

# INTELLIGENT TOPOLOGY PRESERVING GEOMETRIC DEFORMABLE MODEL

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**Abstract:** Geometric deformable models (GDM) using the level sets method provide a very efficient framework for image segmentation. However, the segmentation results provided by these models are dependent on the contour initialization. Moreover, sometimes it is necessary to prevent the contours from splitting and merging in order to preserve topology. In this work, we propose a new method that can detect the correct boundary information of segmented objects while preserving topology when needed. We adapt the stopping function  $g$  in a way that allows us to control the contours' topology. By analyzing the region where the edges of the contours are close we decide if the contours should merge, split or remain the way they are. This new formulation maintains the advantages of standard (GDM). Moreover, the topology-preserving constraint is enforced efficiently therefore, the new algorithm is only slightly computationally slower over standard (GDM).

## 1 INTRODUCTION

The class of geometric deformable models (GDM) introduced in (Caselles et al., 1993; Caselles et al., 1997; Malladi et al., 1995) are deforming contours (curves and surfaces) represented implicitly as level sets of some higher dimensional scalar function. This level sets representation allows these models to have numerous advantages such as providing efficient computational schemes, automatically handling topology changes of the evolving contours and simple implementation. These numerous advantages can be used profitably to provide a very efficient framework for image segmentation, edge detection, shape modeling, and visual tracking. (GDM) level sets formulation can automatically handle topology changes and usually it is a desired property. However, topological flexibility is not always desired especially, when a particular object is sought and its number of components and the homology of each component is known. In past, there have been several postprocessing methods reported to correct the topology of a cortical segmentation that has the wrong topology (Shattuck and Leahy, 2000; B. Fischl and Dale, 2001; X. Han and Prince, 2001; X. Han and Prince, 2003; Alexandrov

and Santosa, 2005). In this and similar applications the topology flexibility of geometric deformable models is considered to be a liability rather than an advantage (X. Han and Prince, 2001). Although "snakes" introduced by (M. Kass and Terzopoulos, 1987) do preserve topology they do not give us the flexibility to change the topology if needed.

In this paper, we develop an intelligent topology-preserving GDM (TPGDM) that can guarantee that the final contour has exactly the same topology as the initial one but also it can let the contours merge or split when judged appropriate.

This paper is organized as follows. In Section 2, we briefly introduce the geometric deformable models. In Section 4, we explain the algorithm for contour initialization. In Section 5 we explain the new TPLSM. An experimental result is also presented. A brief conclusion is given in Section 7.

## 2 GEOMETRIC DEFORMABLE MODELS

Geometric models for active contours have brought tremendous impact to classical problems in imagery

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such as providing ways to devise efficient computational algorithms for automatic segmentation. This is achieved by using the level set methods, which allow handling automatic changes in topology while providing a framework for very fast numerical schemes. These models are based on the theory of curve evolution and geometric flows. The curve/surface is propagating (deforming) by an implicit velocity that contains two terms, one related to the regularity of the deforming shape and the other attracting it to the boundary. The model is given by a geometric flow (PDE), based on mean curvature motion, therefore it's completely intrinsic. When implemented using the level set based numerical algorithm, the model handles topology changes automatically. The geometric model proposed by Caselles *et al* (Caselles et al., 1993) is based on the mean curvature motion equation which describes the propagation of the level set function following the normal direction with speed depending on the mean curvature. Let  $u$  be a level set function  $u : \mathbb{R}^2 \times [0, +\infty) \rightarrow \mathbb{R}$  and curve  $C$  is a level set of  $u$ , such that  $C = \{x \in \mathbb{R}^2 : u(x, t) = r\}, r \in \mathbb{R}$ . The geometric model is defined as follows:

$$\frac{\partial u}{\partial t} = |\nabla u| \left( \text{div}(g(I) \frac{\nabla u}{|\nabla u|}) \right) \quad (1)$$

$$u(x, 0) = u_0(x) \quad (2)$$

where  $u_0$  is the initialized curve. A similar formulation called the geodesic model gives:

$$\frac{\partial u}{\partial t} = g(I)(c + k) |\nabla u| + \nabla g \cdot \nabla u \quad (3)$$

where  $g(I)$  is the stopping function,

$$g(I) = \frac{1}{1 + |\nabla \hat{I}|^2}$$

which will stop the propagation when the evolving front reaches the desired position, the boundary detected.  $\hat{I}$  is a convolved image that ensures the motion of  $C$  is less affected by the noise in the image.  $k$  is the mean curvature. For the added constant term  $c$ , we can think  $cg(I) |\nabla u|$  as an extra speed in the geodesic problem to increase the speed of the convergence. The gradient term  $|\nabla u|$  controls what happens at the interior and exterior of the interface.  $\nabla g \cdot \nabla u$  denotes the projection of an attractive force vector on the normal to the moving interface. This term allows to accurately track boundaries with high variation in their gradient, including boundaries with small gaps. There are many algorithms for numerical implementation of GDM using level sets. Narrow band method and fast marching method are two simple, computationally fast and widely used algorithms. Instead of computing the evolution of all the level sets, which

means all the grid points, narrow band method just updates a small set of points in the neighborhood of the zero level set for each iteration.

However, the results of this model depend on the position of the initialized curve/surface. Different initial positions may lead to totally different result contours. We will discuss in detail and show some examples in section 6.

### 3 THE AVERAGE SQUARED GRADIENT

One of the measures for locally characterizing the image used in (Förstner, 1994) is the average squared gradient defined as follows: with the gradient  $\nabla g = (g_x, g_y)^T$  we obtain the squared gradient  $\Gamma g$  as dyadic product

$$\Gamma g = \nabla g \nabla g^T \begin{bmatrix} g_x^2 & g_x g_y \\ g_y g_x & g_y^2 \end{bmatrix} \quad (4)$$

The Gaussian function with standard deviation  $\sigma$  is denoted by  $G_\sigma(x, y) = G_\sigma(x) * G_\sigma(y)$ . This yields the average squared gradient image

$$\Gamma_{\sigma g}(x, y) = G_\sigma * \Gamma g = \int \int \Gamma g(u, v) G_\sigma(x - u, y - v) dx dy. \quad (5)$$

The three elements of  $\Gamma_{\sigma g}(x, y)$  can be derived by three convolutions.

$$\Gamma_{\sigma g}(x, y) = \begin{bmatrix} G_\sigma * g_{xx} & G_\sigma * g_{xy} \\ G_\sigma * g_{yx} & G_\sigma * g_{yy} \end{bmatrix}$$

$\Gamma_{\sigma g}(x, y)$  can be diagonalised by rotation of the coordinate axes and it gives  $\overline{\Gamma_{\sigma g}} = T \Lambda_g T^T = \lambda_1(g) t_1 t_1^T + \lambda_2(g) t_2 t_2^T$ .

First, the trace  $h = \text{tr} \overline{\Gamma_{\sigma g}} = \lambda_1(g) + \lambda_2(g) = \|\nabla g\|^2 = \sigma_{g_x}^2 + \sigma_{g_y}^2$  gives the total energy of the image function or edge busyness at  $(x, y)$ . We can use  $h = \text{tr} \overline{\Gamma_{\sigma g}}$  for measuring the homogeneity of segment-type features. Second, the ration  $\nu = \frac{\lambda_1}{\lambda_2}$  of the eigenvalues gives us the information about the orientation or isotropy. For example, if  $\lambda_2 = 0$ , we have anisotropic texture of straight general edges with arbitrary cross-section. Third, the largest eigenvalue can give us an estimate for the local gradient of the texture or the edge. Due to the squaring, the phase information is lost (Kass and Witkin, 1987) but, the variance of the orientation is proportional to  $\frac{\lambda_2}{(\lambda_1 - \lambda_2)}$ , giving us an additional interpretation and showing that if  $\lambda_1 \approx \lambda_2$  then the variance of orientation is large.

Therefore, if  $\lambda_1$  and  $\lambda_2$  are eigenvalues of  $\overline{\Gamma g}$  and  $\lambda_1 \geq \lambda_2$  then: (1) if  $\lambda_1$  is large compared to  $\lambda_2$ , the local neighborhood possesses a dominant orientation, (2) if







