ADAPTIVE CUBE TESSELLATION FOR TOPOLOGICALLY CORRECT ISOSURFACES

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Abstract: Three dimensional datasets representing scalar fields are frequently rendered using isosurfaces. For datasets arranged as a cubic lattice, the marching cubes algorithm is the most used isosurface extraction method. However, the marching cubes algorithm produces some ambiguities which have been solved using different approaches that normally implying a more complex process. One of them is to tessellate the cubes into tetrahedra, and by using a similar method (marching tetrahedra), to build the isosurface. The main drawback of other tessellations is that they do not produce the same isosurface topologies as those generated by improved marching cubes algorithms. We propose an adaptive tessellation that, being independent of the isovalue, produces the same topology for all the cases. Moreover the tessellation allows isosurface to evolve continuously when the isovalue is changed smoothly without extra computations.

1 INTRODUCTION

A popular representation of digitized volumes is a regular lattice of points which represent the digitized samples. This set of samples can be used directly or can be previously prefiltered to obtain a reduced dataset. The most used lattice for some fields is a grid of cubic cells. This representation is easily obtained from a computerized axial tomography (CAT scan) or by magnetic resonance imaging (MRI). In both cases several 2D images are obtained that can be easily represented by a 3D grid in which every corner represents a point sample.

Thus, a volume can be represented by the equation:

$$(\{(x,y,z,\gamma)\} \ , \ F(x,y,z) : \mathbb{R}^3 \rightarrow \Gamma \ ) \qquad (1)$$

where $\{(x,y,z,\gamma)\}$ is the set of samples in the property domain Γ, and $F(x,y,z) : \mathbb{R}^3 \rightarrow \Gamma$ is the function that approximates values between samples.

There are several methods to visualize a volume represented by a grid: by slices, direct visualization (Levoy, 1990), or by means of isosurface extraction. By this later method, the focus of this paper, a threshold value γ_0 is set and an isosurface $F(x,y,z) = \gamma_0$ is built and rendered as volume visualization. Usually γ_0 is also called isovalue. Several parts of the volume can be rendered by modifying γ_0 . In this method $F(x,y,z)$ must be extended outside the samples, usually as a linear interpolation function. The classical algorithm in this category is the marching cubes method (Lorensen and Cline, 1987). This method extends F as a trilinear interpolation function and builds the isosurface cube by cube, *marching* through all the *cubes* into the grid of cubic cells.

Every cell is classified, by comparing its eight vertex values with the threshold γ_0, as belonging to one of the fifteen possible equivalence classes. Every class (or case) has an isosurface represented as a triangle mesh. The entire isosurface to be rendered is obtained by joining the pieces of isosurface generated for every cube. Figure 1 shows the fifteen cases with their interior triangle meshes.

However, this method is ambiguous in some topologycal aspects, the ambiguity has been well studied in (Wilhelms and Gelder, 1990; Nielson and Hamann, 1991; Montani et al., 1994; Chernyaev, 1995; Cignoni et al., 2000; Lopes and Brodlie, 2003; Nielson, 2003). For instance, figure 2-a shows that a crack arises on the isosurface between the two

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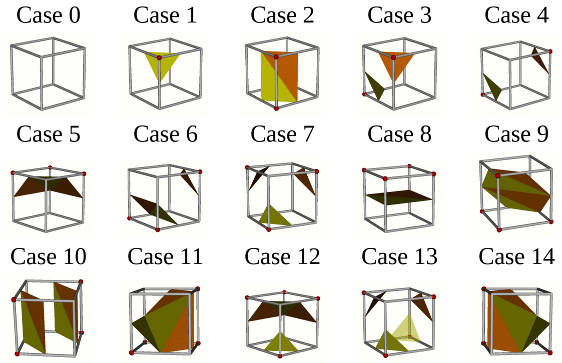


Figure 1: Well known cell cases in marching cubes method.

cells; figure 2-b shows the triangle mesh proposed by marching cubes for the Case 4 however, the one in figure 2-c may also be possible. To find out which one is correct we need information about the interior of the cell.

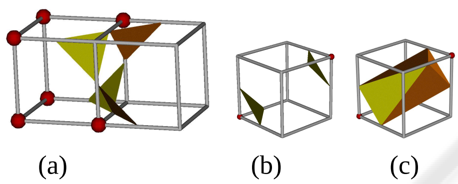


Figure 2: Examples of handicaps of marching cubes.

Lopes, Brodlie and Nielson in (Lopes and Brodlie, 2003; Nielson, 2003) solve the ambiguity studying the interior of the faces and the interior of the cell using the trilinear interpolation equation to define the property variation. As a consequence, the number of different equivalence classes is increased to 31 and 57 cases respectively.

Other authors (Zhou et al., 1995; Gueziec and Hummel, 1995; Chan and Purisima, 1998; Gerstner and Pajarola, 2000) solve the ambiguity using a tessellation of the cell into tetrahedra and building the isosurface by marching tetrahedra (Payne and Toga, 1990). This method is similar to marching cubes but based on a tetrahedron instead of a cube, with the added advantage that, in this method, there are no ambiguities as it is well known and only has 3 equivalence classes.

However, for many classes of extended marching cubes (EMC)(Lopes and Brodlie, 2003; Nielson, 2003), the topology of the isosurface which is built by EMC is not the same as the topology of the isosurface built by marching tetrahedra from tessellations in other published works.

In this paper we propose a tessellation of cells into tetrahedra which produces the same isosurface topology as the one that would be extracted by the extended marching cubes methods.

The next section analyses the sources of ambiguity in a cube. Section three presents some tessellations used to obtain tetrahedra from cubes. In section four we put forth our proposal of tessellation. The paper ends with a section presenting the results and the conclusions.

2 CORRECT ISOSURFACES

In order to determine the correct topology for an isosurface we have to study the interpolation function F, which is a trilinear interpolation function. For a more exhaustive study the paper by Nielson (Nielson, 2003) may be consulted.

Let us begin analyzing a cube face. Taking into account that every face vertex can be positive (its value is greater than the isovalue) or negative and that a face has four vertices, there are $2^4 = 16$ possible configurations of a face, however by rotation or by complementation (positive/negative), there are only 4 equivalence classes (see figure 3).

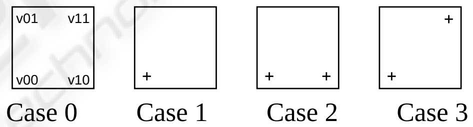


Figure 3: Cases for a face.

Assuming a linear variation along the edges, it is easy to prove that Case 0 does not have isocurve and Cases 1 and 2 have the two isocurve topologies shown in figure 4. However, Case 3 has 2 possible isocurve topologies as is shown in figure 4 depending on which vertices can be connected by a line without intersecting the isocurves, the negative ones (case 3-a) or the positive ones (case 3-b).

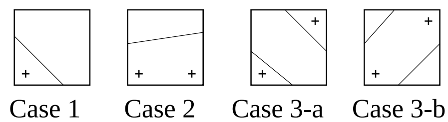


Figure 4: Isocurves for a face.

To discover which one is the correct topology we have to compute the face saddle point, the point where one topology changes into another. This point is computed by using the function that interpolates the interior of the face, which is the bilinear interpolation:

$$F_{face}(x,y) = axy + bx + cy + d \qquad (2)$$

The face saddle point is calculated by making the partial derivates, with respect to x and y, equal zero:

$$\frac{\partial F_{face}}{\partial x} = \frac{\partial F_{face}}{\partial y} = 0 \tag{3}$$

and its value is computed using the equation 2.

So, we have a 5^{th} point which can be positive or negative and allows us to choose the correct topology of the isocurve as it is shown in figure 5. The face saddle point becomes the inflection point between the two configurations; it is a contact point when the isovalue is equal to the face saddle point value (central image in figure 5).

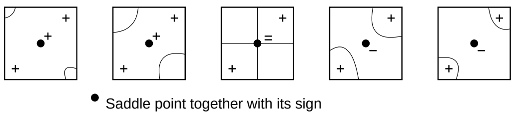


Figure 5: An isocurve which is continuously moved when the isovalue changes.

The face saddle point (FSP), when present and when it is inside the face, can be used to tessellate it into 4 triangles, so the correct topology is directly obtained by processing the triangles instead of the square (see figure 6).

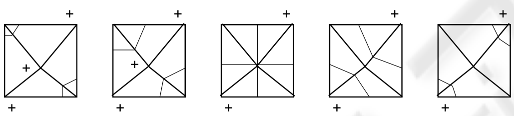


Figure 6: The isocurve topology is preserved when the square is tessellated into triangles.

If the face saddle point is not present or is outside the face, it can be shown that the face will be always classified as belonging to Cases 0, 1 or 2 (figure 3), which are not ambiguous.

For the interior of the cube the process is similar: the body saddle points need to be computed.

The function that interpolates the interior of the cube is the trilinear interpolation:

$$F(x,y,z) = axyz + bxy + cyz + dzx + ex + fy + gz + h \tag{4}$$

The body saddle point (BSP) is also obtained by making 0 the three partial derivates:

$$\frac{\partial F}{\partial x} = \frac{\partial F}{\partial y} = \frac{\partial F}{\partial z} = 0 \tag{5}$$

The result of this equation system gives two possible body saddle points which are used to solve the ambiguities and choose the correct topology for the isosurface. The body saddle points are inflection points between the different configurations for an ambiguous cell. In this way, we can use the body saddle points, when they are present and they are inside the cell, to tessellate the cube into tetrahedra. The correct topology of the isosurface can be obtained by marching tetrahedra as will be shown in sections 4 and 5.

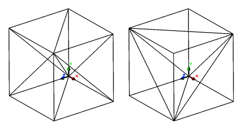
All the possible topologies, numbered following Lopes' methodology in (Lopes and Brodlie, 2003), are shown in figure 7. The first number defines the case taking into account the configuration of positive and negative vertices, the second number defines different solutions for ambiguous faces, and the third one defines different solutions for the ambiguous body. The case 0, without isosurface, is not shown. Nielson presents more cases in (Nielson, 2003), this is due to the use of a different equivalence relationship, but many of the cases are equivalents according to the Lopes' equivalence relationship.

These topologies can be easily obtained by marching tetrahedra with the adequate tessellation. The next section shows related works on tessellations and section 4 puts forth our proposal for the tessellation.

3 PREVIOUS TESSELATIONS

The marching tetrahedra method (Payne and Toga, 1990) for isosurface building is similar to the marching cubes method (Lorensen and Cline, 1987):

This method is non ambiguous by assuming a linear interpolation along the edges. Thus, by tessellating a cell into tetrahedra (figure 8 shows the most used tessellations) the ambiguity problem could be solved as can be appreciated in (Payne and Toga, 1990; Gueziec and Hummel, 1995).



(a) (b)

Figure 8: Two possible tessellations for the basic case.

However, these studies do not take into account that the property variation along a diagonal of a cube face is not linear, but quadratic, so that diagonal which is taken as a tetrahedron edge could have two intersections with the isosurface and not only one. Zhou et al. (Zhou et al., 1995) study this question and define a new way to build isosurfaces inside tetrahedra taking into account the quadratic variation of properties along diagonal edges, but there are 59 different cases! And moreover the method does not produce topologies like that of 4.1.2 in figure 7.

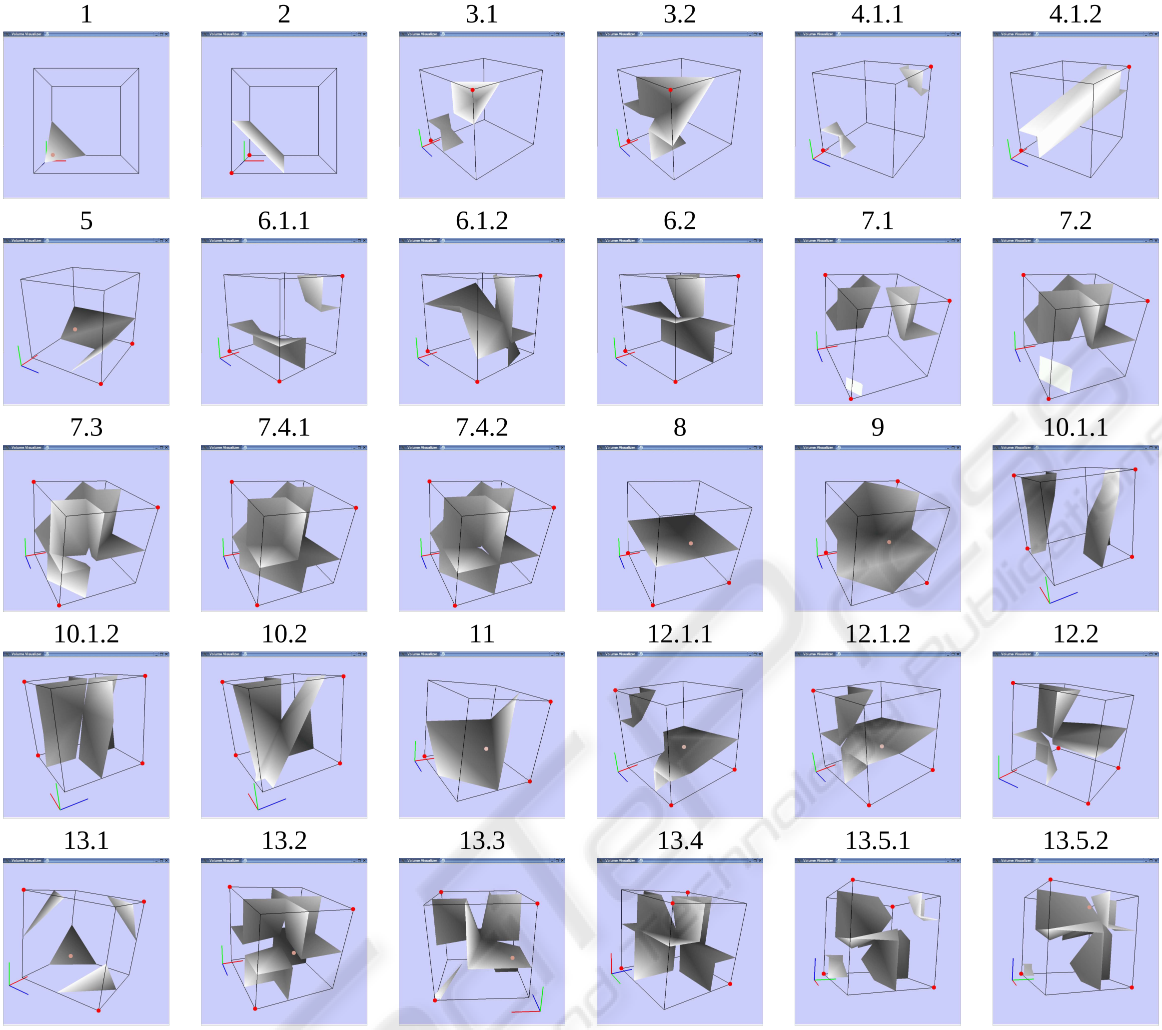


Figure 7: Isosurface topologies for a cell.

Chan and Purisima (Chan and Purisima, 1998) define a different tessellation (see figure 9) by defining tetraedra between adjacent cells, but the method does not produce topologies like that of 13.5.1 in figure 7.

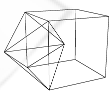


Figure 9: Chan's tessellation.

Recent studies have improved this method. Treece et al. (Treece et al., 1998) reduce the number or triangles by clustering tetrahedra vertices which preserves the topology, the triangles built are more regular, so the Gouraud shading is improved. Gerstner et al. (Gerstner and Pajarola, 2000) achieve a multiresolution tessellation by a recursive bisection of some tetrahedron in two. They take into account several critical points to preserve or not the topology in accordance with certain criteria. Both studies start from a tessellation that does not produce all the topologies of figure 7.

Chiang et al. (Chiang and Lu, 2003) propose a progressive simplification of tetrahedral meshes by using a contour tree, a data structure that represents the relations between connected components of the isosurfaces embedded in a volume dataset. Since the structure used includes critical points, the simplification preserves the topology. The authors focus on an irregular grid, however their method can be also applied to regular grids.

Our proposal produces a tessellation that preserves all the topologies of figure 7 and can be used as an initial tessellation to the work of Chiang.

4 OUR PROPOSAL

We propose to carry out an adaptive tessellation of every cell into tetrahedra in such a way that:

1. The tessellation must be isovalue independent in order to compute it just once, avoiding extra computations every time that the isovalue is changed.
2. The isosurface built inside the cell by joining the pieces of isosurfaces built from every tetrahedron of the tessellation has to be topologically equivalent to the isosurface built by using the extended marching cubes already commented in section 2 and in figure 7.
3. The isosurface has to move smoothly when the isovalue changes smoothly.

It will be done by taking into account the face and body saddle points.

We will use a basic tessellation which is valid for all non ambiguous cell configurations, that is to say, it is valid for those cells without face saddle points nor body saddle points. Then, this basic tessellation will be modified when face or body saddle point exist. These special points will be tetrahedra vertices and will be located at their exact position to improve the isosurface accuracy.

Basic case: 0 ambiguous faces and 0 body saddle points (Case 0-0)
The basic case corresponds to cells without ambiguous faces and without body saddle points. This kind of cell is tessellated as shown on figure 8-a. This tessellation produces 6 tetrahedra. Note that it is possible to build a tessellation which produces just 5 tetrahedra (see figure 8-b) but it is less homogeneous than the one proposed, because it implies the use of 2 possible symmetric tessellations depending on the position of the cell as can be deduced in figure 8-b. In figure 10 you can see an example of this case where the isovalue is smoothly changed.

Case 0-1
This case corresponds to cells without ambiguous faces and just 1 body saddle point. This kind of cell is tessellated as is shown in figure 11-a where the BSP is the central point. In order to see it clearly, one must look at figure 11-b where is shown the tessellation which corresponds to a face, the other 5 faces are tessellated in a similar way. As such, this tessellation produces 12 tetrahedra.

All the tetrahedra share the body saddle point, so the topology around this point is preserved (see figure 12). In this figure the isovalue changes smoothly and so, the isosurface is moved smoothly from one topology to another.

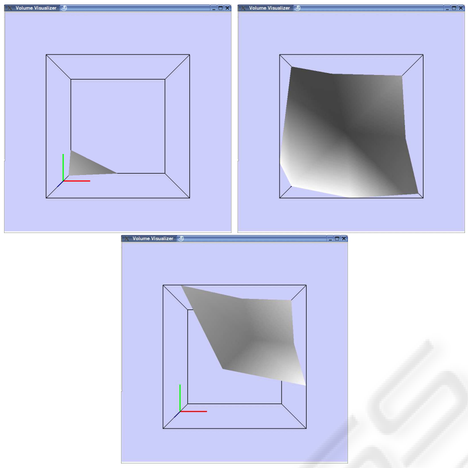
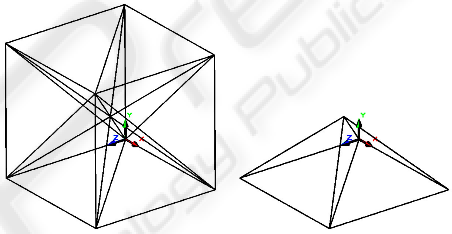


Figure 10: Isosurface of 1 component.



(a) (b)

Figure 11: Tessellation for the case 0-1.

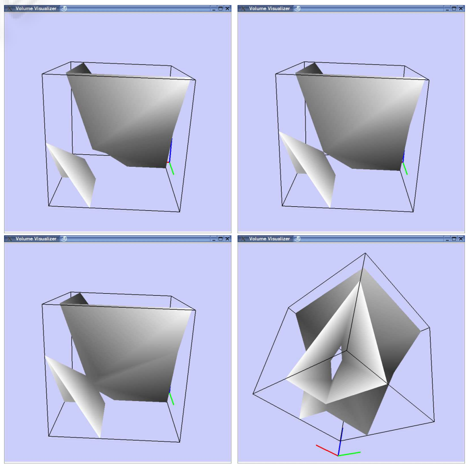
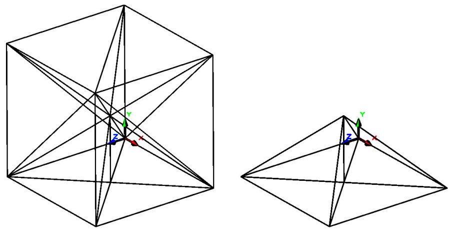


Figure 12: Isosurface in a cell within 1 body saddle point. The right bottom image is the same case than the left bottom one, only the point of view changes.

Case 1-0
In this case, there is just 1 ambiguous face and 0 body saddle points. In this case, we introduce an internal point at $(0.5, 0.5, 0.5)$ together with its property value. As such, the cell is tessellated as is shown in figure

13-a where the FSP is bottom face central point. The ambiguous face is tessellated as shown in figure 13-b and the non ambiguous faces are tessellated as in figure 11-b.



(a) (b)

Figure 13: Analisis for the case 1-0.

Cases x-0 and x-1

As we have already shown, cases x-0 are converted into cases x-1. That is to say, x ambiguous faces ($1 \leq x \leq 6$) and 1 internal point. These cases are tessellated as figure 13-b for ambiguous faces and as figure 11-b for unambiguous faces. This tessellation produces $12+2x$ tetrahedra.

In these cases the topology is preserved around the face and body saddle points as can be easily observed by the fact that all the tetrahedra of an ambiguous face share the face saddle point and all the tetrahedra from the tessellation share the body saddle point (see figures 14, 15 and 16).

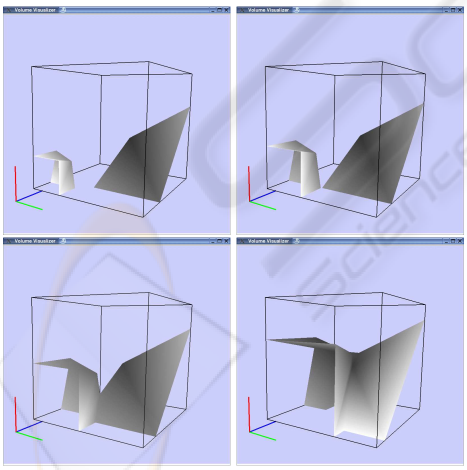


Figure 14: Isosurface on an ambiguous face.

Case 0-2

This represents a cell with no ambiguous faces and 2 body saddle points. These two body saddle points form an edge $\overline{B1B2}$. Using this edge and the 6 edges of the cuboid formed by the vertices V1, V5, V4, V6, V2, V3 (see figure 17-a), 6 tetrahedra are built (see

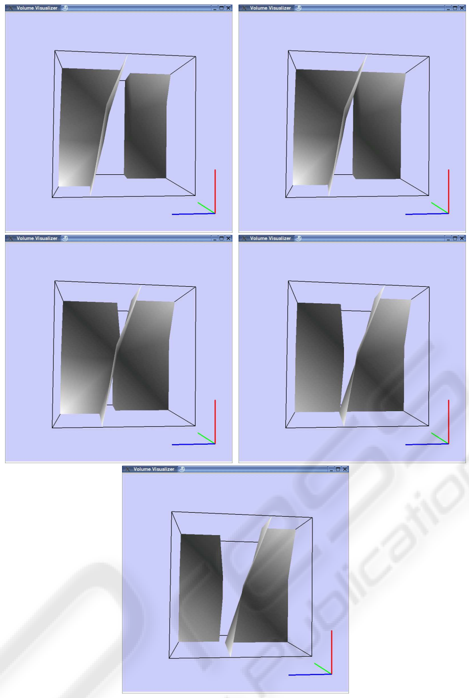


Figure 15: Isosurface on 2 opposite ambiguous faces.

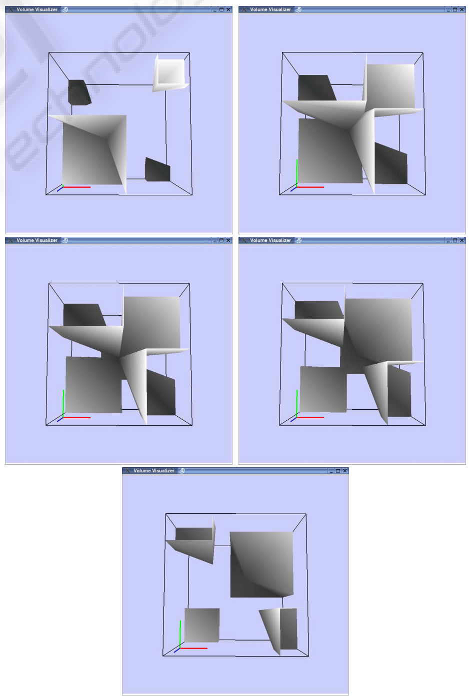
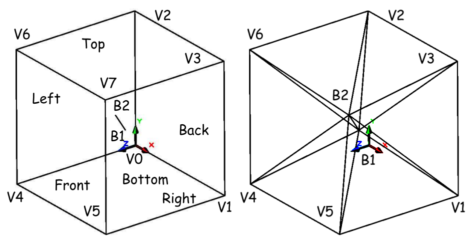


Figure 16: Isosurface in a cell with 6 ambiguous faces but no body saddle points.

figure 17-b). The back, left and bottom faces (which share V0) are tessellated as it was shown in figure 11-b but using B1 as the pyramid vertex (the body sad-

dle point nearest to the cell origin). The front, right and top faces (which share V7) are tessellated in the same way but using B2 as the pyramid vertex, the other body saddle point. This tessellation produces 18 tetrahedra.



(a) (b)

Figure 17: Analisis for the case 0-2.

Case x-2
The tessellation according to this case is carried out as the one shown for the case 0-2 but the ambiguous faces are tessellated as it was shown in figure 13-b instead of the tessellation shown in figure 11-b. This tessellation produces $18+2x$ tetrahedra. The topology is also preserved in theese cases as can be seen in figure 18.

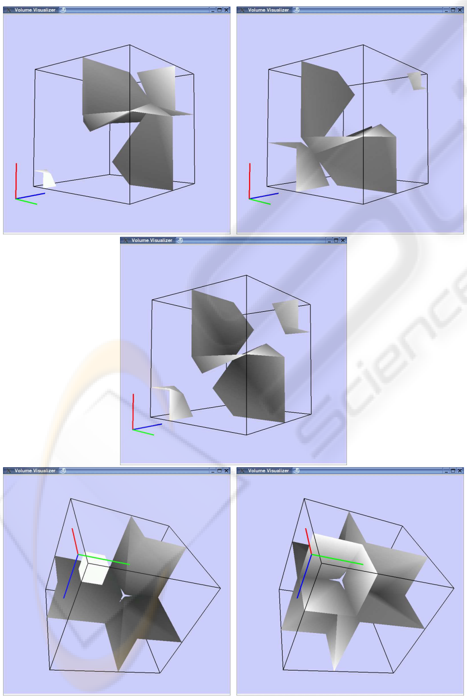


Figure 18: Isosurface in a cell with 2 body saddle points. The images at the 3rd line show the first and last cases of the 1st line from other point of view. The image of the 2nd line shows an intermediate case.

5 RESULTS AND CONCLUSIONS

The proposed method has been tested by using a cell with configurations for all the cases of improved marching cubes. In this section we show the most representative ones. Every example is shown using different isovalues on the basis of the same cell, where one can see how the isosurface changes smoothly when the isovalue is slightly changed.

Example 1 (figure 10): A configuration with 1 component on the basis of the simpliest tessellation.

Example 2 (figure 12): By modifying the isovalue from 26 to 23 we can see how an isosurface of 2 components changes to an isosurface of 1 component (with a tunnel between them) around the body saddle point which is inside this ambiguous cell.

Example 3 (figure 14): By changing the isovalue from 40 to 26 we can see how the isosurface changes smoothly from 2 components to 1 component through the face saddle point on the bottom face.

Example 4 (figure 15): By modifying the isovalue from 36 to 50 the isosurface of 2 components on two opposite ambiguous faces changes continuously to an isosurface of 1 component and again, to an isosurface of 2 components.

Example 5 (figure 16): This example shows a cell with all its faces ambiguous but without body saddle points. The isovalue changes from 43 to 63.

Example 6 (figure 18): This example shows a cell with 2 body saddle points, by changing the isovalue from 42 to 58 we can see how the isosurface forms a tunnel around each body saddle point depending on the particular isovalue.

The vertex values, in addition to the number of tetrahedra, of every example, are shown in table 1.

Table 1: Example's vertex values and number of tetrahedra (last row).

V.	Ex.1	Ex.2	Ex.3	Ex.4	Ex.5	Ex.6
V_0	0	35	50	100	78	33
V_1	10	12	14	71	33	100
V_2	20	0	14	14	33	100
V_3	30	38	14	14	100	0
V_4	40	100	0	14	0	100
V_5	50	15	14	0	78	0
V_6	60	15	100	71	78	0
V_7	100	15	14	71	33	67
	6	14	14	16	24	30

The method produces all Lopes' topologies correctly; in fact, all the topologies in figure 7 have been achieved by using our method.

Our method produces similar results to Lopes (Lopes and Brodlie, 2003). However, every time the isovalue changes, Lopes' method has to compute the new cell configuration (from a total of 31 distinct configurations), the new possible bishoulder points, and the new possible tangent points, and then the triangle configuration is formed and rendered.

As can be seen in his work, the computation of bishoulder points needs to compute a minimum of 2 face shoulder points (or more if a more accurate approximation is needed) so several square roots need to be computed. The computation of tangent points also needs to compute square roots because three quadratic equations have to be solved even though they have the same discriminant. So, every time the isovalue changes, several costly computations have to be performed. That is, it is a time expensive method for interaction.

Our method also needs to compute square roots (just one per cell) to determine which tessellation must be carried out, however, the particular tessellation is computed just once for every cell because our tessellation is isovalue independent. Once the tessellation has been carried out, the volume is represented by a set of tetrahedra and it is visualized by the well known and fast marching tetrahedra.

The drawback of our method is that it is less accurate than Lopes' method, in the sense that all triangle vertices (on edges, on faces and in cell), which are computed by Lopes' method, lie on the real continuous trilinear isosurface inside the cell. Whereas the triangle vertices computed by our method do not all lie on the real continuous trilinear isosurface. The only ones which do lie here are the vertices on cell edges, the face saddle points and the body saddle points. However we think that it is a minor problem once the topology is preserved.

Both methods are topologically valid but our method needs less computations when the isovalue changes because of its isovalue independence. In this way it is faster with regards to the interaction. Moreover, our method also produces smooth changes in the isosurface when the isovalue changes smoothly.

For future work, we want to analyse the possibility of grouping the tetrahedra in order to reduce their global amount and to allow a multiresolution representation of the volume.

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