AN INVESTIGATION OF EXTENDED KALMAN FILTERING IN THE ERRORS-IN-VARIABLES FRAMEWORK
A Joint State and Parameter Estimation Approach

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Abstract: The paper addresses the problem of errors-in-variables filtering, i.e. the optimal estimation of inputs and outputs from noisy observations. While the optimal solution is known for linear time-varying systems of known parameterisation, this paper considers a suboptimal approach where only an approximated set of parameters is available. The proposed filter is derived by the means of joint state and parameter estimation using the extended Kalman filter approach which, in turn, leads to a coupled state-parameter estimation procedure. However, the resulting parameter estimates appear to be biased in the presence of input noise. The novel filter is compared with a previously proposed suboptimal filter.

1 INTRODUCTION

Kalman filtering (Anderson and Moore, 1979) deals with the optimal estimation of states and outputs in the presence of process and output noise. If an errors-in-variables (EIV) framework is adopted, i.e. the inputs are also affected by measurement noise, Kalman filtering cannot directly be applied (Guidorzi et al., 2003). The EIV filtering problem, which deals with the optimal estimation of noise free input and output signals, has been solved in (Guidorzi et al., 2003) and (Markovsky and De Moor, 2005). A unified framework for both, Kalman filtering and EIV filtering has been presented in (Diversi et al., 2005), where the EIV filtering problem is solved by the means of a standard Kalman filter (Kf) applied to a reformulated model. An EIV extended Kalman filter (EIVeKf), which is able to accommodate for model mismatch, in the case where the true system generating the data is unknown, has been presented in (Vinsonneau et al., 2005).

In this paper, the theory of extended Kalman filtering for joint state and parameter estimation (Ljung, 1979) is applied to the reformulated EIV model used in (Diversi et al., 2005). This leads to an algorithm which is shown to be similar to the EIVeKf. The differences and similarities between both approaches are discussed. Essentially, the filters calculate an estimate of the parameters of an assumed model and use this linear time-varying (LTV) model for filtering. It is revealed that these estimates are biased in the presence of input noise.

Section 2 reviews the extended Kalman filter for joint state and parameter estimation, while the existing EIV filtering techniques are summarised in Section 3. The modified algorithm for joint state and parameter estimation in the case of EIV, which is considered to be novel, is presented in Section 4, and an illustrative simulation example is given in Section 5. In Section 6, both EIV extended Kalman filters are compared and the results obtained from simulation are critically appraised . Finally, concluding remarks are given in Section 7.

2 EKF FOR JOINT STATE AND PARAMETER ESTIMATION

Assuming the data is generated by a linear time-invariant (LTI) discrete-time state-space system, its corresponding model may be given by

$$x_{k+1} = A(\theta)x_k + B(\theta)u_k + v_k \quad (1)$$

$$y_k = C(\theta)x_k + D(\theta)u_k + e_k \quad (2)$$

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where x_k denotes the state, u_k the input, y_k the output, v_k process noise, e_k measurement noise and the model matrices $A(\theta)$, $B(\theta)$, $C(\theta)$ and $D(\theta)$ of appropriate dimension are characterised by the parameter vector θ. The noise sequences $\{v_k\}$ and $\{e_k\}$ are assumed to be independent with zero mean and covariance matrices

$$\Sigma_v = E\left[v_k v_{k-\tau}^T\right]\delta(\tau) \tag{3}$$

$$\Sigma_e = E\left[e_k e_{k-\tau}^T\right]\delta(\tau) \tag{4}$$

$$\Sigma_{ve} = E\left[v_k e_{k-\tau}^T\right]\delta(\tau) \tag{5}$$

where $\delta(\tau)$ denotes the Kronecker delta function. Based on an extended Kalman filter (eKf) (Anderson and Moore, 1979) an adaptive estimator for the model parameters can be derived by extending the state with the time dependent parameter vector θ_k, which leads to the following nonlinear state equation

$$\begin{bmatrix} x_{k+1} \\ \theta_{k+1} \end{bmatrix} = \begin{bmatrix} A(\theta)x_k + B(\theta)u_k \\ \theta_k \end{bmatrix} + \begin{bmatrix} v_k \\ d_k \end{bmatrix} \tag{6}$$

The noise term d_k with covariance matrix

$$\Sigma_d = E\left[d_k d_{k-\tau}^T\right]\delta(\tau) \tag{7}$$

allows for variations in the system parameters and is usually set to zero if time-invariance is assumed.

Defining for convenience

$$A_k = A(\hat{\theta}_k) \qquad B_k = B(\hat{\theta}_k)$$
$$C_k = C(\hat{\theta}_k) \qquad D_k = D(\hat{\theta}_k) \tag{8}$$

the eKf for joint state and parameter estimation (jeKf) is given by (Ljung, 1979)

$$\hat{x}_{k+1} = A_k\hat{x}_k + B_k u_k + K_k\left[y_k - C_k\hat{x}_k - D_k u_k\right] \tag{9}$$

$$\hat{\theta}_{k+1} = \hat{\theta}_k + L_k\left[y_k - C_k\hat{x}_k - D_k u_k\right] \tag{10}$$

where

$$K_k = [A_k P_{1_k} C_k^T + F_k P_{2_k}^T C_k^T + A_k P_{2_k} H_k^T + F_k P_{3_k} H_k^T + \Sigma_{ve}]S_k^{-1} \tag{11}$$

$$S_k = C_k P_{1_k} C_k^T + C_k P_{2_k} E_k^T + H_k P_{2_k}^T C_k^T + H_k P_{3_k} H_k^T + \Sigma_e \tag{12}$$

$$L_k = \left[P_{2_k}^T C_k^T + P_{3_k} H_k^T\right] S_k^{-1} \tag{13}$$

$$P_{1_{k+1}} = A_k P_{1_k} A_k^T + A_k P_{2_k} F_k^T + F_k P_{2_k}^T A_K^T + F_k P_{3_k} F_k^T - K_k S_k K_k^T + \Sigma_v \tag{14}$$

$$P_{2_{k+1}} = A_k P_{2_k} + F_k P_{3_k} - K_k S_k L_k^T \tag{15}$$

$$P_{3_{k+1}} = P_{3_k} - L_k S_k L_k^T + \Sigma_d \tag{16}$$

with the Jacobians being defined by

$$F_k = \frac{\partial}{\partial\theta}\left(A(\theta)\hat{x}_k + B(\theta)u_k\right)\big|_{\theta=\hat{\theta}_k} \tag{17}$$

$$H_k = \frac{\partial}{\partial\theta}\left(C(\theta)\hat{x}_k + D(\theta)u_k\right)\big|_{\theta=\hat{\theta}_k} \tag{18}$$

It is shown in (Ljung, 1979) that the above recursive parameter estimator can be interpreted as an attempt to minimise the expected value of squared residuals associated with a constant model θ, i.e. minimising the cost function

$$V(\theta) = E\left|\bar{\varepsilon}_k(\theta)\right|^2 \tag{19}$$

where $\bar{\varepsilon}_k(\theta)$ is the innovation. Hence, this estimator is closely related to a recursive prediction error method (Ljung, 1999). A convergence analysis of this parameter estimator for linear systems is also carried out in (Ljung, 1979) and it is shown that it can produce biased estimates or even diverge. However, the above procedure can be modified to become a stochastic descent-algorithm which is globally convergent by including an approximation of

$$\left[\frac{\partial}{\partial\theta}\bar{K}(\theta)\right]\bar{\varepsilon}_k \tag{20}$$

into the Jacobian F_k (referred to as the coupling term (Ljung, 1979)), where $\bar{K}(\theta)$ is the steady-state Kalman gain. One way to ensure this is to assume an innovation model structure

$$x_{k+1} = A(\theta)x_k + B(\theta)u_k + K(\theta)\varepsilon_k \tag{21}$$

$$y_k = C(\theta)x_k + D(\theta)u_k + \varepsilon_k \tag{22}$$

rather than (1)-(2) and include all elements of the Kalman gain K into the parameter vector θ. Parametrising K and Σ_ε explicitly leads to a modified algorithm given by

$$\hat{x}_{k+1} = A_k\hat{x}_k + B_k u_k + K_k\varepsilon_k \tag{23}$$

$$\hat{\theta}_{k+1} = \hat{\theta}_k + L_k\varepsilon_k \tag{24}$$

where

$$\varepsilon_k = y_k - C_k\hat{x}_k - D_k u_k \tag{25}$$

$$L_k = \left[P_{2_k}^T C_k^T + P_{3_k} H_k^T\right]\Sigma_{\varepsilon_k}^{-1} \tag{26}$$

$$+ F_k P_{3_k} F_k^T - K_k \Sigma_{\varepsilon_k} K_k^T + \Sigma_v \tag{27}$$

$$P_{2_{k+1}} = A_k P_{2_k} + F_k P_{3_k} - K_k \Sigma_{\varepsilon_k} L_k^T \tag{28}$$

$$P_{3_{k+1}} = P_{3_k} - L_k \Sigma_{\varepsilon_k} L_k^T + \Sigma_d \tag{29}$$

$$\Sigma_{\varepsilon_k} = \Sigma_{\varepsilon_{k-1}} + \frac{1}{k}\left(\varepsilon_k\varepsilon_k^T - \Sigma_{\varepsilon_{k-1}}\right) \tag{30}$$

and

$$F_k = \frac{\partial}{\partial\theta}\left(A(\theta)\hat{x}_k + B(\theta)u_k + K(\theta)\varepsilon_k\right)\big|_{\theta=\hat{\theta}_k} \tag{31}$$

$$K_k = K(\hat{\theta}_k) \tag{32}$$

Moreover, a projection facility has to be utilised to ensure that $\hat{\theta}_k$ lies in the compact subset

$$D_s = \{\theta | A(\theta) - K(\theta)C(\theta) \textit{ is exponentially stable.}\} \tag{33}$$

In practice, a step-size reduction might also be necessary to achieve convergence.

3 KALMAN AND EIV FILTERING

Whereas traditional Kalman filtering (Anderson and Moore, 1979) addresses the problem of estimating the optimal states and outputs in the case of process and output noise, EIV filtering deals with the optimal estimation of inputs and outputs, where both quantities are considered to be observations, which are affected by additive noise.

The EIV filtering problem for the LTI case has been solved in (Guidorzi et al., 2003), where the optimal input and output estimates are determined based on the state-space model

$$\xi_{k+1} = \mathcal{A}\xi_k + \mathcal{B}\begin{bmatrix} y_k^T & u_k^T \end{bmatrix}^T \qquad (34)$$

$$\gamma_k = \mathcal{C}\xi_k + \mathcal{D}\begin{bmatrix} y_k^T & u_k^T \end{bmatrix}^T \qquad (35)$$

where ξ_k denotes the state, $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, $\mathcal{D}$ the model matrices and γ_k the residuals. A different approach has been presented in (Markovsky and De Moor, 2005), where the EIV state-space representation is reformulated, such that the new state-space model depends on the measured quantities u_k and y_k with redefined process and measurement noise terms. Subsequently, a modified Kalman filter can be applied to obtain an estimate of the system states, which, in turn, allows the estimation of the true input and output signals. Utilising the latter reformulation of the EIV state-space system, a unified context for both traditional Kalman filtering and EIV filtering has been proposed (Diversi et al., 2005) and this is outlined in the following Subsection.

3.1 Unified Framework for Kalman and EIV Filtering

Consider the discrete-time LTI EIV state-space model given by

$$x_{k+1} = Ax_k + Bu_{0_k} + w_k \qquad (36)$$

$$y_{0_k} = Cx_k + Du_{0_k} \qquad (37)$$

$$u_k = u_{0_k} + \tilde{u}_k \qquad (38)$$

$$y_k = y_{0_k} + \tilde{y}_k \qquad (39)$$

where x_k denotes the state, u_{0_k} and y_{0_k} the unknown inputs and outputs, u_k and y_k the noisy measurements. The noise terms $\tilde{u}_k$, $\tilde{y}_k$ and w_k denote input, output and process noise, respectively, which are assumed to be of zero mean and with covariance matrices

$$E\left[w_k w_{k-\tau}^T\right] = \Sigma_w \delta(\tau) \qquad (40)$$

$$E\left[\tilde{u}_k \tilde{u}_{k-\tau}^T\right] = \Sigma_{\tilde{u}} \delta(\tau) \qquad (41)$$

$$E\left[\tilde{y}_k \tilde{y}_{k-\tau}^T\right] = \Sigma_{\tilde{y}} \delta(\tau) \qquad (42)$$

$$E\left[\tilde{u}_k \tilde{y}_{k-\tau}^T\right] = \Sigma_{\tilde{u}\tilde{y}} \delta(\tau) \qquad (43)$$

$$E\left[w_k \tilde{u}_{k-\tau}^T\right] = 0 \qquad (44)$$

$$E\left[w_k \tilde{y}_{k-\tau}^T\right] = 0 \qquad (45)$$

The model equations (36)-(39) can be rewritten as

$$x_{k+1} = Ax_k + Bu_k + v_k \qquad (46)$$

$$z_k = Cx_k + e_k \qquad (47)$$

where z_k, v_k and e_k are the redefined measurements, process noise and measurement noise, respectively, which are given by

$$z_k = y_k - Du_k \qquad (48)$$

and

$$v_k = w_k - B\tilde{u}_k \qquad (49)$$

$$e_k = \tilde{y}_k - D\tilde{u}_k \qquad (50)$$

The covariance matrices are readily obtained via

$$\Sigma_v = \Sigma_w + B\Sigma_{\tilde{u}}B^T \qquad (51)$$

$$\Sigma_e = \Sigma_{\tilde{y}} - \Sigma_{\tilde{u}\tilde{y}}^T D^T - D\Sigma_{\tilde{u}\tilde{y}} + D\Sigma_{\tilde{u}}D^T \qquad (52)$$

$$\Sigma_{ve} = B\left[\Sigma_{\tilde{u}}D^T - \Sigma_{\tilde{u}\tilde{y}}\right] \qquad (53)$$

A standard Kalman filter is then utilised to determine the optimal state estimate

$$\hat{x}_{k+1|k} = Ax_{k|k-1} + Bu_k + K_k\varepsilon_k \qquad (54)$$

$$K_k = \left[AP_{k|k-1}C^T + \Sigma_{ve}\right]\Sigma_\varepsilon^{-1} \qquad (55)$$

$$P_{k+1|k} = AP_{k|k-1}A^T + \Sigma_v - \left[AP_{k|k-1}C^T + \Sigma_{ve}\right] \times \Sigma_\varepsilon^{1-}\left[AP_{k|k-1}C^T + \Sigma_{ve}\right]^T \qquad (56)$$

where

$$\varepsilon_k = z_k - C\hat{x}_{k|k-1} = C\left(x_k - \hat{x}_{k|k-1}\right) + e_k \qquad (57)$$

$$\Sigma_\varepsilon = E\left[\varepsilon_k\varepsilon_k^T\right] = CP_{k|k-1}C^T + \Sigma_e \qquad (58)$$

are the innovations and its corresponding covariance matrix. The filtered inputs and outputs are then given by (Diversi et al., 2005)

$$\hat{u}_{0_k} = u_k - E\left[\tilde{u}_k|z_k\right] = u_k - \left[\Sigma_{\tilde{u}\tilde{y}} - \Sigma_{\tilde{u}}D^T\right]\Sigma_\varepsilon^{-1}\varepsilon_k \qquad (59)$$

$$\hat{y}_{0_k} = y_k - E\left[\tilde{y}_k|z_k\right] = y_k - \left[\Sigma_{\tilde{y}} - \Sigma_{\tilde{u}\tilde{y}}^T D^T\right]\Sigma_\varepsilon^{-1}\varepsilon_k \qquad (60)$$

Hence, a traditional Kalman filter can be utilised to achieve both, the optimal estimation of states and input/output sequences.

3.2 Extended EIV Kalman Filtering

A drawback of the linear filter described in Subsection 3.1, is that it relies on exact information of the noise characteristics and an exact model of the process generating the data. In an attempt to compensate for the latter requirement, an extended EIV Kalman filter (EIVeKf), based on the EIV Kalman filter given in (Guidorzi et al., 2003), has been proposed in (Vinsonneau et al., 2005). Instead of an exact process representation, the EIVeKf requires only an approximate parametrisation of a linear default model, characterised by θ_d, to achieve acceptable results. Under certain conditions, use of the EIVeKf can lead to a superior filter performance with respect to the linear counterpart in cases where the system parametrisation is only approximately known. Moreover, the EIVeKf is also able to accommodate, to a certain degree, the case of LTV systems.

The idea of the EIVeKf is very similar to the jeKf approach; the state vector is augmented with the compensating parameters θ_c such that the new state vector becomes

$$\begin{bmatrix} \xi_k \\ \theta_{c_k} \end{bmatrix} \tag{61}$$

where ξ_k is the original state vector in (34) for the calculation of the residual sequence $\{\gamma_k\}$. The resulting system equations are thus nonlinear and the EIVKf filter can be modified using first order Taylor approximations for the predicting step, which results to the EIVeKf equations.

4 EIV EXTENDED KALMAN FILTER FOR JOINT PARAMETER ESTIMATION

Since the EIV filtering problem can be solved by the means of a standard Kalman filter, as outlined in Section 3.1, one could apply well known modifications of traditional Kalman filtering techniques to estimate u_{0_k} and y_{0_k}. The approach proposed here is to apply the idea of joint state and parameter estimation, as summarised in Section 2, to EIV systems. This is expected lead to a similar filter as the one presented in (Vinsonneau et al., 2005) with the difference that the estimate $\hat{\theta}_k$ is not only used for the prediction step, but in the overall filter equations. In addition, the changes in the parameters can be tracked by the means of Σ_d defined in (7).

4.1 Algorithm

Assuming the data is generated by a Eiv system of structure (36)-(39) and the assumed model structure is given by

$$x_{k+1} = Ax_k + Bu_k + v_k \tag{62}$$
$$y_k = Cx_k + Du_k + e_k \tag{63}$$

with v_k and e_k as defined in (49)-(53). Then the EIV extended Kalman filter for joint parameter estimation (EIVjeKf) is readily given by (23)-(30) together with the estimated inputs and outputs as defined in (59) and (60), whereas A, B, C, D are replaced by A_k, B_k, C_k, D_k, respectively.

However, it it found in simulations, that this form of the EIVjeKf can suffer from outliers in terms of overall EIV filter performance as illustrated in Section 5. Therefore, a slight different formulation will be preferred in the subsequent: while the Kalman gain is still to be estimated, in order to assure the existence of the terms $\left[\frac{\partial}{\partial\theta}K(\theta)\right]\big|_{\theta=\hat{\theta}_k}\bar{\varepsilon}_k$ within F_k, these estimates are not further utilised in the algorithm but rather K_k as given by (11)-(16). For clarification, the algorithm is summarised as follows.

Algorithm 4.1 *Assuming an EIV system of the form* (36)-(39) *and the model structure of* (62)-(63) *with noise characteristics* (49)-(53), *the EIVjeKf is given by*

1. *Augment the state vector x_k with the model parameters θ_k and Kalman gain K_k*
2. *Determine the time-varying model matrices A_k, B_k, C_k and D_k as given in* (8)
3. *Compute the innovation given by* (25) *and its covariance matrix as defined in* (30)
4. *Determine the Jacobians F_k and H_k as given in* (31) *and* (18), *respectively*
5. *Determine the reformulated covariance matrices* (51)-(53) *with B and D replaced by B_k and D_k*
6. *Compute $\hat{x}_{k+1}$ and $\hat{\theta}_{k+1}$ using* (9)-(16)
7. *Determine $\hat{u}_{0_k}$ and $\hat{u}_{0_k}$ given by* (59) *and* (60), *where D is replaced with D_k*
8. *Increment k and continue with step 2*

Remark 4.1 *As outlined in Section 2, the parameter estimator resulting from the EIVjeKf can be interpreted as an recursive prediction error method with the correction inspired by the eKf algorithm. However, it is known, that the application of standard prediction error methods to EIV systems does not yield consistent estimates as demonstrated in (Söderström, 1981). Therefore, the estimated $\hat{\theta}_m$ is expected to be biased with respect to the true parametrisation.*

5 SIMULATION

Consider the single-input single-output LTV state-space system given by (36)-(39) with

$$A = \begin{bmatrix} 0 & 0.1 \\ -0.2 & 0.3 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$
$$C = \begin{bmatrix} 0.9 & b_k - 1.35 \end{bmatrix} \qquad D = -4.5 \tag{64}$$

and

$$\Sigma_{\tilde{u}} = 0.2 \quad \Sigma_{\tilde{y}} = 5 \quad \Sigma_{\tilde{u}\tilde{y}} = 0.8 \quad \Sigma_w = 0 \tag{65}$$

where the time-varying parameter b_k with mean value $E[b_k] = 4.7$ slowly varies as illustrated in Figure 1. The input u_{0_k} is a zero mean white noise process with unity variance, the signal-to-noise ratios (SNR) are given by

$$\mathrm{SNR}_u = 10 \log \left(\frac{\mathrm{var}(u_0)}{\mathrm{var}(\tilde{u})} \right) = 16.0 \tag{66}$$

$$\mathrm{SNR}_y = 10 \log \left(\frac{\mathrm{var}(y_0)}{\mathrm{var}(\tilde{y})} \right) = 19.7 \tag{67}$$

whereas u_0, $\tilde{u}$, y_0, $\tilde{y}$ without index denote the sequences to the corresponding signals, i.e. $\tilde{u} = \{\tilde{u}_k\}_{k=1}^{N}$ and so forth. The number of samples is set to $N = 5000$. While the covariance matrices (65) are assumed to be known, the system parametrisation is approximated by the default parameter

$$\theta_d = \begin{bmatrix} -0.5 & 0.3 & -3.1 & 4.1 \end{bmatrix}^T \tag{68}$$

while θ_k is given by

$$\theta_k = \begin{bmatrix} -0.3 & 0.2 & -4.5 & b_k \end{bmatrix}^T \tag{69}$$

In order to model the variation in the system parameters, the covariance matrix (7) corresponding to θ_k is chosen to be

$$\Sigma_d = \begin{bmatrix} 0_{3\times 3} & 0_{3\times 1} \\ 0_{1\times 3} & 1 \cdot 10^{-3} \end{bmatrix} \tag{70}$$

where $0_{m\times n}$ denotes the $m \times n$ zero matrix. The performance index of interest is the EIV filter performance, i.e ‘how much’ noise can be removed from the noisy observations u_k and y_k. This can be quantified by

$$P_u = 100 \frac{||u_0 - u||_2 - ||u_0 - \hat{u}_0||_2}{||u_0 - u||_2} \tag{71}$$

$$P_y = 100 \frac{||y_0 - y||_2 - ||y_0 - \hat{y}_0||_2}{||y_0 - y||_2} \tag{72}$$

giving a relative measure for the removed noise in percentage, i.e. a value of 100 indicates a perfect filtering (estimate and true signal are identical), while a value of 0 corresponds to no filtering performance (estimate and noisy signal are identical). All simulations are verified by the means of 100 Monte-Carlo runs.

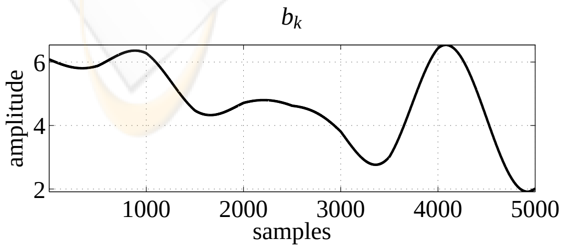

Figure 1: Time-varying parameter b_k.

5.1 Results

The three filters are applied to the above simulation, that is, the Kf presented in Section 3.1, the EIVeKf of (Vinsonneau et al., 2005) and the new approach, the EIVjeKf discussed in the previous Section. The mean and variances of P_u and P_y for the different Monte-Carlo runs are summarised in Table 1. It is observed that the Kf exhibits the worst EIV filter performance by removing only around 25% and 26% of input and output noise, respectively, while the best performance is achieved applying the EIVjeKf, which removes on average approximately 43% and 50% of the noise contamination. In contrast, the variances of the performance indices with respect to the Monte-Carlo simulation are smaller in the case of the Kf. The results of the EIVeKf lie in between of the other two filters for both, mean and variance of the filter performance.

Table 1: Filter performance for the different filters.

	Kf	EIVeKf	EIVjeKf
$E[P_u]$	25.2	33.7	42.9
$E[P_y]$	26.1	38.5	50.0
$\mathrm{var}(P_u)$	0.9	1.4	2.1
$\mathrm{var}(P_y)$	0.8	0.8	2.5

6 DISCUSSION

Since only a default time-invariant model (and not the true system parametrisation) is available to the Kf, a negative impact on the filter performance is not surprising. If the true LTV system parametrisation, and hence, the true time-varying covariance matrices (51)-(53) are known, the Kf would be optimal and may well outperform the eKf approaches considered here. In fact, the only reason that the nonlinear approaches yield superior performance is that they attempt to compensate for the parameter-mismatch by estimating the the model parameters, which are then used for filtering (at least in-part).

The different performance results of the EIVeKf and the EIVjeKf can be explained by regarding the estimate of b_k, which is shown in Figure 2. Since the EIVjeKf models the variations of b_k by means of Σ_d, it is able, to a certain degree, to track b_k, while the EIVeKf uses no adaptivity to estimate θ_k. However, it would be a straightforward step to include Σ_d into the EIVeKf algorithm. In such a case, both estimates become nearly identical and the corresponding filter performance ($E[P_u] = 41.8$ and $E[P_y] = 48.6$) is very similar to the results of the EIVjeKf (cf. Table 1). The remaining differences may be explained by the fact that while the prediction phase of the EIVeKf utilises the estimates of θ_k, the default parameter set θ_d is still used for the correction.

Another point to be observed in Figure 2 is that the estimate for b_k produced by the EIVjeKf is biased. This was expected, as mentioned in Remark 4.1, since the parameter estimator resulting from the eKf approach is not adjusted for the EIV case. Hence, the EIV filter performance of the nonlinear approaches can deteriorate if the bias is large with respect to the model mismatch characterised by θ_d. In such a situation, the EIVeKf is expected to perform better than the EIVjeKf, since the latter relies completely on $\hat{\theta}_k$. This can be verified by modifying the above simulation and increasing the input noise to $\Sigma_{\tilde{u}} = 1$, which corresponds to $\text{SNR}_u = -0.1$. The filter performance[1] is given in Table 2 and the time-varying parameter b_k and its estimates are shown in Figure 3. It can be observed, that the estimate produced by the EIVjeKf becomes more biased as $\Sigma_{\tilde{u}}$ increases resulting in a decreased filter performance, while the estimate given by the EIVeKf is less biased, hence, by opposition, yielding a better filter performance.

[1]Note, that in this case, Σ_d is incorporated into the EIVeKf algorithm.

Table 2: Filter performance for the different filters for the case $\Sigma_{\tilde{u}} = 1$.

	Kf	EIVeKf	EIVjeKf
$E[P_u]$	39.5	36.1	16.7
$E[P_y]$	18.4	24.9	20.0
$\text{var}(P_u)$	0.8	0.7	3.7
$\text{var}(P_y)$	0.8	0.8	5.2

7 CONCLUSIONS

The solution of the EIV filtering problem as a special case of traditional Kalman filtering in extended noise environments (Diversi et al., 2005) has been reviewed. Since the optimal estimation of noise-free inputs and outputs can be achieved by applying the well known Kalman filter to a reformulated model, a joint state and parameter estimation procedure via extended Kalman filtering (Ljung, 1979) is investigated for the EIV case. The resulting algorithm is very similar to the approach presented in (Vinsonneau et al., 2005). In fact, these nonlinear EIV filter approaches attempt to estimate the model parameters by means of a recursive prediction error method. In turn, this means that these estimates are generally biased in the presence of input noise and this may be considered as the main limitation of these approaches. The difference between both nonlinear filters is that the EIVeKf in (Vinsonneau et al., 2005) uses the estimated model parameters only for the prediction phase of the filter, and whilst more investigation may be necessary, it appears to lead to more robustness if the SNR of the input is low.

Some potentially interesting further work would aim to investigate other suboptimal filters, again with

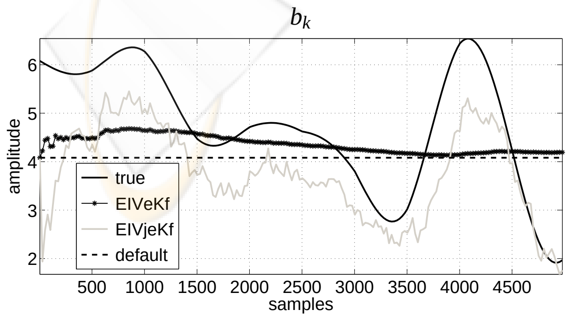

Figure 2: Time-varying parameter b_k, its estimates and the default value.

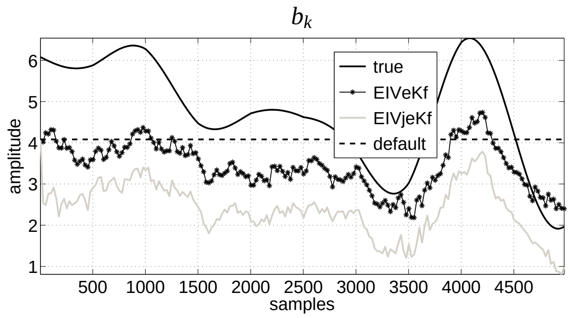

Figure 3: Time-varying parameter b_k, its estimates and the default value for the case $\Sigma_{\tilde{u}} = 1$.

coupled state and parameter estimation, but where the parameter set is obtained via a recursive EIV identification technique.

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