HIGHER ORDER SLIDING MODE STABILIZATION OF A CAR-LIKE MOBILE ROBOT

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Abstract: This paper deals with the robust stabilization of a car-like mobile robot given in a perturbed chained form. A higher order sliding mode control strategy is developed. This control strategy switches between two different sliding mode controls: a second order one (super-twisting algorithm) and a new third order sliding mode control that performs a finite time stabilization. The proposed third sliding mode controller is based on geometric homogeneity property with a discontinuous term. Simulation results show the control performance.

1 INTRODUCTION

In the recent years, the control of nonholonomic systems has received a considerable attention, and in particular the stabilization problem. Due to the peculiar nature of nonholonomic kinematics, the stabilization problem addressed in control design for wheeled mobile robots (WMR) is in general quite difficult. In fact, it is known that nonholonomic WMR with restricted mobility (such as unicycle-type and car-like vehicles) cannot be stabilized to a desired configuration (or posture) via differentiable, or even continuous, pure-state feedback control despite they are open loop controllable (Brockett, 1983). Several nonlinear control designs have been proposed to achieve the stabilization for such systems. Time-varying feedbacks (Samson, 1995) or (open loop) sinusoidal and polynomial controls (Murray and Sastry, 1993) can be developed. Other alternatives consist in using the backstepping recursive techniques (Jiang and Nijmeijer, 1999), (Huo and Ge, 2001), flatness (M. Fliess *et al.*, 1995), or discontinuous approaches (Astolfi, 1996), (Floquet *et al.*, 2003). The robustness property is an important aspect for stabilizing tasks of uncertain systems, especially when there exist disturbances or errors dynamics in the system. It is well known that the standard sliding mode features are high accuracy and robustness with respect to various internal and external disturbances. The basic idea is to force the state via a discontinuous feedback to move on a prescribed manifold called the sliding manifold. A specific drawback involved by sliding mode technique is the well known chattering effect (undesirable vibrations), which limits the practical relevance. To overcome this drawback, the Higher Order Sliding Mode (HOSM) approach has been proposed (Emel'yanov *et al.*, 1993). The main objective is to keep the sliding variable and a finite number of its successive time derivatives to zero through a discontinuous function acting on some high order time derivative of the sliding variable. This technique generalizes the basic sliding mode idea and can be implemented for systems with arbitrary relative degree. Keeping the main advantages of the standard sliding mode control, the chattering effect is avoided and finite time convergence together with higher order precision are provided. Actually, the problem of higher order sliding mode control is equivalent to the finite time stabilization of an integrator chain with nonlinear uncertainties. In (Floquet *et al.*, 2003), it is shown that the HOSM theory is efficient to design control laws which robustly stabilizes in finite time a chained form system. Second order sliding mode controllers were proposed to stabilize a three-dimensional system (unicycle type vehicle). It should be pointed out that, in the case of the four dimensional car-like robot system, the proposed procedure requires the finite time stabilization of a third order integrator chain. Thus, a

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third order sliding mode control is at least necessary and a new type of third order sliding mode algorithm is introduced in this paper.

The aim of this paper is to present a high order sliding mode control strategy for the robust stabilization problem of a car-like mobile robot. First, the perturbed one-chain form of the robot is derived. Then, second order sliding mode controllers based on the so-called super twisting algorithm and a third order sliding mode controller are developed. The latter controller is a combination of a finite time controller based on geometric homogeneity and a discontinuous term that ensures robustness properties. By switching between these sliding mode controllers, a finite time stabilization to the origin is obtained.

The organization of this paper is as follows: Section 2 presents the perturbed chained form model of the car-like vehicle and states the problem under interest. Section 3 deals with the design of the hybrid control law strategy via higher order sliding mode technique. Simulation results are presented in Section 4.

2 CAR-LIKE ROBOT MODEL AND PROBLEM STATEMENT

As mentioned in (Murray and Sastry, 1993), many nonlinear mechanical systems that belong to the class of driftless nonholonomic systems (the knife-edge, articulated vehicles, a car towing several trailers, etc...) can be transformed via change of coordinates in the state and control spaces into a so-called chained form. In this paper, we are particularly concerned with nonholonomic systems whose trajectories can be written as the solution of the driftless system:

$$\dot{x} = g_1(x)u_1 + g_2(x)u_2 \tag{1}$$

where $x \in \Re^n$ is the state vector, u_1, $u_2 \in \Re$ are the two control inputs, g_1, g_2 are smooth linearly independent vector fields. We are interested in the case of $n = 4$ which is the example of a car-like mobile robot. The kinematic model of the robot in a single drive mode is given by:

$$\begin{cases} \dot{x} = \cos(\theta)\ u_1 \\ \dot{y} = \sin(\theta)\ u_1 \\ \dot{\theta} = \frac{\tan(\phi)}{L} u_1 \\ \dot{\phi} = u_2 \end{cases} \tag{2}$$

where (x,y) are the Cartesian coordinates of the center of the rear axle, θ is the orientation angle of the vehicle with respect to a fixed frame, ϕ is the steering angle relative of the car body and u_1, u_2 are respectively the driven and the steering velocities.

For $\theta \in \left]-\frac{\pi}{2}, \frac{\pi}{2}\right[$, let us consider the following transformation on the control input vector:

$$w = \begin{pmatrix} \cos(\theta) & 0 \\ 0 & 1 \end{pmatrix} u$$

and let us introduce perturbations in the model. Then, the behavior of the robot can be described by the following system:

$$\dot{q} = g_1 w_1 + g_2 w_2 + \gamma(x,t) \tag{3}$$

with $q = (x, y, \theta, \phi)^T$, $g_1(q) = \left(1, \tan(\theta), \frac{\tan(\phi)}{L\cos(\theta)}, 0\right)^T$, $g_2(q) = (0,0,0,1)^T$. $\gamma(x,t) \in \Re^n$ is an additive perturbation assumed to be smooth enough. In (Murray *et al.*, 1994), conditions are given for a nonholonomic systems (1) to be transformed into a so-called one-chained form.

By using the diffeomorphism and the control input space transformation given in (Murray *et al.*, 1994)

$$\begin{cases} z_1 = x \\ z_2 = \frac{\tan(\phi)}{L\cos(\theta)^3} \\ z_3 = \tan(\theta) \\ z_4 = y \end{cases} \tag{4}$$

$$\begin{cases} v_1 = w_1 = u_1 \cos(\theta) \\ v_2 = \frac{3\tan(\phi)^2 \sin(\theta)}{L^2\cos(\theta)^3} w_1 + \frac{1}{L\cos(\theta)^3\cos(\phi)^2} w_2 \end{cases}, \tag{5}$$

for $\phi \in \left]-\frac{\pi}{2}, \frac{\pi}{2}\right[$, the system (3) is transformed in the following perturbed one-chained form

$$\begin{cases} \dot{z}_1 = v_1 + \gamma^1(z,t) \\ \dot{z}_2 = v_2 + \gamma^2(z,t) \\ \dot{z}_3 = z_2\ (v_1 + \gamma^1(z,t)) \\ \dot{z}_4 = z_3\ (v_1 + \gamma^1(z,t)) \end{cases} \tag{6}$$

if and only if $\gamma(x,t)$ belongs to the distribution spanned by $g_1(x)$ and $g_2(x)$ (Floquet *et al.*, 2000).

In the following, γ^1 and γ^2 are supposed to be bounded for all $z \ \varepsilon\ \Omega$ (an open set in $\Re^4$) and for all t as follows:

$$\begin{aligned} \left|\gamma^1(z,t)\right| &\leq \sigma_1 \\ \left|\gamma^2(z,t)\right| &\leq \sigma_2 \\ \left|\dot{\gamma}^1(z,t)\right| &\leq \sigma_1^{'} \end{aligned}$$

where σ_1, σ_2, $\sigma_1^{'}$ are positive constants.

Problem: Find a robust control law for the car-like robot model (6) guaranteeing the finite time stabilization at the origin $(x = 0,\ y = 0,\ \theta = 0,\ \phi = 0)$ in the presence of matched disturbances.

3 FINITE TIME STABILIZATION VIA HIGHER ORDER SLIDING MODES

3.1 Higher Order Sliding Modes

In this approach, the designed control law will switch between second order and third order sliding mode algorithms in order to obtain the finite time stabilization of (6). It is assumed that the reader is familiar with the sliding mode theory (see (Emel'yanov *et al.*, 1993), (Fridman and Levant, 2002) or (Perruquetti and Barbot, 2002) for further details). Let us just briefly recall that the principle of higher order sliding mode control is to constrain, by the mean of a discontinuous control acting on the r^{th} time derivative of a suitably chosen sliding variable $S : \Re^{+} \times \Re^{n} \rightarrow \Re$, the system trajectories to reach and stay, after a finite time, on a given sliding manifold S^r in the state space defined by:

$$\mathcal{S}^{r} = \left\{ S(t,x) = \dot{S}(t,x) = \ldots = S^{(r-1)}(t,x) = 0 \right\},$$

where $x \in \Re^n$ is the state system. A control law leading to such a behavior will be called a r^{th} order ideal sliding mode algorithm with respect to S.

Arbitrary-order sliding mode controllers with finite time convergence have been proposed in (Levant, 2001) and (Levant, 2003). As the control algorithm proposed in (Levant, 2001) requires the knowledge of high order time derivatives of the output, the author in (Levant, 2003) proposes the use of a robust exact finite time convergence differentiators based on the super twisting algorithm. The implementation of these controllers is not easy since some singularities in the time derivatives of the sliding variable may appear. In (Laghrouche *et al.*, 2004), a third order sliding mode controller that combines a standard sliding mode control with a linear quadratic one has been proposed. However, it directly depends on the initial conditions of the system and complex off-line computations are needed before starting the control action. A higher order sliding mode control strategy with smooth manifold leading to a practical convergence was developed in (Djemai and Barbot, 2002). Based on this strategy, a real third order sliding mode controller with time varying smooth manifolds was designed for the practical stabilization of a unicycle-type mobile robot (Barbot *et al.*, 2003).

3.2 Finite Time Stabilization of the Car-like Robot

The stabilization of (6) is made in three steps by switching between different types of sliding mode controllers:

First step:

The control algorithm is first to constrain the subsystem:

$$\dot{z}_1 = v_1 + \gamma^1(z,t) \tag{7}$$

to evolve after a finite time on the sliding manifold

$$s_1 = z_1 - at = 0,\ a > 0.$$

The subsystem (7) has relative degree one with respect to s_1 and the second time derivative of s_1 is given by:

$$\ddot{s}_1 = \dot{v}_1 + \dot{\gamma}^1(z,t).$$

The chosen sliding mode algorithm is the super twisting algorithm which has been developed for systems with relative degree one to avoid chattering. The control law v_1 is given as follows:

$$\begin{array}{l} v_1 = -\lambda_1 |s_1|^{\frac{1}{2}} \operatorname{sign}(s_1) + v_{11}, \\ \dot{v}_{11} = -\alpha_1 \operatorname{sign}(s_1), \end{array} \tag{8}$$

where α_1, λ_1 are positive constants that satisfy the following conditions (Levant, 2003):

$$\begin{array}{rcl} \alpha_1 & > & \sigma_1' \\ \lambda_1^2 & > & 4\sigma_1' \dfrac{\alpha_1 + \sigma_1'}{\alpha_1 - \sigma_1'} \end{array}$$

This ensures that the trajectories reach the sliding manifold $\Gamma_1 = \{ z \in \Omega : s_1 = \dot{s}_1 = 0\}$ in a finite time T_1 and stay it after T_1. Thus, for $t \geq T_1$, the resulting dynamics, in sliding motion, is given by:

$$\left\{ \begin{array}{l} \dot{z}_1 = a, \\ \dot{z}_2 = v_2 + \gamma^2(z,t), \\ \dot{z}_3 = az_2, \\ \dot{z}_4 = az_3. \end{array} \right. \tag{9}$$

Second step:

When the state trajectory evolves in Γ_1, the dynamics of the subsystem

$$\left\{ \begin{array}{l} \dot{z}_2 = v_2 + \gamma^2(z,t), \\ \dot{z}_3 = az_2, \\ \dot{z}_4 = az_3. \end{array} \right. \tag{10}$$

is equivalent to a perturbed triple chain of integrator. The finite time stabilization of (10) can be obtained using a 3^{rd} order sliding mode algorithm for v_2. A new kind of algorithm is presented here. The control law v_2 is made of two terms:

$$v_2 = v_{2,id} + v_{2,vss}$$

where $v_{2,id}$ is an ideal control, based on the geometric homogeneity approach, and that ensures the finite time stabilization of the system (9) without perturbations. $v_{2,vss}$ is a discontinuous part of the control v_2 allowing to reject the uncertainties.

a. Control design of $v_{2,id}$

Consider the system (10) without perturbations:

$$\begin{cases} \dot{z}_2 = v_{2,id} \\ \dot{z}_3 = az_2 \\ \dot{z}_4 = az_3 \end{cases} \quad (11)$$

and let us define a control law $v_{2,id}$ stabilizing $\bar{z}=(z_2,\ z_3,\ z_4)^T$ to zero in finite time.

To this end, let $k_1, k_2, k_3 > 0$ be such that the polynomial $p^3 + k_3p^2 + k_2p + k_1$ is Hurwitz. From the works (Bhat and Bernstein, 2005), there exists $\varepsilon \in (0,1)$ such that, for every, $\beta \in (1-\varepsilon, 1)$, the origin is a globally finite time stable equilibrium for (11) via the state feedback:

$$\begin{aligned} v_{2,id} &= -k_1\text{sign}(z_4)\,|z_4|^{\beta_1} - k_2\text{sign}(z_3)\,|z_3|^{\beta_2} \\ &\quad -k_3\text{sign}(z_2)\,|z_2|^{\beta_3} \end{aligned} \quad (12)$$

with

$$\begin{cases} \beta_{i-1} = \frac{\beta_i\beta_{i+1}}{2\beta_{i+1}-\beta_i},\ i = 2,3 \\ \beta_3 = \beta,\ \beta_4 = 1 \end{cases}$$

This ensures that the following equalities hold after a finite time $T_{2,i}$:

$$z_2 = z_3 = z_4 = 0.$$

b. Control design of $v_{2,vss}$

For perturbation rejection, the following sliding variable $s \in \Re$ is introduced:

$$s = s_0(z_2, z_3, z_4) + s_2 \quad (13)$$

Here, s_0 is a quite conventional sliding mode variable, selected such that $\frac{\partial s_0}{\partial z}[1,0,0]^T \neq 0$ (relative degree one requirement). One can choose

$$s_0 = z_2.$$

s_2 is an additional term that enables integral control to be included such that

$$\dot{s}_2 = -v_{2,id}.$$

Let us show that a sliding motion can be induced on $s = 0$ by using the discontinuous control

$$v_{2,vss} = -D\text{sign}(s) \quad (14)$$

where the switching gain satisfies:

$$D > \sigma_2.$$

For this, define a Lyapunov function V as follows:

$$V = \frac{1}{2}s^2$$

The time derivative of this function is given by:

$$\begin{aligned} \dot{V} &= s(v_2 + \gamma^2 + \dot{s}_2) \\ &= s(v_{2,id} + v_{2,vss} + \gamma^2(z,t) - v_{2,id}) \\ &= s(\gamma^2(z,t) - D\text{sign}(s)) \\ &\leq \sigma_2\,|s| - D\ |s| \\ &\leq -G\sqrt{V}, \quad G > 0 \end{aligned}$$

Hence, the trajectories of the system converge in a finite time $T_{2,v}$ on the sliding manifold given by $\{s = 0\}$. When sliding, the equivalent control denoted by $v_{2,eq}$ (see (Edwards and Spurgeon, 1998), p. 34 for a definition), required to maintain the sliding motion on the surface $s = 0$, is obtained by writing that $\dot{s} = 0$:

$$\begin{aligned} \dot{s} &= \dot{s}_0 + \dot{s}_2 \\ &= v_{2,eq} + v_{2,id} + \gamma^2(z,t) - v_{2,id} \\ &= 0 \end{aligned} \quad (15)$$

Thus:

$$v_{2,eq} = -\gamma^2(z,t)$$

and the equivalent dynamics on $s = 0$ is given by the system (11). This implies that the trajectory enters the set

$$\Gamma_2 = \{\ z \in \Omega : z_2 = z_3 = z_4 = 0\}$$

after a finite time $T_2 = T_{2,i} + T_{2,v}$.

Third step:

When $z \in \Gamma_1 \cap \Gamma_2$, the controls are switched to:

$$\begin{cases} v_1 = -\lambda_{11}\,|z_1|^{\frac{1}{2}}\,\text{sign}(z_1) + w_{11} \\ \dot{w}_{11} = -\alpha_{11}\text{sign}(z_1) \end{cases} \quad (16)$$

$$\begin{cases} v_2 = -\lambda_{22}\,|z_2|^{\frac{1}{2}}\,\text{sign}(z_2) + w_{22} \\ \dot{w}_{22} = -\alpha_{22}\text{sign}(z_2) \end{cases} \quad (17)$$

With a suitable choice of the positive constants α_{11}, λ_{11}, α_{22} and λ_{22} (see (Levant, 2003)), a sliding motion is obtained on the manifolds $\{z_1 = \dot{z}_1 = 0\}$ and $\{z_2 = \dot{z}_2 = 0\}$ after a finite time T_3. This allows to maintain z_2 equals to zero (and so $z_3 = z_4 = 0$), and to reach the manifold $z_1 = 0$, so that the global system is stabilized in a finite time lower than $T_1 + T_2 + T_3$.

4 SIMULATION RESULTS

In this section, simulation results for the finite time stabilization of the car-like robot are presented. The sampling time is set to be $\tau = 0.01s$ with the physical parameter $L = 1.2m$. The design parameters of the second order sliding mode controllers are:

$$a = 3,\ \lambda_1 = 5,\ \lambda_{11} = \lambda_2 = 1,\ \alpha_1 = \alpha_{11} = \alpha_{22} = 0.5$$

while for the third order one, they are:

$$k_1 = 1, k_2 = k_3 = 1.5, D = 0.97,$$
$$\beta_1 = 2/3, \beta_2 = 3/5, \beta_3 = 3/4.$$

The perturbations are taken as band limited white noises with unit variance. The results of the stabilization are given in Figures 1, 2, 3 with the initial conditions:

$$z_1 = x = 0.5m, z_2 = z_3 = 0, z_4 = y = 1.4m.$$

Figures 1, 2 and 3 show that the control problem is fulfilled since the state trajectory of the robot converges to zero in a robust manner. One can note (Figure 1) that z_4 tends to zero $(\approx 5s)$ faster than z_3 $(\approx 5.2s)$, and z_2 $(\approx 5.4s)$. Once $z_2 = 0$, the control v_1 switches in the third step and z_1 reaches the origin in finite time $(13s)$. Figure 2 gives the angles behaviour and the trajectory of the robot in the phase plan (x, y), while Figure 3 shows the movement of the robot. The choice of a second order sliding mode controller (super twisting algorithm) in the first and third steps allows to overcome the chattering phenomenon since the discontinuity is acting on the first time derivative of the control v_1. Indeed, the trajectory of the system is smoother as there are few uncertainties on the information injected in the equivalent dynamics in the second step. Also, it can be seen on the behavior of the actual control inputs (Figure 4), that the driven velocity is not affected by the chattering effect. However, the steering velocity still exhibits some chattering in the second step due to the discontinuous part of the control v_2. In practice, the chattering phenomenon can be reduced by using sigmoïd functions instead of the signum function. Another solution would be the use of a second order sliding mode algorithm in (14).

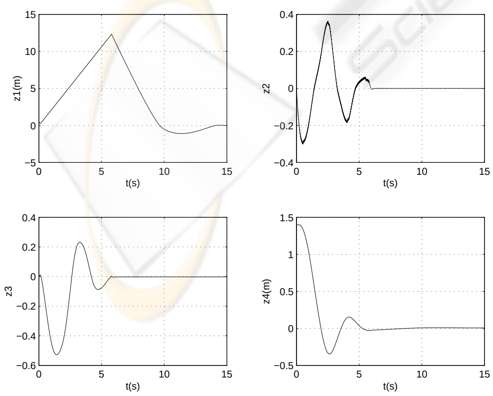

Figure 1: Coordinates z_1, z_2, z_3, z_4.

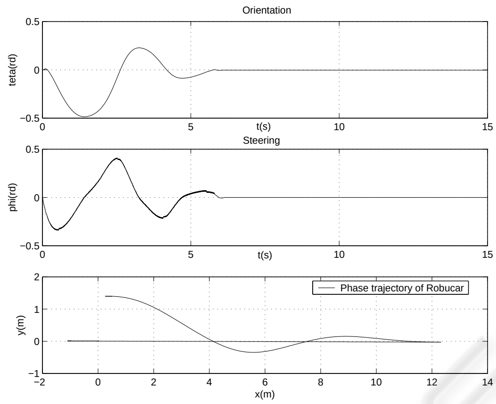

Figure 2: Angles and phase trajectory.

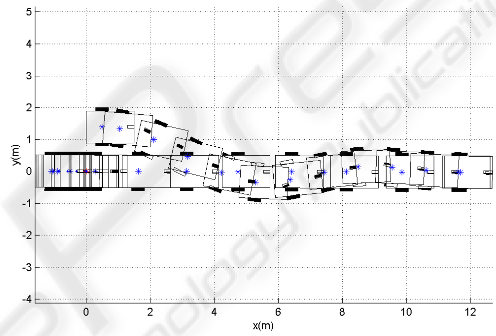

Figure 3: Movement of the Robot.

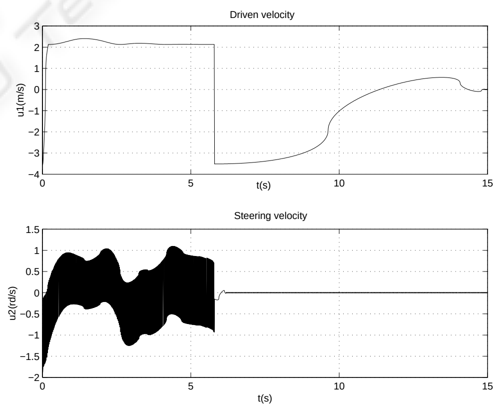

Figure 4: Actual control inputs.

5 CONCLUSION

This paper has presented a higher order sliding mode control solution for the robust stabilization problem applied to a car-like robot. Based on the perturbed chain form of the robot, control laws, switching between different higher order sliding mode controllers, have been developed to obtain a robust finite time stabilization. One contribution of this paper is the design of a 3^{rd} order sliding mode control based on geometric homogeneity property with a discontinuous term. Future work concerns the experimental test of the proposed control approach.

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