PEAK-TO-AVERAGE POWER RATIO OF MULTITONE-HOPPING CDMA SIGNALS USING FEEDBACK-CONTROLLED HOPPING PATTERNS

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Abstract: We present the characteristics of peak-to-average power ratio (PAR) for multitone-hopping code-division multiple access (MH-CDMA) signals using feedback-controlled hopping patterns (FCHPs) (FCHP/MH-CDMA). In FCHP/MH-CDMA, since each chip of transmitted signals consists of plural tones, energy consumption due to large PAR may not be negligible at the transmitter. Therefore, it is important to investigate the PAR characteristics of FCHP/MH-CDMA signals. It is shown that limiting the number of tones per chip, and the number of quantization bits, and clipping on FCHP are effective in reducing the PAR at almost identical bit-error rate (BER).

1 INTRODUCTION

Intersymbol interference (ISI) and multiple access interference (MAI) are two primary factors that reduce wireless communication performance. To greatly reduce ISI and MAI, feedback-based systems have been studied. For uplink channels, a method in which a base station (BS) employs an adaptive filter at a receiver to produce an analog pseudo-noise (PN) sequence, which is assigned to a new user, was proposed (Hamada et al., 1998) in direct-sequence code-division multiple access (DS-CDMA). Analog PN sequences can be orthogonal to each other under arbitrary asynchronous conditions. For a synchronous DS-CDMA, an iterative construction method that produces signature sequences using a minimum mean-squared error (MMSE) filter was proposed (Ulukus and Yates, 2001). It has been shown that this method produces a set of Welch bound equality (WBE) sequences (Welch, 1994; Rupf and Massey, 1994) using an MMSE filter whose size is identical to the length of the signature sequence. In contrast, we have proposed another DS-CDMA using feedback-controlled spreading sequences (FCSS/DS-CDMA) (Miyatake et al., 2004; Miyatake et al., 2008). In the FCSS/DS-CDMA, the receiver employs an adaptive filter whose size is larger than the length of the signature sequence and returns part of the filter coefficients to a transmitter. It has been shown that this method yields superior performance in terms of bit-error rate (BER) over time-invariant multipath channels. Furthermore, we have proposed multitone-hopping CDMA (MH-CDMA) using a feedback-controlled hopping pattern (FCHP) (FCHP/MH-CDMA), which combines the frequency-hopping CDMA (FH-CDMA) with the FCSS/DS-CDMA, to increase signal-to-interference plus noise ratio (SINR) (Chiba and Hamamura, 2007). Each receiver of the FCHP/MH-CDMA is composed of a time-frequency, two-dimensional, adaptive finite-duration impulse response (FIR) filter, which is larger than the hopping pattern. The receiver returns part of the filter coefficients to a transmitter. Since the signals transmitted in the FCHP/MH-CDMA consist of FCHP-coded multiple frequency tones, which usually result in large peak-to-average power ratio (PAR) that increases energy consumption at the transmitter, it is important to investigate the characteristics of PAR. In this paper, the impact of limiting the number of tones, the number of quantization bits, and tone level on PAR and BER is clarified.

2 FCHP/MH-CDMA

2.1 Transmitter

We assume uplink multiple access illustrated in Fig. 1.

A signal received at the position of BS can be

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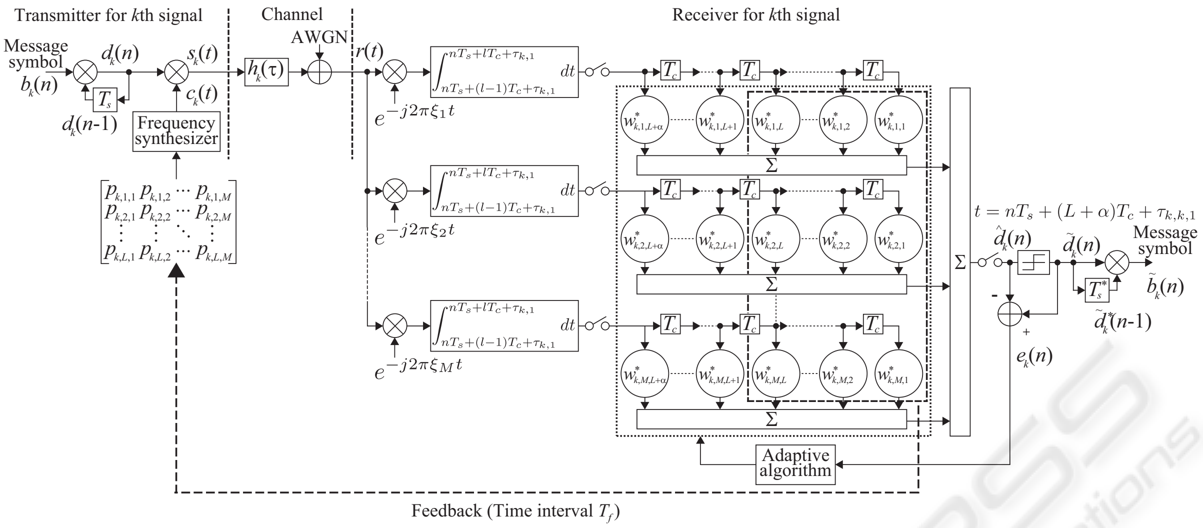


Figure 2: Transmitter and receiver for kth signal.

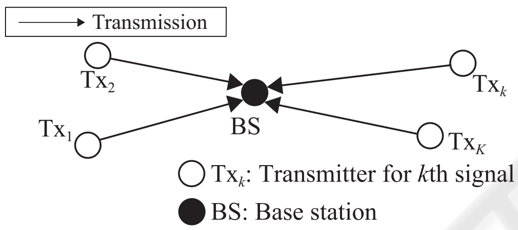


Figure 1: Uplink multiple access (asynchronous transmission).

modeled as a sum of K signals that are independently transmitted through distinct channels. The transmitter and receiver for the kth signal ($k = 1, 2, \cdots, K$) of the FCHP/MH-CDMA are shown in Fig. 2.

The signature waveform $c_k(t)$ for the kth signal is given by

$$c_k(t) = \sum_{l=1}^{L} a_{k,l}(t - (l-1)T_c), \tag{1}$$

where $a_{k,l}(t)(0 < t < T_c; T_c[s]$ is the chip duration) is the lth chip waveform ($l = 1, 2, \cdots, L$) for $c_k(t)$, given by

$$a_{k,l}(t) = g(t) \sum_{m=1}^{M} p_{k,l,m} e^{j2\pi\xi_m t}, \tag{2}$$

where $j = \sqrt{-1}$, $p_{k,l,m} (= A_{k,l,m} e^{j\phi_{k,l,m}})$ is the complex amplitude of the mth tone of frequency ξ_m[Hz] ($m = 1, 2, \cdots, M$) for the lth chip of $c_k(t)$, and $g(t) = \{1(0 < t < T_c), 0(\text{otherwise})\}$. In this paper, we choose $\xi_m = \frac{m-1}{T_c}$.

Let $\mathbf{P}_k$ be an $L \times M$ matrix that contains $p_{k,l,m}$ such that

$$\mathbf{P}_k = \begin{bmatrix} p_{k,1,1} & p_{k,1,2} & \cdots & p_{k,1,M} \\ p_{k,2,1} & p_{k,2,2} & \cdots & p_{k,2,M} \\ \vdots & \vdots & \ddots & \vdots \\ p_{k,L,1} & p_{k,L,2} & \cdots & p_{k,L,M} \end{bmatrix}. \tag{3}$$

The matrix $\mathbf{P}_k$ is the hopping pattern for the kth signal.

The kth signal transmitted by the transmitter is given by

$$s_k(t) = \sum_{n=0}^{\infty} d_k(n) c_k(t - nT_s), \tag{4}$$

where $d_k(n) = b_k(n) d_k(n-1)$ is a differentially encoded complex symbol transmitted in $nT_s < t < (n+1)T_s$ ($n = 0, 1, \cdots$), $b_k(n)$ is a complex message symbol, and T_s[s] is the symbol duration ($T_s = LT_c$). In this paper, we assume that $b_k(n)$ is a quaternary phase-shift keying (QPSK) symbol.

2.2 Channel

Let $h_k(t)$ be the impulse response of the channel through which the kth signal ($k = 1, 2, \cdots, K$) is transmitted to the BS, given by

$$h_k(t) = \sum_{i=1}^{I_k} h_{k,i} \delta(t - \tau_{k,i}), \tag{5}$$

where $h_{k,i} (= |h_{k,i}| e^{j\theta_{k,i}})$ is the complex gain constant for the ith path of the channel, $\tau_{k,i}$ ($0 \le \tau_{k,i} < T_s$) is the delay for the ith path, and I_k is the number of paths of the channel.

The received signal $r(t)$ at the position of the

BS is given by

$$r(t) = \sum_{k=1}^{K} (s_k(t) * h_k(t)) + n(t) \quad (6)$$

$$= \sum_{k=1}^{K} \sum_{n=0}^{\infty} \sum_{i=1}^{I_k} h_{k,i} d_k(n) c_k(t - nT_s - \tau_{k,i}) + n(t), \quad (7)$$

where $n(t)$ is an additive white Gaussian noise (AWGN) with a double-sided power spectral density of $N_0/2$ [W/Hz].

2.3 Receiver

The receiver for the kth signal is composed of the adaptive FIR filter, which has $(L+\alpha) \times M$ complex weights ($0 \leq \alpha \leq L$). Let $\mathbf{W}_k$ be an $(L+\alpha) \times M$ matrix whose (l,m)th entry is the complex weight $w_{k,l,m}$ of the receiver. The weight matrix $\mathbf{W}_k$ is updated by an adaptive algorithm. In this paper, we adopt a normalized least-mean-square (N-LMS) algorithm (Haykin, 1996). For simplicity, we assume that the receiver for the kth signal is synchronized with the first path of the channel $h_k(t)$. The kth receiver obtains discrete-time samples of every frequency and chip from the received signal $r_k(t)$. The mth frequency component $r_{k,l,m}$, detected at $t = nT_s + lT_c + \tau_{k,1}$ ($l = 1, 2, \cdots, L+\alpha$), is given by

$$r_{k,l,m}(n) = \int_{nT_s+(l-1)T_c+\tau_{k,1}}^{nT_s+lT_c+\tau_{k,1}} r_k(t) e^{-j\frac{2\pi(m-1)}{T_c}t} dt. \quad (8)$$

We define the $(L+\alpha) \times M$ matrix $\mathbf{R}_k(n)$ that contains the samples detected in $nT_s + \tau_{k,1} < t < nT_s + (L+\alpha)T_c + \tau_{k,1}$ as

$$\mathbf{R}_k(n) = \begin{bmatrix} r_{k,1,1}(n) & r_{k,1,2}(n) & \cdots & r_{k,1,M}(n) \\ r_{k,2,1}(n) & r_{k,2,2}(n) & \cdots & r_{k,2,M}(n) \\ \vdots & \vdots & \ddots & \vdots \\ r_{k,L,1}(n) & r_{k,L,2}(n) & \cdots & r_{k,L,M}(n) \\ r_{k,1,1}(n+1) & r_{k,1,2}(n+1) & \cdots & r_{k,1,M}(n+1) \\ r_{k,2,1}(n+1) & r_{k,2,2}(n+1) & \cdots & r_{k,2,M}(n+1) \\ \vdots & \vdots & \ddots & \vdots \\ r_{k,\alpha,1}(n+1) & r_{k,\alpha,2}(n+1) & \cdots & r_{k,\alpha,M}(n+1) \end{bmatrix}. \quad (9)$$

The FIR filter output $\hat{d}_k(n)$ can be represented as

$$\hat{d}_k(n) = \mathrm{tr}[\mathbf{W}_k^{\mathcal{H}}(n)\mathbf{R}_k(n)], \quad (10)$$

where the superscript $^{\mathcal{H}}$ denotes the complex conjugate and transpose of the matrix, and $\mathrm{tr}[\cdot]$ denotes the trace of the matrix. To recover the message symbol $b_k(n)$, the receiver determines the sign for the real and imaginary parts of $\hat{d}_k(n)$, such that

$$\tilde{d}_k(n) = \mathrm{sgn}[\mathrm{Re}[\hat{d}_k(n)]] + j\mathrm{sgn}[\mathrm{Im}[\hat{d}_k(n)]], \quad (11)$$

where $\mathrm{sgn}[\cdot]$ is the signum function, $\mathrm{Re}[\cdot]$ is the real part of the complex value, and $\mathrm{Im}[\cdot]$ is the imaginary part of the complex value. Using $\tilde{d}_k(n)$, the estimate $\tilde{b}_k(n)$ of the complex message symbol $b_k(n)$ is given by

$$\tilde{b}_k(n) = \tilde{d}_k(n)\tilde{d}_k^*(n-1) \quad (12)$$

$$= (\mathrm{sgn}[\mathrm{Re}[\hat{d}_k(n)]] + j\mathrm{sgn}[\mathrm{Im}[\hat{d}_k(n)]])$$
$$\times (\mathrm{sgn}[\mathrm{Re}[\hat{d}_k(n-1)]] - j\mathrm{sgn}[\mathrm{Im}[\hat{d}_k(n-1)]]), \quad (13)$$

where the superscript * denotes the complex conjugate.

The weight matrix $\mathbf{W}_k(n)$ is updated as

$$\mathbf{W}_k(n+1) = \mathbf{W}_k(n) + \frac{\mu}{\mathrm{tr}[\mathbf{R}_k^{\mathcal{H}}(n)\mathbf{R}_k(n)]}\mathbf{R}_k(n)e_k^*(n), \quad (14)$$

where μ is the step size parameter and $e_k(n)$ is

$$e_k(n) = \tilde{d}_k(n) - \mathrm{tr}[\mathbf{W}_k^{\mathcal{H}}(n)\mathbf{R}_k(n)]. \quad (15)$$

In this paper, the initial value $\mathbf{W}_k(0)$ of the weight matrix $\mathbf{W}_k(n)$ for the kth receiver is chosen to be a set of weights that consists of the corresponding initial hopping pattern $\mathbf{P}_k(0)$ and the zero matrix $\mathbf{0}_{\alpha \times M}$ of size $\alpha \times M$, that is,

$$\mathbf{W}_k(0) = [\mathbf{P}_k^{\mathcal{T}}(0)\ \mathbf{0}_{\alpha\times M}^{\mathcal{T}}]^{\mathcal{T}}, \quad (16)$$

where the superscript $^{\mathcal{T}}$ denotes the transpose of the matrix.

2.4 Feedback

Part of the FIR filter weights of the receiver for the kth signal are fed back to the corresponding transmitter, in which they are used as an updated version of the hopping pattern $\mathbf{P}_k$. In this paper, no delay time and no error for the feedback are assumed. Therefore, the hopping pattern $\mathbf{P}_k(\lambda)$ updated at $t = \lambda T_f + \Delta_k + \alpha T_c + \tau_{k,1}$ ($\lambda = 1, 2, \cdots, N_f$; N_f is the number of iterations of the feedback, T_f is the feedback time interval, Δ_k is the preassigned offset of the feedback timing ($0 \leq \Delta_k < T_f$)) is represented as

$$\mathbf{P}_k(\lambda) \left(\triangleq \begin{bmatrix} p_{k,1,1}(\lambda) & p_{k,1,2}(\lambda) & \cdots & p_{k,1,M}(\lambda) \\ p_{k,2,1}(\lambda) & p_{k,2,2}(\lambda) & \cdots & p_{k,2,M}(\lambda) \\ \vdots & \vdots & \ddots & \vdots \\ p_{k,L,1}(\lambda) & p_{k,L,2}(\lambda) & \cdots & p_{k,L,M}(\lambda) \end{bmatrix} \right) \quad (17)$$

$$= \begin{bmatrix} w_{k,1,1}(\hat{n}_k) & w_{k,1,2}(\hat{n}_k) & \cdots & w_{k,1,M}(\hat{n}_k) \\ w_{k,2,1}(\hat{n}_k) & w_{k,2,2}(\hat{n}_k) & \cdots & w_{k,2,M}(\hat{n}_k) \\ \vdots & \vdots & \ddots & \vdots \\ w_{k,L,1}(\hat{n}_k) & w_{k,L,2}(\hat{n}_k) & \cdots & w_{k,L,M}(\hat{n}_k) \end{bmatrix}, \tag{18}$$

where $\hat{n}_k \triangleq \lfloor(\lambda T_f + \Delta_k + \alpha T_c + \tau_{k,1})/T_s\rfloor$ ($\lfloor x \rfloor$ is the maximum positive integer less than or equal to x). The FIR filter receiver produces the filter weights that are the MMSE solution to the reference $\tilde{d}_k(n)$. As a result, the FIR filter receiver obtains the minimum ISI and MAI for the present received signals.

2.4.1 Limited Number of Tones

The hopping pattern generates multicarrier signals at the transmitter. In general, the PAR of multicarrier signals is larger than that of single carrier signals. Therefore, we discuss the impact of the limited number of tones per chip. In this paper, we consider the FCHP/MH-CDMA signals consisting of $M_{limited}$ ($\leq M$) tones per chip. This can be realized by the hopping pattern $\mathbf{P}_k(\lambda)$ every row of which contains $M_{limited}$ nonzero elements and $M - M_{limited}$ zero elements.

2.4.2 Quantization

Since the FIR filter weights take continuous values, the updated version of the hopping pattern contains elements that have continuous values. Therefore, the receiver requires quantization for feedback. We employ uniform quantization. Let $a_k^{(max)}(\lambda)$ be the maximum absolute value of the real and imaginary parts in all the elements of the hopping pattern $\mathbf{P}_k(\lambda)$, that is,

$$a_k^{(max)}(\lambda) = \max_{l,m}\{|\mathrm{Re}[p_{k,l,m}]|, |\mathrm{Im}[p_{k,l,m}]|\}. \tag{19}$$

The elements $p_{k,l,m}^{(quantized)}$ of a quantized hopping pattern are given by

$$p_{k,l,m}^{(quantized)} = \begin{cases} \lceil \mathrm{Re}[\rho_{k,l,m}(\lambda)]\rceil + j\lceil \mathrm{Im}[\rho_{k,l,m}(\lambda)]\rceil & (0 \leq \phi_{k,l,m} < \frac{\pi}{2}) \\ \lfloor \mathrm{Re}[\rho_{k,l,m}(\lambda)]\rfloor + j\lceil \mathrm{Im}[\rho_{k,l,m}(\lambda)]\rceil & (\frac{\pi}{2} \leq \phi_{k,l,m} < \pi) \\ \lceil \mathrm{Re}[\rho_{k,l,m}(\lambda)]\rceil + j\lfloor \mathrm{Im}[\rho_{k,l,m}(\lambda)]\rfloor & (\pi \leq \phi_{k,l,m} < \frac{3\pi}{4}) \\ \lfloor \mathrm{Re}[\rho_{k,l,m}(\lambda)]\rfloor + j\lfloor \mathrm{Im}[\rho_{k,l,m}(\lambda)]\rfloor & (\frac{3\pi}{4} \leq \phi_{k,l,m} < 2\pi) \end{cases}, \tag{20}$$

where $\lceil x \rceil$ is the minimum integer greater than or equal to x, and

$$\rho_{k,l,m}(\lambda) = \frac{p_{k,l,m}(\lambda)}{a_k^{(\max)}(\lambda)} 2^{q-1}, \tag{21}$$

and q is the number of quantization bits.

2.4.3 Clipping

In general, every chip of FCHP has a different energy. This may cause a large variation in the amplitude of the FCHP/MH-CDMA signal, which results in a large PAR. Therefore, to reduce this variation, we employ a technique of clipping a hopping pattern. Let $A^{(clipping)}$ be a clipping level, given by

$$A^{(clipping)} = \frac{\sqrt{\mathrm{tr}[\mathbf{P}_k^{\mathcal{H}}(\lambda)\mathbf{P}_k(\lambda)]}}{L}\beta, \tag{22}$$

where β is a real constant. Using $A^{(clipping)}$, the values $p_{k,l,m}^{(clipped)}$ of elements for the clipped hopping pattern are given by

$$p_{k,l,m}^{(clipped)}(\lambda) = \begin{cases} p_{k,l,m}(\lambda) & (|p_{k,l,m}| \leq A^{(clipping)}) \\ A^{(clipping)} e^{j\phi_{k,l,m}} & (|p_{k,l,m}| > A^{(clipping)}) \end{cases}. \tag{23}$$

3 PERFORMANCE EVALUATION

3.1 Specifications

3.1.1 Multipath Model

We assume a six-path model (i.e., $I_k = 6$ for all $k's$) that has a delay profile of exponential decay, where the relative intensities of $|h_{k,i}|$ are $20\log_{10}(|h_{k,i+1}|/|h_{k,i}|) = -3$dB ($i = 1, 2, \cdots, I_k - 1$), the path delays $\tau_{k,i}$ are $\tau_{k,i+1} - \tau_{k,i} = \frac{L+1}{16}T_c$ ($\approx \frac{1}{16}T_s$ for $L = 7$), and $\tau_{k,1}$ (for all $k's$) and $\theta_{k,i}$ (for all $k's$ and $i's$) are mutually statistically independent, uniformly distributed random variables in the intervals of $[0, T_s)$ and $[0, 2\pi)$, respectively.

3.1.2 Other Specifications

The FCHP/MH-CDMA requires an initial training period during which the receiver returns part of the filter weights to the corresponding transmitter to construct a suitable hopping pattern for the current channel state. In this paper, we define the initial training period as $t < (N_f + 1)T_f + \Delta_k + \tau_{k,1}$ and discuss the BER performance in the steady period, which is defined as the period after the initial training period, that is, $t \geq (N_f + 1)T_f + \Delta_k + \tau_{k,1}$. In the steady period, only the filter weights are updated at the receiver (i.e., no feedback). We assume that the reference $\tilde{d}_k(n)$ used for updating the filter weights is $\tilde{d}_k(n) = d_k(n)$ during the initial training period, which implies that the receiver has prior knowledge of the pilot data symbols used for the initial training. Since both BER and

PAR slightly depend on the randomly chosen values of $\tau_{k,1}$ and $\theta_{k,i}$, all plots show the average of five simulation trials. Other common specifications are listed in Table 1.

Table 1: Common specifications.

Data	Differentially encoded QPSK
E_b/N_0	9.9dB
L	7
α	7
M	8
T_f	$10^4 T_s$
N_f	10
Δ_k	Uniform distribution in $[0, T_f)$
Adaptive algorithm	N-LMS ($\mu = 0.1$)

3.2 Simulation Results

3.2.1 Par Vs Limited Number of Tones

It is easily expected that a small number of tones yields a small PAR; however, this causes a large BER. Therefore, we evaluate both BER and PAR. Figure 3(a) shows the characteristics of BER vs the number of active signals, K, for different limited numbers of tones, $M_{limited}$.

It is observed that the BER values for $4 \leq M_{limited} \leq 7$ are almost identical to those for $M_{limited} = 8 (= M)$. Figure 3(b) shows a comparison of the PAR for $M_{limited} = 4$ with $M_{limited} = 8$. It is seen that $M_{limited} = 4$ reduces the PAR for $M_{limited} = 8$.

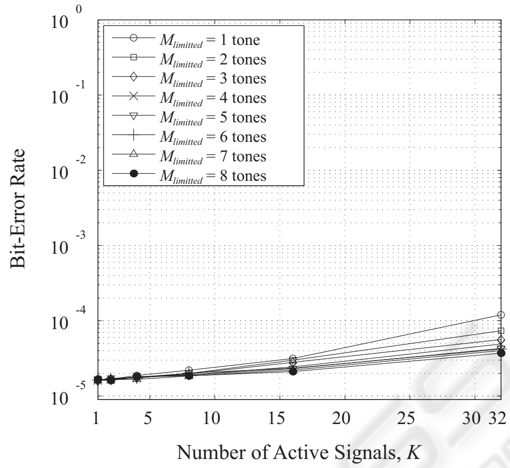
3.2.2 Par Vs Number of Quantization Bits

Figure 4(a) shows the characteristics of BER vs the number of active signals, K, for different numbers of quantization bits, q. $M_{limited} = 4$ is assumed. It is observed from Fig. 4(a) that the BER values are identical for $q \geq 4$. Figure 4(b) shows the characteristics of PAR for $q = 4$ and $M_{limited} = 4$, from which the effectiveness of PAR reduction can be confirmed.

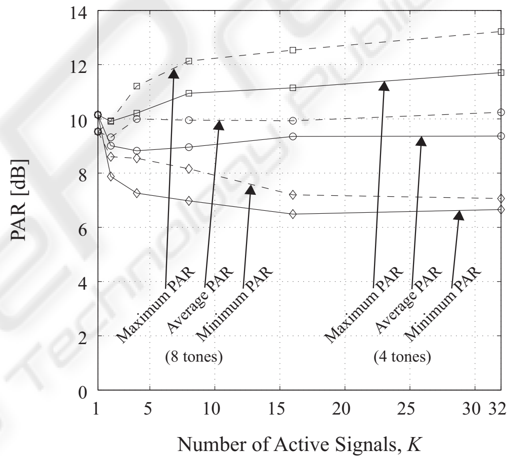
3.2.3 Clipping

Figure 5(a) shows the characteristics of BER vs the number of active signals, K, for different clipping levels, β.

$M_{limited} = 4$ and $q = 4$ are assumed. It is observed from Fig. 5(a) that the BER values are identical for $\beta \geq 1.5$. Figure 5(b) shows the characteristics of PAR for $\beta = 1.5$, $M_{limited} = 4$ and $q = 4$, from which the effectiveness of PAR reduction can be confirmed. Note that the technique of clipping discussed in this section does not cause any inter-tone interference, because only the elements of hopping pattern are clipped, that is, waveforms are not clipped.



(a) BER vs limited number of tones (no quantization, noclipping).

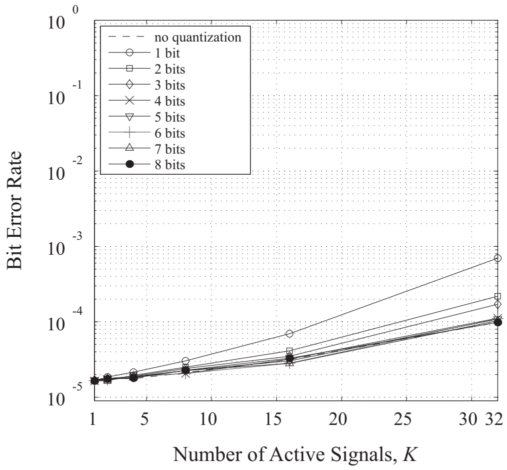


(b) PAR vs limited number of tones (no quantization, no clipping).

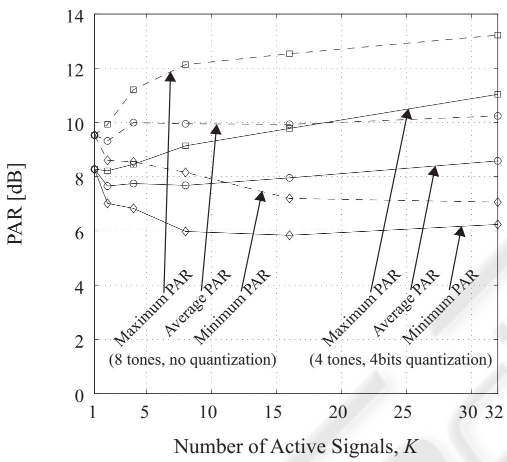
Figure 3: Impact of limited number of tones.

4 CONCLUSIONS

In this paper, we have investigated the PAR of FCHP/MH-CDMA signals and shown that tone selection by limiting the number of tones per chip, quantization for reducing the overhead for feedback, and clipping an FCHP are effective in reducing the PAR.

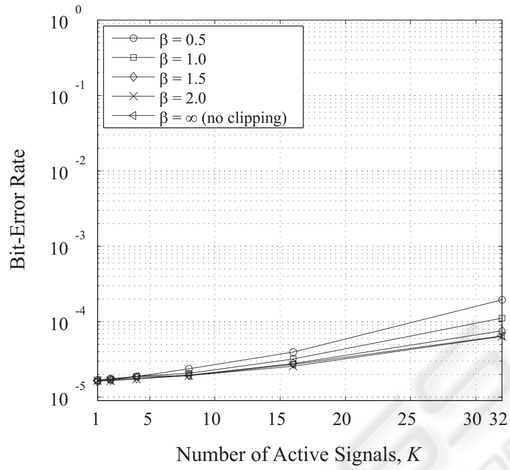


(a) BER vs number of quantization bits ($M_{limited}$ = 4 tones, no clipping).

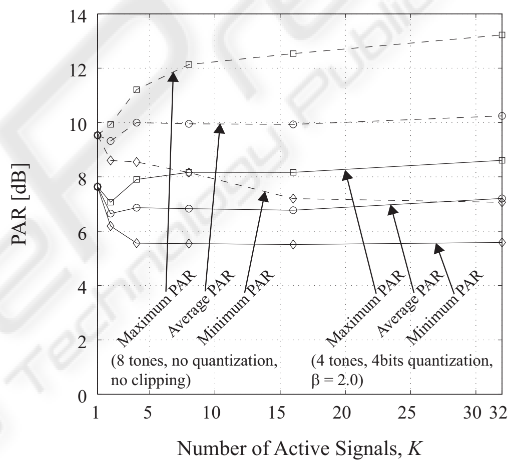


(b) PAR vs number of quantization bits (no clipping).

Figure 4: Impact of number of quantization bits.



(a) BER vs clipping level ($M_{limited}$ = 4 tones, q = 4 bits).



(b) PAR vs clipping level.

Figure 5: Impact of clipping.

REFERENCES

Chiba, K. and Hamamura, M. (2007). Performance of multitone hopping CDMA using feedback-controlled hopping pattern over multipath channel. In *IEICE Technical Report*, volume WBS2007-12. IEICE.

Hamada, S., Hamamura, M., Suzuki, H., and Tachikawa, S. (1998). A proposed DS/CDMA system using analog PN sequences produced by adaptive filters. *IEICE Trans. Fundamentals*, E81-A(11):2261–2268.

Haykin, S. (1996). *Adaptive Filter Theory*. Prentice Hall, New Jersey, 3rd edition.

Miyatake, T., Chiba, K., Hamamura, M., and Tachikawa, S. (2008). Asynchronous, decentralized DS-CDMA using feedback-controlled spreading sequences for time-dispersive channels. *IEICE Trans. Commun.*, E91-B(1):53–61.

Miyatake, T., Hamamura, M., and Tachikawa, S. (2004). Performance of DS/SS system using feedback controlled spreading sequence over a multipath channel. pages 567–571. Proc. ISITA2004. Parma.

Rupf, M. and Massey, J. L. (1994). Optimum sequence multisets for synchronous code-division multiple-access channels. *IEEE Trans. Inform. Theory*, 40(4):1261–1266.

Ulukus, S. and Yates, R. D. (2001). Iterative constructions of optimum signature sequence sets in synchronous CDMA systems. *IEEE Trans. Inform. Theory*, 47(5):1989–1998.

Welch, L. R. (1994). Lower bounds on the maximum cross correlation of signals. *IEEE Trans. Inform. Theory*, 20(3):397–399.