PATH PLANNING USING DISCRETIZED EQUILIBRIUM PATHS
A Robotics Example

Cornel Sultan
Aerospace and Ocean Engineering Department, Virginia Tech, 215 Randolph Hall, Blacksburg, VA, U.S.A.

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Abstract: A collision avoidance path planning problem is considered and a simple solution which uses piecewise constant controls generated by discretizing a feasible equilibrium path is presented and investigated.

1 INTRODUCTION

A new methodology has been recently proposed (Sultan, 2007) for the control of nonlinear ODEs,

$$\dot{x} = \frac{dx}{dt} = f(x,u), \quad x \in X \subset R^n, \; u \in U \subset R^m, \; t \in T \subset R. \tag{1}$$

Here f is a function of class C^k in $X \times U$ ($k > 0$), x, u, and t are the state, control vectors, and time, whereas X, U, and T are *open sets* in the n, m, and one dimensional real spaces.

The key idea is to control (1) such that its *state space trajectory is close to an equilibrium path* obtained by solving

$$0 = f(x,u). \tag{2}$$

If (x_i, u_i) is a solution of (2) and $J_i = \frac{\partial f}{\partial x}(x_i, u_i)$ is *not singular*, there exist an open set U_e and a *unique* function g of class C^k such that

$$x = g(u), x_i = g(u_i), \quad f(g(u),u) = 0, \mathrm{g}: U_e \to X_e. \tag{3}$$

Here U_e is the *largest domain* in U in which (2) can be solved uniquely for x as in (3) and $\frac{\partial f}{\partial x}(g(u),u)$ is not singular. If (x_f, u_f), $u_f \in U_e$ is a different solution of (3), u_i and u_f can be connected by a curve $u_e(s)$ in U_e, parameterized by $s \in [0, \tau]$,

$$u_e(0) = u_i, u_e(\tau) = u_f, \tag{4}$$

which is g-mapped onto an *equilibrium path,* $x_e(s) = g(u_e(s))$, $x_e(0) = x_i$, $x_e(\tau) = x_f$.

The control problem is to develop control laws which guarantee that *the state space trajectory of the system is close to the equilibrium path*, as illustrated in Figure 1. In order to achieve this goal, the strategy described next was proposed in (Sultan, 2007). The controls are initially fixed at u_i and when the transition begins, at $t=0$, they start to vary along u_e, $u(t) = u_e(t)$, $t \in [0, \tau] \subset T$. When t reaches τ the controls are frozen at the final, desired value:

$$u(t) = \begin{cases} u_i, t < 0 \\ u_e(t), 0 \le t \le \tau \\ u_f, t > \tau \end{cases} \cdot \tag{5}$$

The corresponding state space trajectory, $x_d(t)$, called the *deployment path*, is the solution of

$$\dot{x}_d = f(x_d, u(t)), x_d(0) = x_i. \tag{6}$$

If $x_d(\tau)$ belongs to the basin of attraction of x_f then the system's trajectory will settle down, asymptotically in time, to the desired final value, x_f. *Asymptotical stability of* x_f *is crucial* for the application of this methodology.

Sultan C. (2008).
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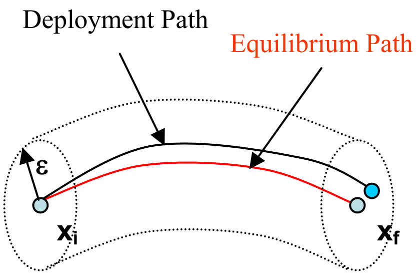

Figure 1: Deployment and equilibrium paths.

In this paper an example of a collision avoidance path planning problem is considered. An equilibrium path which satisfies the constraints is found and discretized to generate *piecewise constant* controls which are used to drive the system. It is important to remark that this strategy is different from the one proposed in (Sultan, 2007), where continuous controls are used. Here, the parameterization of the equilibrium path, originally continuous, is discretized. One justification for this approach is the easiness of discrete controls implementation.

2 THEORETICAL RESULTS

In the following two important results are given (the proofs are omitted for brevity).

Theorem 1. If the equilibrium path is composed only of *asymptotically stable* equilibria then, for $\forall \varepsilon > 0$ there exists a piecewise constant control $u(t)$, *obtained by discretizing the equilibrium path*, such that the distance between the corresponding segments of the deployment and equilibrium paths is less than ε (i.e. the deployment and equilibrium paths are arbitrarily close).

Theorem 2. If the equilibrium path is composed only of asymptotically stable equilibria and for any u, $f(x,u)$ is Taylor series expandable in x, for $\forall \eta > 0$ there exists a piecewise constant control $u(t)$, *obtained by discretizing the equilibrium path* such that $\|\dot{x}_d(t)\| < \eta, \forall t \in [0,\tau]$.

3 A PATH PLANNING PROBLEM

Consider a two link robotic manipulator in the vertical plane (Figure 2). The links are rigid, the system is placed in a constant gravitational field, control torques and damping torques proportional to the relative angular velocity between the moving parts act at the joints. The equations of motion are:

$$(m_1c_1^2 + m_2l_1^2 + I_1)\ddot{\theta}_1 + m_2l_1c_2\cos(\theta_1-\theta_2)\ddot{\theta}_2 + d_1\dot{\theta}_1 + m_2l_1c_2\sin(\theta_1-\theta_2)\dot{\theta}_2^2 + (m_1c_1+m_2l_1)g\sin(\theta_1) = u_1 \quad (7)$$

$$m_2l_1c_2\cos(\theta_1-\theta_2)\ddot{\theta}_1 + (m_2c_2^2+I_2)\ddot{\theta}_2 + d_2(\dot{\theta}_2-\dot{\theta}_1) - m_2l_1c_2\sin(\theta_1-\theta_2)\dot{\theta}_1^2 + m_2c_2g\sin(\theta_2) = u_2 \quad (8)$$

where angles θ_1, θ_2 describe the motion, m_i, l_i, c_i, I_i, are the mass, length, center of mass (CM) position, transversal moment of inertia of the i-th link, d_i and u_i are the damping coefficient and control torque at joint i, respectively, g is the gravitational constant. These equations can be easily cast into the first order form (1). The numerical values (SI units) used are:

$$m_1 = 10, m_2 = 5, l_1 = l_2 = 1/\sqrt{3}, c_1 = c_2 = 0.5, \\ I_i = m_i l_i / 12, d_1 = d_2 = 0.5, g = 9.81. \quad (9)$$

The system must transition between two equilibria, $\theta_{1i} = 70, \theta_{2i} = 0, \theta_{1f} = -70, \theta_{2f} = 0$. Collision with a circular sector obstacle, of radius *R=1*, described below, must be avoided:

$$\begin{aligned} &\theta_2 > 60 + \theta_1, if \quad -60 < \theta_1 \le 0 \\ &\frac{\sin(30-\theta_2)}{\sin(\theta_1-\theta_2)} - \frac{1}{\sqrt{3}} < 0, if \quad 0 < \theta_1 < 30 \\ &\theta_2 > 60 - \theta_1, if \quad 30 \le \theta_1 < 60. \end{aligned} \quad (10)$$

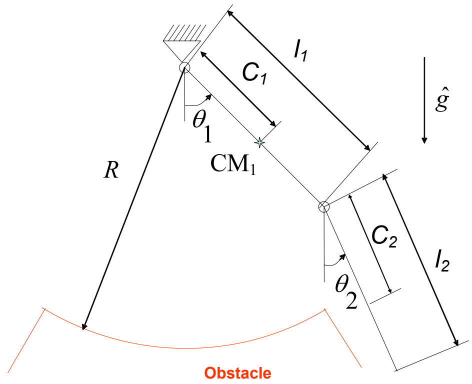

Figure 2: Two link robotic manipulator.

An equilibrium path which satisfies (10) is

$$\theta_{2e} = 62 \cos\left(\frac{\theta_{1e}}{\theta_{1f} - \theta_{1i}}\right), \theta_{1e} \in [\theta_{1i}, \theta_{1f}]. \quad (11)$$

and the equilibrium controls are easily found,

$$\begin{aligned} u_{1e} &= g(m_1 c_1 + m_2 l_1)\sin(\theta_{1e}), \\ u_{2e} &= m_2 c_2 g \sin(\theta_{2e}). \end{aligned} \quad (12)$$

The equilibrium path is parameterized using the following class C^2 function

$$\theta_{1e}(t) = \theta_{1i} + \frac{30}{\tau^5}(\theta_{1f} - \theta_{1i}) \left(\frac{t^5}{30} - \frac{t^4}{6}(t-\tau) + \frac{t^3}{3}(t-\tau)^2\right), 0 \le t \le \tau, \quad (13)$$

which is further discretized to obtain piecewise constant controls using (11) and (12).

Consider $\tau = 10$ ("fast deployment"). Piecewise constant controls are generated using N equal time intervals. Figure 3 shows the deployment and equilibrium paths.

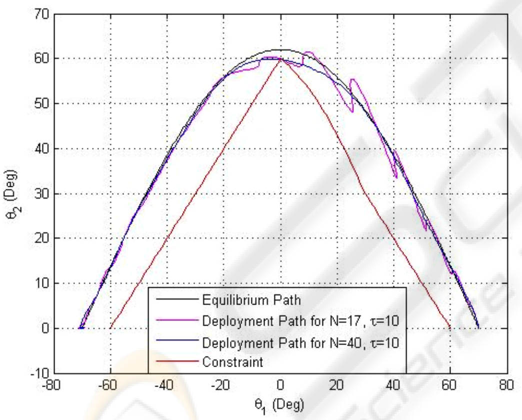

Figure 3: Deployment paths for "fast" deployment.

Figures 4 and 5 give the time histories of the controls and angles for N=17 and N=40. *The deployment error cannot be made small enough to avoid the obstacle* regardless of how large N is (higher values of N were considered). Thus τ *should be increased and the controls refined* for the deployment error to be sufficiently small.

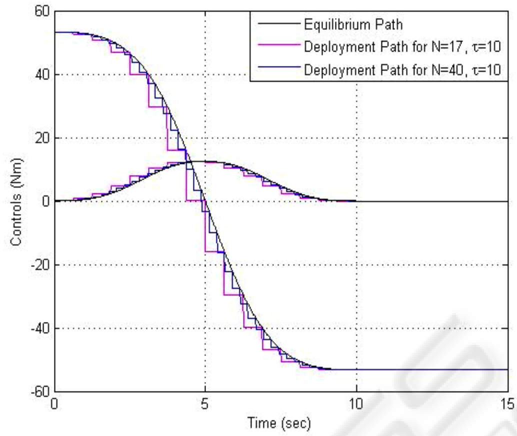

Figure 4: Controls variation for "fast" deployment.

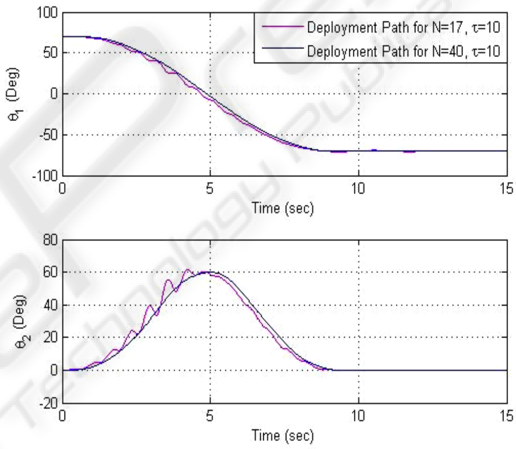

Figure 5: Generalized coordinates variation for "fast" deployment.

In the second scenario, called "slow" deployment, the deployment time is $\tau = 20$ and piecewise constant controls are generated by discretizing (11-13) with N=34 and N=80. Figures 8-10 show that collision is avoided. The deployment error is smaller because the deployment time is longer and finer controls are used. It is important to mention that if only the deployment time is increased the desired result is not obtained; if N=17 or N=40 are used in conjunction with $\tau = 20$, the deployment error is still big and collision with the obstacle occurs.

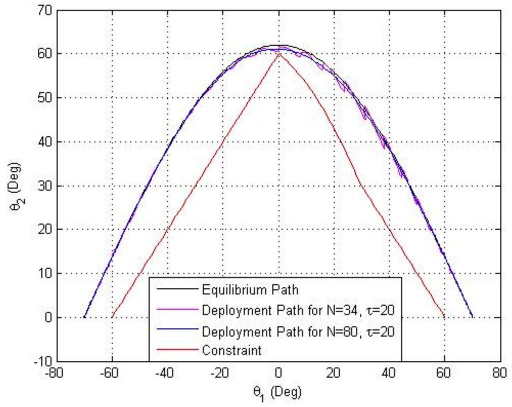

Figure 6: Deployment paths for “slow” deployment.

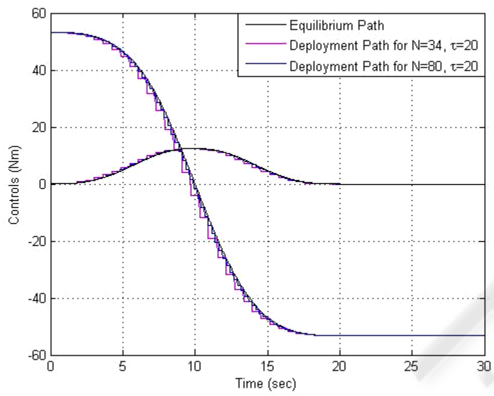

Figure 7: Controls variation for “slow” deployment.

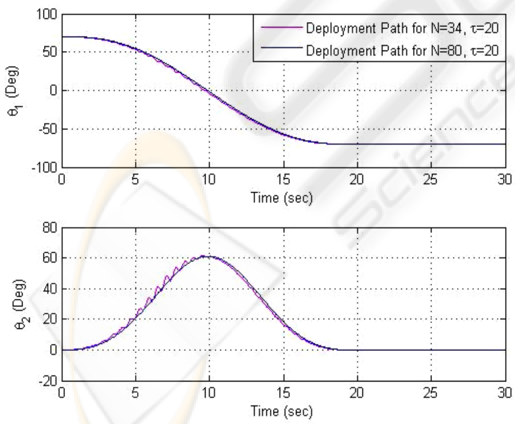

Figure 8: Generalized coordinates variation for “slow” deployment.

4 CONCLUSIONS

An example of a path planning problem is used to illustrate the control of nonlinear systems using equilibrium paths. The idea is to find an equilibrium path which satisfies the collision avoidance constraints, which is a much easier problem than finding a dynamic path which satisfies the constraints. Then the equilibrium path is discretized to build piecewise constant controls which are used to drive the system. Simulations indicate that for the deployment and equilibrium paths to be close the deployment time should be sufficiently long and the controls sufficiently refined.

It is important to remark that the solution investigated here uses discretizations of an equilibrium path which satisfies the collision avoidance constraints as opposed to continuous parameterizations and hence continuous controls. One justification for this approach is the easiness of practical implementation of discrete controls.

REFERENCES

Sultan, C., 2007. Nonlinear systems control using equilibrium paths. In *Proceedings of the Conference on Decision and Control, New Orleans, LA, USA.*