

# FEEDING A GENETIC ALGORITHM WITH AN ANT COLONY FOR CONSTRAINED OPTIMIZATION

## *An Application to the Unit Commitment Problem*

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Abstract: In this paper, a new optimisation strategy for the solution of the classical Unit Commitment problem is proposed. This problem is known to be an often large scale, mixed integer programming problem. Due to high combinatorial complexity, the exact solution is often intractable. Thus, a metaheuristic based method has to be used to compute a very often suitable solution. The main idea of the approach is to use ant colony algorithm, to explicitly deal with the feasibility of the solution, and to feed a genetic algorithm whose goal is to intensively explore the search space. Finally, results show that the proposed method leads to the tractable computation of satisfying solutions for the Unit Commitment problem.

## 1 INTRODUCTION

The Unit Commitment problem is a classical optimization mixed integer problem, referring to the optimal scheduling computation of several production units while satisfying consumer's demand. Due to temporal coupling of constraints (time up and time down constraints), a long temporal horizon is required, implying a large number of binary variables. Numerous methods have already been applied to tackle the difficulties of the problem (Sen and Kothari, 1998). Roughly speaking, the following classification can be made: Exact methods (exhaustive enumeration, "Branch and Bound" (Chan and Wang, 1993), dynamic programming (Ouyang and Shahidehpour, 1991)); deterministic approximated methods (priority lists (Senjyu, *et al.*, 2004)); Lagrangian relaxation (Zhai and Guan, 2002); Stochastic methods, also called metaheuristics (simulated annealing (Yin Wa Wong, 1998), tabu search (Rajan and Mohan, 2004), genetic algorithms (Swarup and Yamashiro, 2002)).

All these approaches have pros and cons: exact methods suffer from combinatorial complexity,

deterministic approaches are very easy and tractable, but can be strongly suboptimal, Lagrangian relaxation allows taking into account constraints and can be used to medium scale cases, but, due to the non convexity of the objective function, no guarantee can be given on the actual optimality. In the case of metaheuristics methods, there is no guarantee on the actual optimality of the solution, but one can very often compute a very satisfying suboptimal with low computation times. This kind of methods is very interesting, especially for large scale cases. However, one of the problems of such stochastic methods is the management of the feasibility of solutions. As the algorithm "walks" randomly in the search space, there is no guarantee that the final solution is in the feasible set. This is particularly the case for the Unit Commitment problem, which is a strongly constrained problem.

In this paper a new optimization strategy is defined for the use of metaheuristics optimization methods, leading to a satisfying solution and the guarantee of its actual feasibility. The first step of the procedure is to use an ant colony algorithm as a "feasible solutions generator". As such an algorithm is constructive it is possible to explicitly manage the

constraints of the problem to create a set of feasible solutions. The second step is to define a criterion to optimize the problem with a genetic algorithm from the initial population created by ant colony. In section 2, the optimization strategy is depicted and then is adapted to the Unit Commitment problem in section 3. Numerical results are given in section 4. Forthcoming works are given in section 5 and concluding remarks are drawn in section 6.

## 2 OPTIMIZATION STRATEGY

### 2.1 Optimization Method Synopsis

The synopsis of the algorithm is depicted on figure 1. The idea is to firstly sample the feasible space with an ant colony algorithm. This algorithm is constructive. Thus, solutions are explicitly built as feasible ones (see section 2.2). However, ant colony may fail to find a very good solution: because of its positive feedback structure, it may be trapped in a local minimum. Thus, a genetic algorithm will be used to intensively explore the search space (see section 2.4) from the population computed by ants. For that purpose, a new criterion is defined, for which feasibility is guaranteed (see section 2.3).

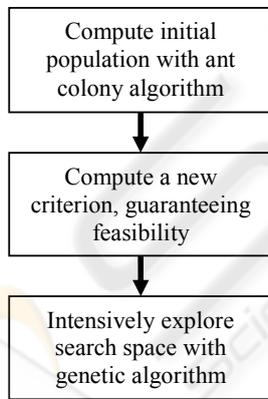


Figure 1: Optimization strategy synopsis.

The proposed methodology can be used to solve various and large scale optimization problems, the satisfaction of all constraints being guaranteed.

### 2.2 Ant Colony Optimization

Ant colony optimization was introduced by Marco Dorigo (Dorigo, *et al.*, 1996). It is based on the way ants are looking for food. The metaphor is used to solve graph exploration problems. Various criterions

can be optimized. For instance, a cost may be associated to each node. The goal is to minimize the sum of costs while exploring the graph. This is the well known Travelling Salesman Problem, for which ant colony algorithm has been firstly used (Dorigo, *et al.*, 1997). During iteration  $t$  of the algorithm,  $F$  ants walk on the graph of figure 2.

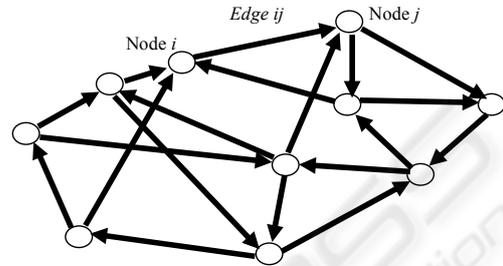


Figure 2: Graph exploration.

If ant  $f$  has reached node  $i$ , the probability that it chooses the next node  $j$  is:

$$P_i^{(f)}(j) = \frac{\eta_{ij}^\alpha \tau_{ij}(t)^\beta}{\sum_{m \in J_f(i)} \eta_{im}^\alpha \tau_{im}(t)^\beta} \quad (1)$$

- $\tau_{ij}(t)$  is the pheromone trail on edge  $ij$  during iteration  $t$ . Its value depends on the results of previous ants;
- $\eta_{ij}$  is the attractiveness. It refers to the « local choice ». For the Traveling Salesman Problem,  $\eta_{ij} = 1/d_{ij}$ , where  $d_{ij}$  is the cost associated to the edge  $ij$  of the graph;
- $\alpha$  and  $\beta$  are weighting factors;
- $J_f(i)$  is the feasible set (for the travelling Salesman problem, this feasible set contains all nodes connected to node  $i$ , but those nodes which have been already explored by the ant are to be removed).

At the end of iteration  $t$  of the algorithm,  $F$  ants have computed  $F$  potential solutions, which are evaluated. The pheromone trail which laid on the graph of figure 2 is then updated:

$$\tau_{ij}(t+1) = (1 - \rho) \tau_{ij}(t) + \Delta\tau_{ij}(t) \quad (2)$$

$\Delta\tau_{ij}$  is the update coefficient whose value depends on the results of ants in iteration  $t$ . The more the ants which have walked on edge  $ij$  have obtained good results, the higher is  $\Delta\tau_{ij}$ . Several strategies can be

used for this update.  $\rho$  is the evaporation coefficient. This coefficient is an analogy with physical evaporation of pheromone in nature. Usually, there are about 20 to 30 ants, and  $\alpha \approx 1$  and  $\beta \approx 2$ . It is often necessary to bound the pheromone trail on each edge ( $\sigma \in [\sigma_{\min}, \sigma_{\max}]$ ), to avoid premature convergence; see (Stützle, *et al.*, 2000). Note that ant colony optimisation is a constructive algorithm. As results, it can explicitly take into account all the constraints of the problem, with the help of the feasible sets  $J_f(i)$  which are built for each ant.

### 2.3 Defining a Feasibility Criterion

Consider the following optimization problem:

$$\min_x f(x) \quad (3)$$

$$s.t. \begin{cases} h(x) = 0 \\ g(x) \leq 0 \end{cases}$$

If a feasible solution with cost  $c^f$  is known, the following feasibility criterion can be defined:

$$\min_x f(x) + ((1 + \varepsilon) c^f + h(x)).B(x) \quad (4)$$

where

- $\varepsilon$  is a small positive real;
- $h(x)$  is a penalty function.
- $B(x)$  is a boolean function with value 1 for non feasible solutions and 0 for feasible ones.

Any unfeasible solution has a higher cost than the feasible known solution. Then the criterion can be optimized by any unconstrained optimization algorithm, and for instance by genetic algorithm. In the proposed strategy,  $c^f$  is the cost of the best solution found by the ant algorithm.

### 2.4 Genetic Algorithm

Genetic algorithm is a classical global optimization method. The general flow chart of this algorithm is given in figure 3. Classical cross-over and mutation operators are represented in figure 4 and 5 for a binary optimization problem. The aim of the crossover operator is to create 2 new potentially efficient individuals from 2 parents by mixing their variables.

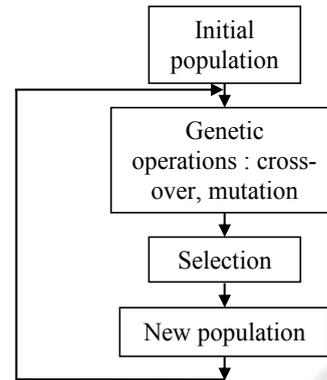


Figure 3: General flow chart of a genetic algorithm.

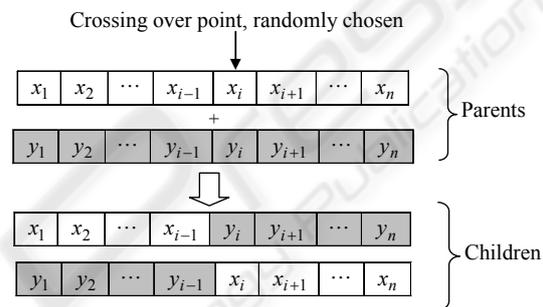


Figure 4: Crossing-over operator.

The mutation operator allows the introduction of new genes in the population by randomly changing one of the variables. The selection operator is made via the classical biased roulette selection.

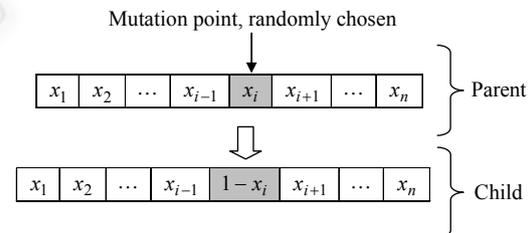


Figure 5: Mutation operator.

## 3 APPLICATION TO THE UNIT COMMITMENT

### 3.1 Unit Commitment Problem

The Unit Commitment refers to the minimization of global costs of  $K$  production units, over a time horizon  $N$ , satisfying a consumer's demand:

$$\min_{\{u_n^k, Q_n^k\}} \sum_{n=1}^N \left( \sum_{k=1}^K \left( c_{prod}^k(Q_n^k, u_n^k) \right) \right) + c_s^k(u_n^k, u_{n-1}^k) \quad (5)$$

where:

- $Q_n^k$  is the produced power of unit  $k$  at time  $n$ ;
- $u_n^k$  is the on/off status of unit  $k$  at time  $n$  (binary variable);
- $c_{prod}^k(Q_n^k, u_n^k) = (\alpha_1^k Q_n^k + \alpha_0^k) u_n^k$  is the cost function of unit  $k$  ( $\alpha_i^k$  are technical data).
- $c_s^k(u_n^k, u_{n-1}^k) = c_{on}^k u_n^k (1 - u_{n-1}^k) + c_{off}^k u_{n-1}^k (1 - u_n^k)$  is the start up and shut down cost of unit  $k$ .

Constraints of the problem are:

- Capacity constraints:

$$\underline{Q}_n^k u_n^k \leq Q_n^k \leq \bar{Q}_n^k u_n^k \quad (6)$$

- Satisfaction of consumer's demand  $Q_n^{dem}$ :

$$\sum_{k=1}^K Q_n^k \geq Q_n^{dem} \quad (7)$$

- Time-up  $T_{up}^k$  and time-down  $T_d^k$  constraints:

$$(u_{n-1}^k = 0, u_n^k = 1) \Rightarrow (u_{n+1}^k = 1, u_{n+2}^k = 1, \dots, u_{n+T_{up}^k-1}^k = 1) \quad (8a)$$

$$(u_{n-1}^k = 1, u_n^k = 0) \Rightarrow (u_{n+1}^k = 0, u_{n+2}^k = 0, \dots, u_{n+T_d^k-1}^k = 0) \quad (8b)$$

### 3.2 Graph Exploration Formulation for Ant Colony

The Unit Commitment problem can be depicted by the graph represented in figure 6. Nodes of the graph are all the possible states of production system:  $\{u_n^1, \dots, u_n^K\}$ . The goal is to go from one of the possible states at time 1, to one of the possible states at time  $N$ , while satisfying all constraints and minimising global costs. Start up/shut down are associated to edges of the graph; production costs are associated to nodes.

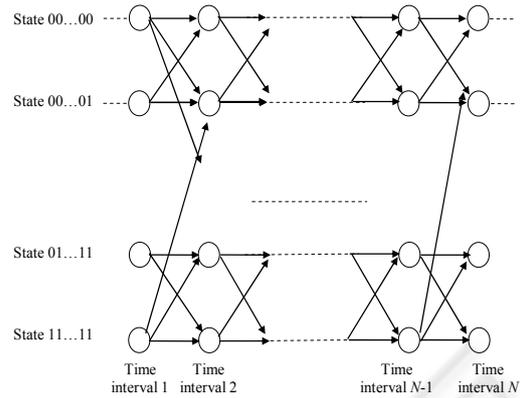


Figure 6: Graph exploration formulation of Unit Commitment.

### 3.3 Computation of Real Variables

Ant colony and genetic algorithm are mostly dedicated to integer programming. The problem can be reformulated in a purely integer programming problem. Consider that binary variables are given and refer to a feasible solution. Real variables are computed as the solution of the following optimization problem:

$$\arg \min_{\{Q_n^k\}} \sum_{n=1}^N \left( \sum_{k=1}^K \left( c_{prod}^k(Q_n^k, u_n^k) \right) \right) + c_s^k(u_n^k, u_{n-1}^k) \quad (9)$$

$$= \arg \min_{\{Q_n^k\}} \left( \sum_{n=1}^N \sum_{k=1}^K \alpha_1^k Q_n^k u_n^k \right)$$

The optimal solution is to produce as much as possible with low-cost units, while satisfying capacity constraints:

$$Q_n^1 = \min \left( \max \left( Q_n^{dem} - \sum_{i=2}^K \underline{Q}_n^i u_n^i, \underline{Q}_n^1 \right), \bar{Q}_n^1 \right) u_n^1$$

$$\vdots$$

$$Q_n^k = \min \left( \max \left( \begin{array}{l} Q_n^{dem} - \sum_{i=1}^{k-1} \underline{Q}_n^i u_n^i \\ - \sum_{i=k+1}^K \underline{Q}_n^i u_n^i, \underline{Q}_n^k \end{array} \right), \bar{Q}_n^k \right) u_n^k \quad (10)$$

$$\vdots$$

$$Q_n^K = \min \left( \max \left( Q_n^{dem} - \sum_{i=1}^{K-1} \underline{Q}_n^i u_n^i, \underline{Q}_n^K \right), \bar{Q}_n^K \right) u_n^K$$

### 3.4 Enhanced Genetic Algorithm

It has been observed that the classical genetic algorithm can be more efficient by using the a priori knowledge of the system (Sandou, *et al.*, 2007). A “selective mutation operator” is added to the classical genetic operators. Consider figure 7, with a particular unit scheduling. Very often, a random mutation leads to an infeasible solution, because of time-up and time-down constraints. To increase the probability of reaching a feasible point with a mutation, the muted gene has to be located at switching times of the planning.

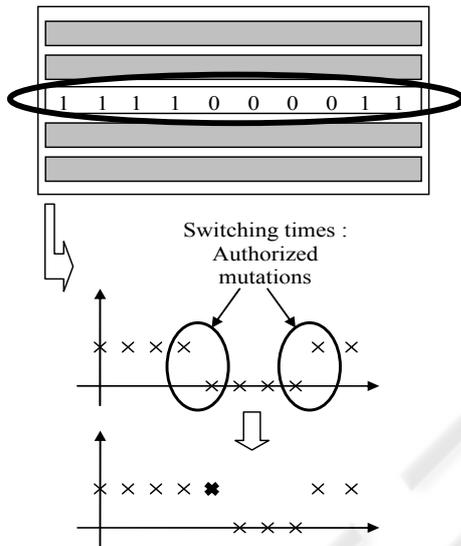


Figure 7: Selective mutation operator.

## 4 NUMERICAL RESULTS

The proposed strategy has been tested with Matlab 6.5 with a Pentium IV 2.5GHz. Optimization horizon is 24 hours with a sampling time of one hour. A comparison is made with pure genetic algorithm. A “four unit” case is considered. Characteristics are given in table 1.  $c_{on}$  is 2€ and  $c_{off}$  is 10€ for all units.

Table 1: Characteristics of the “4 unit case”.

$k$	$Q_{min}$ (MW)	$Q_{max}$ (MW)	$\alpha_0$ (€)	$\alpha_1$	$T_d$ (h)	$T_{up}$ (h)
1	10	40	25	2.6	2	4
2	10	40	25	7.9	2	4
3	10	40	25	13.1	3	3
4	10	40	25	18.3	3	3

For this small case example, a “Branch and Bound” method has been developed so as to get the global optimum and validate the method. Parameters of the the ant colony algorithm are:  $\alpha = 1$ ;  $\beta = 2$ ,  $\rho = 0.2$ ,  $\tau_{min} = 1$ ;  $\tau_{max} = 5$ . For the genetic algorithm, parameters are: Crossover probability 70%, classical mutation probability 5%, selective mutation: 10%. Results are given in table 2. As stochastic algorithms are considered, 100 tests are performed for each case, and statistical data about the results are given: mean cost (compared to the global optimum), worst case, rate of success (number of times that the global optimum is found) and computation times. The population are set to 50 individuals for genetic algorithm. When it is fed by ant colony, 5 iterations of 10 ants are performed to compute 50 initial solutions.

Table 2: Results for the “4 unit” case.

	Mean	Worst	Success	Comp. times
Ants + GA 100 iter	+2.5%	+9.4%	30%	15 s
GA 100 iter	+3.1%	+13.4%	20%	13 s
Ants + GA 200 iter	+0.4%	+3.5%	80%	25 s
GA 200 iter	+0.5%	+4.5%	77%	24 s

Computation times are very low for all cases. Results show that the use of ant colony as a “feasible solutions generator” leads to an increase in the quality of the solution. In particular, the worst case cost is better with the cooperative method. Thus, the cooperative method is very satisfying, especially for the “50 Ants – GA 200 iterations” case, as the mean result is just 0.4% higher than the optimal solution, and the worst case leads to a slight increase (less than 4%). Computation times are about 25 seconds.

For 200 generations, results seem to prove that the ant generation has no influence anymore, compared with the pure genetic algorithm. However, for successful tests, it is interesting to have a look on the iteration number for which the best solution has been found. For pure generic algorithm, the best solution is found after 126 generations (average number), whereas it is found after 96 generations for the ant colony/genetic algorithm. Convergence is achieved earlier with the cooperative method. Note that results of the pure genetic algorithm are still very satisfying, thanks to the selective mutation operator, as shown in (Sandou, *et al.*, 2007).

## 5 DISCUSSION

### 5.1 Very Large Scale Cases

The interest of a feasible initial population has been shown in previous results. In this paper, this feasible population is computed by an ant algorithm. The ant colony algorithm can be seen as a stochastic dynamic programming algorithm. The size of the state space is  $2^K$ . This is the main limiting point of the proposed method. Thus, for high values of  $K$ , the computation times of the ant colony algorithm grows very quickly. Thus, one of the main points is the application of ant colony to very large scale cases.

### 5.2 Global Optimization and Cooperation

Ant colony algorithm and genetic algorithm are two global optimization techniques and it may be astonishing to use them as a cooperative method. The goal of this hybridising was to couple the feasibility properties of ant colony algorithm and the intensive exploration of genetic algorithm. The cooperation is a sequential procedure, and a more alternated procedure could be profitable. For instance, results of genetic algorithms can be used to define the attractiveness parameters in a new iteration of ant colony algorithms. Furthermore, it will be interesting to couple the method with a local search.

## 6 CONCLUSIONS

In this paper, an optimization strategy has been defined and applied to solve the Unit Commitment. The main idea is to use an ant algorithm as a feasible solutions generator. These feasible solutions are brought together in an initial population for a genetic algorithm. To guarantee the feasibility of the final solution, a special criterion is computed from the results of the ant algorithm. To increase the efficiency of the classical genetic algorithm, a knowledge-based operator is defined (selective mutation). Finally, the proposed method leads to high quality solution, with guarantees of feasibility and with low computation times. The main limiting point appears to be the computation times of ant colony algorithm for very large scale cases. However, the use of feasible solutions in the initial population of a genetic algorithm is an interesting way to decrease the number of iterations required to

find near optimal solutions. Forthcoming works deal with the use of such algorithm for predictive control of non linear hybrid systems.

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