion across main edges (Tauber et al., 2004). Acton introduced *deconvolutional* SRAD (DeSpeRADo) filter (Acton, 2005), where a deblurring is performed at at the same time with diffusion. DeSperado showed significant improved results when applied to synthesized images. However, the poor estimation of the *point spread function* (PSF) of the imaging device (assumed to cause the boundary broadening effect) limited its application on real data. Yu et. al. developed a *regularized* SRAD (Reg-SRAD) for enhancing point, linear and regional features (Yu and Yadegar, 2006). Reg-SRAD required the correct estimation of a threshold value for bright image features.

In this paper, we propose *memory-based* SRAD (MSRAD) where memory is integrated into the diffusion process through the reaction term. The incorporated memory provides feedback between diffusion stages, reminding the newly diffused image with the correct edge location found in previously diffused images. MSRAD will enhance the diffusion process providing a balance between diffusion and correct edge localization by maintaing features' sizes.

The organization of this paper is as follows; in Section 2, we first give a brief introduction to the diffusion process, its physical background and the Perona-Malik diffusion model. Then, we outline the original SRAD and ICOV models, and previous refinements made to them. In Section 3, we introduce our MSRAD technique. In Section 4, we outline the results obtained by MSRAD. Finally, In Section 5, we conclude our work.

2 DIFFUSION FILTERING

Diffusion is a physical process that equilibrates concentration differences without creating or destroying mass. One of the well known physical diffusion equation is Fick's law (Weickert, 1997) stating that a concentration gradient causes a flux in order to compensate for this gradient. A diffusion tensor (D) governs the relation between concentration gradient and the produced flux.

In image processing, the concentration gradient can be expressed as image gradient. A constant diffusion tensor (D) applied over the whole image domain causes homogenous diffusion or isotropic diffusion. In addition, a space-dependant D on the image domain causes inhomogeneous (anisotropic) diffusion. Linear diffusion happens when D is a function of the differential structure (image gradient) of the original image, while non-linear diffusion has the diffusivity matrix D dependant on the successively diffused image differential structure (Weickert, 1997).

Throughout this proposal, the notation used for diffusion time is t, where a time dependent variable will have t as its superscript. I indicates the original image, u refers to the diffused image, u^t indicates the diffused image at time t, where $u^{t=0}$ is the original image I. The subscript x is used to represent the pixel coordinates (i,j) of the image in the Cartesian domain, and it is assumed to exist wherever I or u terms are used.

The general diffusion equation is given by (1),

$$\frac{\partial u^t}{\partial t} = div\ (D \times \nabla u^t), \tag{1}$$

where div is the divergence operator, D is the diffusivity tensor, u is the diffused image, ∇u is the image gradient. Changing the diffusivity tensor defines the kind of diffusion applied to the image whether linear, nonlinear, isotropic, or anisotropic.

The Perona-Malik model uses a rapidly decreasing diffusivities D as shown in (2),

$$D = g(\|\nabla u^t\|) = \frac{1}{1+\left(\frac{\|\nabla u^t\|}{\lambda}\right)^2} \quad (\lambda \neq 0), \tag{2}$$

where λ is the edge magnitude parameter, D is a function that gives low values (near zero) for gradient values >> λ inhibiting diffusion near edges (Perona and Malik, 1990). Using (2) as the diffusivity coefficient of (1), the model sharpens edges if their gradient is larger than the edge magnitude parameter λ by inhibiting diffusion. For gradient values << λ, D approaches one and isotropic diffusion smoothes homogenous regions of the image converging equation (2) to a linear homogenous diffusion similar. The correct choice of λ greatly affects the filter operation. As for large values of λ, D will be always close to one independent on the gradient value. While for smaller values of λ, D will be nearly equal to zero inhibiting diffusion.

2.1 Instantaneous Coefficient of Variation (ICOV)

Yu and Acton (2002) (2004) introduced ICOV as the edge detector operator. ICOV operator is given by (3),

$$ICOV(u^t) = \frac{\sqrt{\left|\delta \times \left\|\nabla u^t\right\|^2 - \omega \times (\nabla^2 u^t)^2\right|}}{(u^t + \chi \times \nabla^2 u^t)} \tag{3}$$

where |.| is the absolute operator, ||.|| is the magnitude operator, ∇ is the gradient operator, ∇^2 is the Laplacian operator, *δ, ω*, and, *χ* are weighting parameters responsible for sharpening edge response and reduce edge position bias. They are usually taken to be equal 1/2, 1/16 and, 1/4, respectively.

ICOV is an edge detector utilizing the normalized gradient and Laplacian operators. It optimizes edge detection in speckle imagery through decreasing the probability of false edge detection and improving the edge localization accuracy.

2.2 Speckle Reducing Anisotropic Diffusion (SRAD)

Yu et. al. incorporated Lee (Lee, 1980) and Frost (Frost et. al., 1982) filters along with the anisotropic diffusion filter of Perona-Malik to come up with a novel speckle de-noising partial differential equation called *speckle reducing anisotropic diffusion* (SRAD) filter (Yu and Acton, 2002). SRAD is given by (4),

$$\begin{aligned} SRAD(u^t) &= u^{t+\Delta t} \\ &= div\,[g(ICOV(u^t)) \times \nabla u^t], \end{aligned} \tag{4}$$

where, t is diffusion time index where $u^{t=0}$ is the original image *I*. Δt is the time step (usually taken in the range from 0.05 to 0.25) and it is responsible for the convergence rate of the diffusion process, *g(.)* is the diffusion tensor function and is given by (5),

$$D = g(ICOV(u^t)) = e^{-(P)}, \tag{5}$$

P is a function in the ICOV of the diffused image as shown in (6),

$$P = \frac{\left(\frac{ICOV(u^t)}{q^t}\right)^2 - 1}{1 + (q^t)^2}, \tag{6}$$

where q^t is the measure of speckle coefficient of variation in a homogenous region of the image.

ICOV serves as the edge detector for the diffusion process. It gives high response at edges and low response in homogenous regions. q^t weights the amount of diffusion applied by SRAD to the image similar to *λ* in (2). For simplicity, the form in (7) is used for *D,*

$$D = g(ICOV(u^t)) = \frac{1}{1+P}, \tag{7}$$

The behaviour of SRAD allows diffusion in the direction parallel to the edge. Negative diffusion is allowed in the direction normal to the edge. SRAD outperforms normal anisotropic diffusion filters by enhancing edge strength and reducing speckle noise along image contours. However, SRAD still suffers from ordinary diffusion drawbacks distorting the size of image features with the increase of diffusion. In the following section, we introduced our modification to SRAD to lower its smoothing effect.

3 MEMORY-BASED SPECKLE REDUCING ANISOTROPIC DIFFUSION (MSRAD)

SRAD efficiently reduces speckle noise from images, where the incorporation of ICOV as the diffusion coefficient provides clear edge maps.

Memory-based SRAD provides features tracking feedback between the generated set of images through diffusion varying from coarse to fine scale. At the beginning of the diffusion process, the coarse images produce noisy edge maps and provide correctly located edges, as the effect of feature broadening is not yet severe. As the diffusion proceeds with time, the finer images are smoother and generate more enhanced, highly connected edge maps but they suffer from dislocated edges due to feature broadening.

MSRAD introduced memory reminds each diffused image with the correct edge location and feature size from previously diffused images. MSRAD enhance the diffusion process providing memory feedback balancing diffusion (smoothing), edge localization, and, feature allocation throughout different diffusion stages.

MSRAD equation is given in 0,

$$\begin{aligned} &u^0 = I, \\ &u^1 = SRAD(u^0), \\ &MSRAD(u^t) = u^{t+1} \\ &= \alpha \times u^t + (1-\alpha) \times SRAD(u^t), \quad t > 0 \end{aligned} \tag{8}$$

where α is a weighting parameter. Comparing 0 to memory-less SRAD in (4), MSRAD incorporates the weighted average of the currently diffused image with the set of the previously generated diffused images. It requires the determination of a single weighting parameter α.

The proper choice of α favours either more diffusion or more adhering to image features. The original and successively the coarse images exhibit correct edge locations and feature sizes. As diffusion proceeds with time towards the finer set of images α provides coupling between the fine and coarse images. We empirically choose α to be in the range from 0.15 to 0.85 depending on the amount of diffusion needed.

3.1 MSRAD as Diffusion-Reaction Term

Reformulating MSRAD as a diffusion-reaction term 0 can be rewritten as 0,
where MSRAD resembles the diffusion-reaction

$$\begin{aligned} & MSRAD\ (u^t) = u^{t+1} \\ & = SRAD\ (u^t) + \alpha \times (u^t - SRAD\ (u^t)),\ t > 0 \end{aligned} \tag{9}$$

model (Weickert, 1997). Memory-less SRAD and consequently ICOV extracted edge maps are highly sensitive to the time step Δt determining SRAD rate of convergence (stopping criteria). MSRAD alleviate this reliance by incorporating memory to the diffusion process through the reaction term as shown in 0.

3.2 MSRAD versus DeSpeRADo and Reg-SRAD

MSRAD along with DeSpeRADo (Acton, 2005) and Reg-SRAD (Yu and Yadegar, 2006) tackled the problem of feature broadening and edge dislocation exhibited by normal SRAD.

DeSpeRADo required the exact estimation of the PSF of the imaging device assumed to cause speckle noise. This estimation makes the real utilization of DeSpeRADo impractical and dependant on the imaging device.

Reg-SRAD depends on the determination of a threshold value along with other two weighting parameters. The threshold value depends on the bright regions intensity of the image. Thus, the correct choice of the threshold value is highly dependant on the processed image.

MSRAD requires only the determination of a single weighting parameter. This parameter is independent neither of the imaging device used nor of the image features' intensities. Thus, MSRAD provides more convenient and easy to determine weighting parameter providing balance between diffusion and features perseverance. The lack of code and/or test data for both DeSpeRADo and Reg-SRAD limited our ability to compare our results with theirs. However, in Section 4 we give a thoroughly measure of MSRAD performance.

4 RESULTS

In this section, the performance of MSRAD is compared to adaptive linear noise reduction filters of Lee (Lee, 1980), Frost (Frost et. al., 1982), and, Weiner (Wiener, 1976). Also, MSRAD is compared to the diffusion filters of Perona-Malik and normal SRAD. The evaluation will be made in terms of feature perseverance and noise reduction.

For evaluating the MSRAD performance, we generated a synthesized image shown in Figure 1(a). The synthesized image is of 150 column width and 150 column height. It consists of a unit step function in the range from column 15 to column 65 and a ramp function from column 85 to column 135. A speckled version of the synthesized image is shown in Figure 1(b), where a Gaussian distributed speckle noise of zero mean and variance of 0.1 is added.

In terms of noise reduction and feature perseverance, Figure 1(c), (d), (e), and (f) shows the results of de-noising the synthesized speckled image shown in Figure 1(b) by Lee, Frost, Wiener, and Perona-Malik filters, respectively. The results where obtained using 3×3 window for Lee and Frost filters and 5×5 for Weiner filter. For Perona-Malik filter the edge magnitude parameter λ, was taken equal to 0.03, with a time step $\Delta t = 0.1$. MSRAD, SRAD, and, Perona-Malik results were obtained after 200 iterations, where SRAD result is shown in Figure 1(g), and MSRAD result shown in Figure 1(h). Both MSRAD and SRAD results were obtained using a time step $\Delta t = 0.25$.

Compared to adaptive linear filters (i.e. Frost, Lee, Wiener) and Perona-Malik filter, MRSAD showed superior noise reduction effect. Original SRAD suffer from boundary broadening and distortion of features. MSRAD result showed significant perseverance of the features' sizes.

Figure 2 inspects the results of applying Lee, Frost, Wiener, Perona-Malik, SRAD, and MSRAD over a horizontal scan line extracted from the images at row 71 in Figure 1. The results show that MSRAD virtually approximated the original signal shown in

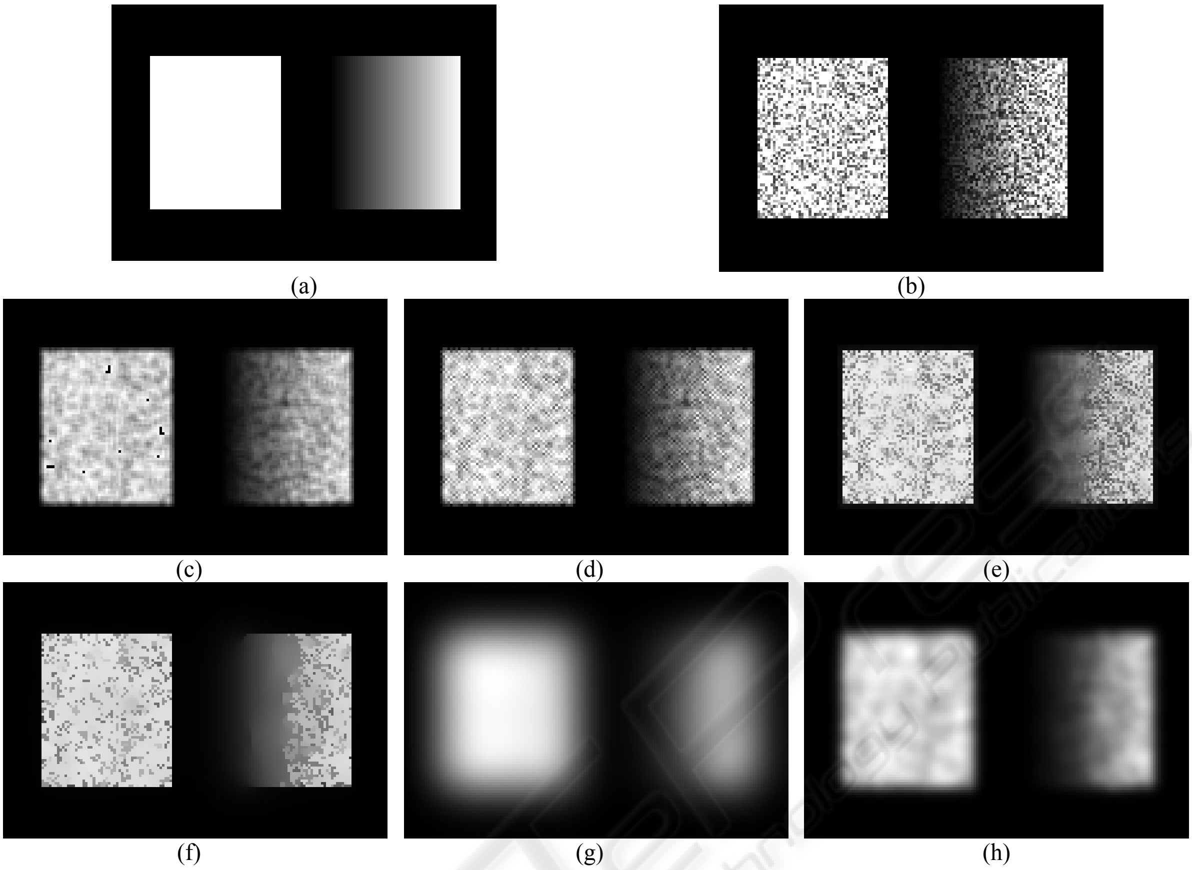

Figure 1: Synthesized image along with the results of applying various de-noising filters and MSRAD. (a) Original synthesized image. (b) Speckled synthesized image. (c) Lee filter result (d) Frost filter result. (e) Wiener filter result (f) Perona-Malik filter result. (g) SRAD result (h) MSRAD result.

Figure 2(a). Lee, Frost, Wiener and, Perona-Malik filters have limited noise reducing responses. Yet, they do not suffer from feature broadening effects. While SRAD suffer from severe boundary broadening and feature merging effect. MSRAD shows more consistent features along with good approximation of original signal.

The adaptive linear filters of Lee, Frost, Weiner depend totally on the window (mask) size. Perona-Malik filter depends on the edge magnitude parameter λ, while SRAD depends on the diffusion step Δt. MSRAD depends only on a single weighting parameter, α, maintaining a good balance between image smoothing and boundary allocation.

5 CONCLUSIONS

In this paper, memory-based SRAD was introduced as feature perseverance SRAD. The introduced memory through the reaction term balanced the effect of diffusion and correct boundaries allocation. MSRAD showed significant noise reduction effect over linear filters of Lee, Frost, and, Wiener, as well as over the diffusion filter of Perona-Malik. Compared to the original SRAD, MSRAD maintained the correct sizes of features and reduced speckle noise. MSRAD requires the determination of a single weighting parameter compared to estimating PSF of DEspeRADo or the image dependant threshold parameter controlling Reg-SRAD.

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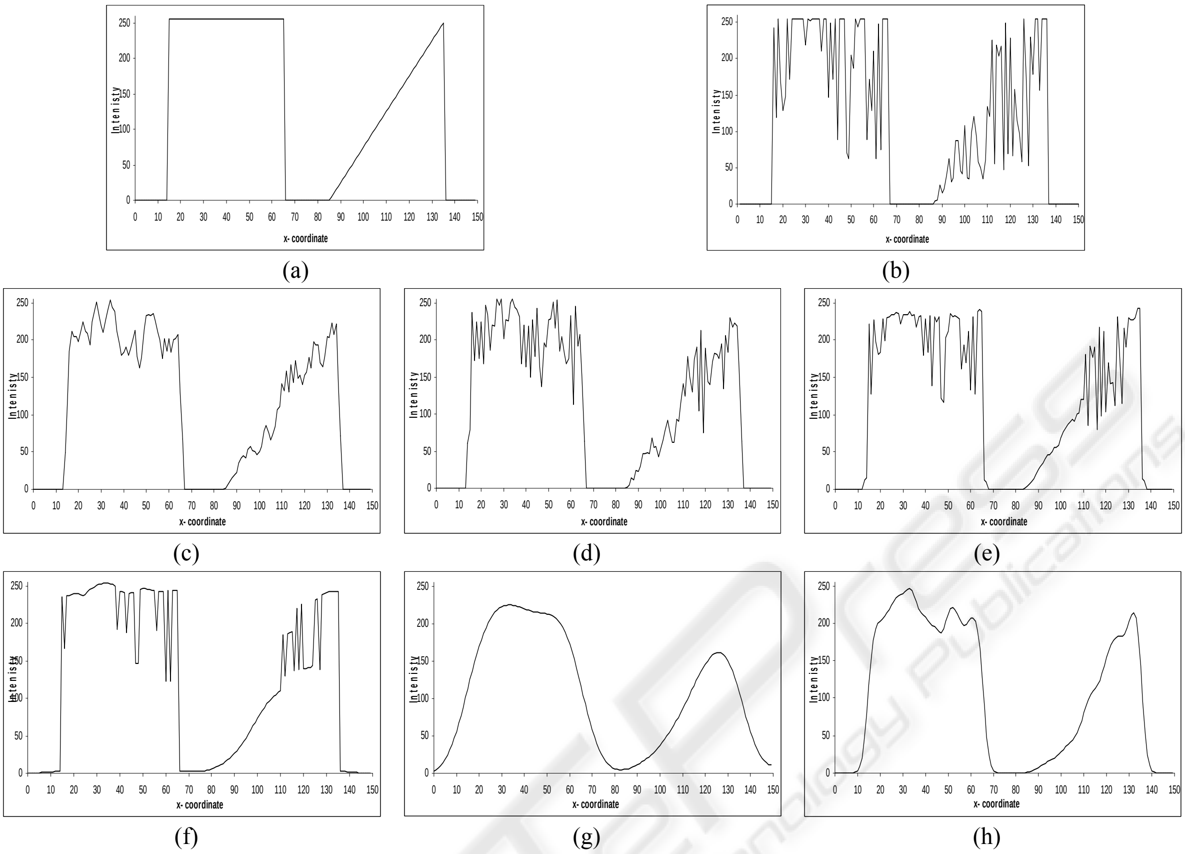

Figure 2: MSRAD versus various de-noising filters in terms of smoothing over a horizontal scan line of the images in Figure 1. (a) Original signal. (b) Speckled signal. (c) Lee filter signal result. (d) Frost filter signal result. (e) Wiener filter signal result. (f) Perona-Malik signal result. (g) SRAD result signal. (h) MSRAD signal result.

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