SLIDING MODE CONTROL FOR HAMMERSTEIN MODEL BASED ON MPC

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Abstract: This paper addresses discrete sliding mode control of nonlinear systems. The nonlinear system is identified as a Hammerstein model firstly to isolate the nonlinearity from the sliding surface design. An MPC law is employed to design the sliding surface. Then Utkins method of equivalent control is used. The method illustrates the effect of the nonlinearity on reaching control. The ball and beam system is adopted as an example. Simulation and on-line results are provided.

1 INTRODUCTION

Variable structure systems (VSS) have been extensively used for control of dynamic industrial processes. The essence of variable structure control (VSC) (Raymond et al., 1988) is to use a high speed switching control scheme to drive the nonlinear plant's state trajectory onto a specified and user chosen surface in the state space which is commonly called the sliding surface or switching surface, and then to keep the plant's state trajectory moving along this surface. The surface is chosen to produce specified dynamic behaviour. Once the state trajectory intercepts the sliding surface, it remains on the surface for all subsequent time, sliding along the surface, leading to the term "sliding mode". Sliding mode controller design comprises two stages. The first is the design of sliding surface, while the second forces the state to approach the sliding surface from any other region of the state space, and remain on it.

The ball and beam system is a widely used laboratory process. It reflects typical control problems which include a double integrating factor, nonlinearity, time delay and noise. In the ball and beam system, a conductive ball lies on the beam comprised of two parallel rods, and is free to roll along the beam. A resistive strip, with impedance proportional to length, covers one of the rods. The other rod is conductive. The position of the ball can be determined by introducing a small current through the rods and measuring the resulting voltage, which varies with impedance as the ball moves. One end of the beam is fixed and the other is mounted on the output shaft of a DC servo motor so the beam is tilted as the motor shaft rotates. The control task is to regulate the position of the ball by altering the angular shaft position of the DC motor.

Design from an identified model has potential advantages in nonlinear control for the ball and beam, and more generally. It relies on mathematical tools and algorithms that build dynamical models from measured data. Relatively simple structures of nonlinearity may be used to describe complex nonlinear systems or ones for which models are difficult to derive. In this paper, discrete sliding mode control of a system described by a Hammerstein model will be addressed. This provides a simple method to deal with nonlinear systems using VSC. The ball and beam system will be used as an example to illustrate the design procedure.

The control of a Hammerstein model has been addressed in the past by several authors (cite15,cite16). Satisfying performance has been derived. In (Hwang and Hsu), Hwang and Hsu talked about nonlinear control profile based on Hammerstein model in case of model uncertainty. They also introduced an inverse block into the system. Meanwhile, they spent a lot effort on designing an observer.

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2 HAMMERSTEIN MODEL

Hammerstein models are amongst those most commonly used for nonlinear identification. They are capable of providing simple nonlinear models for a wide range of engineering problems. The model is characterized by a static nonlinearity followed by a linear time invariant (LTI) block. A typical Hammerstein model for a process is shown in Figure 1:

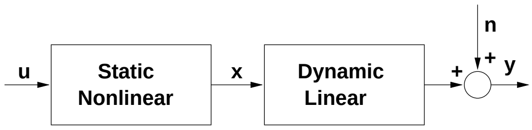

Figure 1: Typical Hammerstein model.

$$y(t) = Tx(t) + n(t) \quad (1)$$
$$x(t) = f(u(t)) \quad (2)$$

where $x(t)$ and $y(t)$ are the inputs and outputs respectively, $n(t)$ is additive noise, f is the nonlinear mapping, T is the transfer function of linear part which can be written as

$$T = \frac{b_0 + b_1 q^{-1} + b_2 q^{-2} + \cdots + b_m q^{-m}}{1 + a_1 q^{-1} + a_2 q^{-2} + \cdots + a_n q^{-n}}$$

with q^{-1} representing the unit delay operator.

In this way, the nonlinearity of system is separated from the linear block. This leads to the possibility of ignoring the nonlinearity during key steps in the controller design. Also, the nonlinear block in the Hammerstein model is a polynomial, which is a relatively simple form. This reduces problems introduced by complex nonlinearities such as exponentials and sinusoids.

3 DISCRETE SLIDING MODE CONTROL DESIGN

3.1 Sliding Surface Design

Suppose the state space model of the above Hammerstein model is:

$$z(t) = Az(t-1) + Bx(t) \quad (3)$$
$$y(t) = Cz(t) + \bar{n}(t) \quad (4)$$

where $x(t) = f(u(t))$.

Performing a similarity transformation defined by an orthogonal matrix P:

$$z_l = Pz = [z_1 : z_2]^T, A_l = PAP^T, B_l = PB = \begin{bmatrix} 0 \\ B_2 \end{bmatrix}, \quad (5)$$

where z_1 does not have direct dependence on the input nonlinearity. Sliding surface design may be undertaken considering only z_1, treating z_2 as an "input" to the partitioned equations. In this way, the nonlinearity may be ignored while determining the sliding surface, which is linear.

The partitioned state equations corresponding to (3) and (4) may now be expressed in the following way:

$$z_{l1}(t+1) = A_{l11} z_{l1}(t) + A_{l12} z_{l2}(t) \quad (6)$$
$$z_{l2}(t+1) = A_{l21} z_{l1}(t) + A_{l22} z_{l2}(t) + B_{l2} x(t). \quad (7)$$

Suppose

$$sz_l(t) = [\begin{matrix} s_1 & s_2 & \cdots & s_v \end{matrix}] z_l(t) = w_1 z_{l1}(t) + w_2 z_{l2}(t)$$

in which v is the dimension of the corresponding state vector, and s is the sliding surface, then the sliding condition is

$$w_1 z_{l1}(t) + w_2 z_{l2}(t) = 0,$$

which yields

$$z_{l2}(t) = -w_2^{-1} w_1 z_{l1}(t). \quad (8)$$

Substitute (8) into (6) then we have,

$$z_{l1}(t+1) = A_{l11} z_{l1}(t) - A_{l12} w_2^{-1} w_1 z_{l1}(t) \quad (9)$$
$$= (A_{l11} - A_{l12} w_2^{-1} w_1) z_{l1}(t). \quad (10)$$

Any standard design algorithm which produces a linear state feedback controller for a linear dynamic system can be used to determine $(A_{l11} - A_{l12} w_2^{-1} w_1)$ and achieve desired performance through selection of sliding mode dynamics (Spurgeon, 1992). Pole placement is an obvious way of assigning closed loop eigenvalues, but for systems of higher order the method has attendant difficulties.

MPC is a widely-used method for calculating closed-loop feedback controller gains. It is suitable for systems with high order. It is employed to determine the sliding surface in this paper. Considering $z_{l1}(t+1) = A_{l11} z_{l1}(t) + A_{l12} z_{l2}(t)$, $z_{l2}(t)$ can be viewed as the input to a new system the state vector of which is $z_{l1}(t)$. An MPC criterion minimizes the cost function which is defined to be:

$$\mathbf{J} = M_{t+1}^{\top} M_{t+1} + \lambda \mathbf{U}_t^{\top} \mathbf{U}_t. \quad (11)$$

where

$$M_{t+1} = \begin{bmatrix} z_{l1}(t+1) \\ z_{l1}(t+2) \\ \cdots \\ z_{l1}(t+N) \end{bmatrix}, U_t = \begin{bmatrix} z_{l2}(t) \\ z_{l2}(t+1) \\ \cdots \\ z_{l2}(t+N-1) \end{bmatrix}.$$

The goal is to fix the relationship between $z_{l2}(t)$ and $z_{l1}(t)$ to prescribe desirable performance for the nominal sliding mode dynamics. The controller gain derived is:

$$z_{l2}(t) = -k z_{l1}(t) \quad (12)$$

which means that

$$\sigma(\mathbf{z}_l(t)) = \left[\begin{array}{ccc} k & \vdots & I \end{array}\right] z_l(t) \tag{13}$$

Note that inversion of the similarity transformation (using P) is needed to recover $z(t)$ from $\mathbf{z}_l(t)$. Then $sz(t)$ is the sliding surface.

3.2 Sliding Mode Controller Design

The reaching law still applies for discrete systems. However, the state trajectory may overshoot the sliding surface repeatedly, so that true sliding does not occur. The switching manifold of a discrete VSC system is called an ideal switching manifold because in all practical situations, switching seldom occurs on it. The size of each successive overshoot is non-increasing and the trajectory stays within a specified band which is called a *quasi-sliding mode* (QSM). The specified band is called *quasi-sliding mode band* (QSMB) (Gao et al., 1995) and is defined by

$$\{x \mid -\Delta < s(x) < \Delta\} \tag{14}$$

where 2Δ is the width of the band.

Consider the single input linear system with switching manifold s, a common type of sliding mode controller is:

$$u(t) = u_{eq}(t) + u_2(t) \tag{15}$$

where $u_{eq}(t)$ represents the equivalent control which ensures sliding and $u_2(t)$ drives the state onto the sliding surface, (termed reaching control).

According to the definition of sliding mode, we have

$$\sigma(t+1) = sz(t+1) = sAz(t) + sBu_{eq}(t) = \sigma(t). \tag{16}$$

and

$$\sigma(t) = 0.$$

From the above, the equivalent control can be described as follows:

$$u_{eq}(t) = -(sB)^{-1} sAz(t). \tag{17}$$

Then let us consider the reaching control law. For continuous SMC problem, a simple Lyapunov function $V(\sigma(z)) = 0.5\sigma^T(z)\sigma(z)$ is considered. The corresponding reaching condition is

$$\frac{\partial V}{\partial t} = \sigma^T \dot{\sigma} < 0. \tag{18}$$

In discrete system design, the equivalent form of this condition is

$$[\sigma(t+1) - \sigma(t)]\sigma(t) < 0. \tag{19}$$

Substitute (3), (4), (15) and (17) into (19) then:

$$\begin{aligned}
[\sigma(t+1) - \sigma(t)]\sigma(t) &= (sAz(t) + sBu(t) - sz(t))sz(t) \\
&= ((sB)((sB)^{-1} sAz(t) \\
&\quad + u(t)) - sz(t))sz(t) \\
&= (sB(u(t) - u_{eq}(t)) \\
&\quad - sz(t))sz(t). \\
&= (sBu_2(t) - sz(t))sz(t).
\end{aligned}$$

$u_2(t)$ should be selected to ensure that:

$$sBu_2(t) < sz(t) \quad when \;\; sz(t) > 0 \tag{20}$$

$$sBu_2(t) > sz(t) \quad when \;\; sz(t) < 0. \tag{21}$$

As mentioned before, in a discrete sliding mode control system, the switching manifold is actually an ideal one. To eliminate the overshoot, the reaching law should be modified. Once the state trajectory enters a specified band around the manifold, the reaching control action ceases and only sliding control applies. The goal is to keep the state trajectory within the specified band.

The modified reaching control law is:

$$sBu_2(t) < sz(t) \text{ when } sz(t) > \Delta \tag{22}$$

$$sBu_2(t) > sz(t) \text{ when } sz(t) < -\Delta. \tag{23}$$

Considering equations (18)-(21) and absolute values of $\sigma(t+1)$ and $\sigma(t)$, if

$$\|\sigma(t+1)\| < \|\sigma(t)\|, \tag{24}$$

it can be concluded that the state trajectory is towards the sliding surface. On the contrary, if

$$\|\sigma(t+1)\| > \|\sigma(t)\|, \tag{25}$$

the trajectory is away from the sliding surface.

Note that (24) is equivalent to

$$\|sBu_2(t)\| < \|\sigma(t)\|, \tag{26}$$

and (25) is equivalent to

$$\|sBu_2(t)\| > \|\sigma(t)\|. \tag{27}$$

The conclusion may be drawn that while the trajectory is outside the ($\varepsilon = \|sBu_2(t)\|$), the trajectory will approach the surface. While the state is within this specified neighborhood, it moves in the direction of leaving the surface (Hui and Zak, 1999). Thus $u_2(t)$ has to be carefully chosen because the value of $\varepsilon = \|sBu_2(t)\|$ is the crucial factor which determines the radius attraction around the sliding surface.

Figure 2. shows the structure of closed loop sliding mode control system: where P is the plant and W represents the reaching controller.

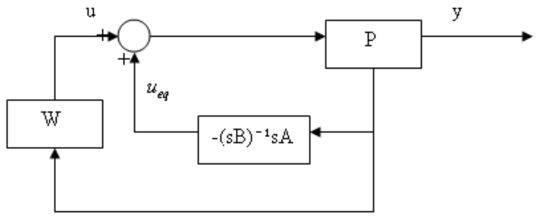

Figure 2: Structure of closed loop sliding mode control system.

3.3 Sliding Mode Control for a Hammerstein Model

In a Hammerstein model, the nonlinearity has been separated from the linear block already. The nonlinear mapping f is a smooth polynomial with respect to its input, hence it is invertible. Thus in the controller design stage, the nonlinearity can be ignored and the controller is only designed based on the linear block and afterwards, the control signal is filtered through an inverse of the nonlinearity before being sent to the plant. This may not be necessary if the nonlinearity is taken into account in the Lyapunov function used for determination of the reaching control.

As far as the description of the linear block is concerned, a non-minimal state space model is employed (refer to (Xi and Hesketh) for details of derivation of non-minimal state space models). The motivation for this is that in a non-minimal state space model, the error signal sequence is contained in state vector (the error signal being the difference between the plant output and the desired trajectory). Sliding mode control results in a regulator, where the state variables will be constrained to move along the sliding surface and eventually reach and stay at zero. Thus the error signal will be regulated to approach zero and stay there if the sliding mode control is based on a non-minimal state space model. This is the aim of tracking control. Figure 3. shows the structure of such a control system.

In the figure, r represents the set point and e is the difference between filtered set point and the output of plant (Xi and Hesketh). For our example, the "filter" is selected simply as q^{-1}. Here both equivalent and reaching controller are related to the tracking error and the set point. The control action force the states to reach the sliding manifold and move along it. This action continues until the plant output is equal to the filtered set point, which is equivalent to the error signal being zero. Then the states will be kept at the

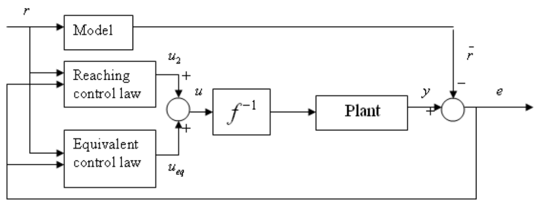

Figure 3: Structure of sliding mode control system based on Hammerstein model.

origin and the system achieves steady state.

4 EXAMPLE AND SIMULATION RESULTS

4.1 Identification of Ball and Beam

Deriving an approximate model of the ball and beam system involves determining the transfer function between the input signal (the shaft angle of the motor) and the output signal (the position of the ball). In an identification experiment, a pseudo-random sequence is applied to the input signal, and both input and output signals are sampled (For the ball and beam the sampling interval selected was 1 second). Here Captain Toolbox which is written for Matlab is used to realize the identification ((Young et al., 2001), http://www.es.lancs.ac.uk/cres/captain/). The result of the Hammerstein model identification is:

$$y(t) = 0.962y(t-1) + 0.396u(t-1) - 0.036u^2(t-1)$$

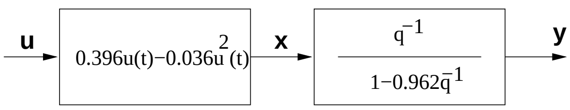

Figure 4: Identification result of Hammerstein model.

The resultant non-minimal state space model of the linear block is:

$$z(t) = Az(t-1) + B\Delta x + Q\Delta r(t) \quad (28)$$
$$y(t) = Cz(t) \quad (29)$$

where

$$A = \begin{bmatrix} 1.962 & -0.962 & -1 & 0.962 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}, Q = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, z(t) = \begin{bmatrix} e \\ eq^{-1} \\ \Delta r \\ \Delta rq^{-1} \end{bmatrix}.$$

Suitable differencing is undertaken to introduce $\Delta = 1 - q^{-1}$. Note the way in which the setpoint is introduced within the state vector. This results in feedforward action, achieved with the sliding mode control.

4.2 Controller Design and Simulation

This system is typically unobservable. Performing of observable/unobservable decomposition prevents singularity occurrence later. The system model becomes:

$$\bar{z}(t) = \bar{A}\bar{z}(t-1) + B\Delta\bar{x}(t-1) + Q_{ob}\Delta r(t) \quad (30)$$

$$\bar{y}(t) = \bar{C}\bar{z}(t) \quad (31)$$

where the transformation matrix is T and

$$\bar{A} = \begin{bmatrix} 0 & 0.5698 & 0.4188 & 0.7071 \\ 0 & 0.3374 & 0.2480 & -0.4188 \\ 0 & -0.4591 & -0.3374 & 0.5698 \\ 0 & 0 & -1.6885 & 1.962 \end{bmatrix},$$

$$\bar{B} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \bar{C} = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}.$$

Extracting the observable part we have:

$$z_{ob}(t) = A_{ob}z_{ob}(t-1) + B_{ob}\Delta u_{ob}(t-1) \quad (32)$$

$$y_{ob}(t) = C_{ob}z_{ob}(t) \quad (33)$$

where

$$A_{ob} = \begin{bmatrix} 0.3374 & 0.2480 & -0.4188 \\ -0.4591 & -0.3374 & 0.5698 \\ 0 & -1.6885 & 1.962 \end{bmatrix}, B_{ob} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

In this case, the requirement of equation (5) has already been satisfied so that no further transformation P is needed. Then

$$A_{ob11} = \begin{bmatrix} 0.3374 & 0.248 \\ -0.4591 & -0.3374 \end{bmatrix}, A_{ob12} = \begin{bmatrix} -0.4188 \\ 0.5698 \end{bmatrix},$$

$$A_{ob21} = \begin{bmatrix} 0 & -1.6885 \end{bmatrix}, A_{ob22} = [1.962]$$

The result of optimization is $\begin{bmatrix} k_1 & k_2 \end{bmatrix}$.

Note that an inversion of the observable/unobservable decomposition is to be performed to recover the state vector $z_{ob}(t) = Tz(t)$ after sliding surface design,

$$\sigma(z_{ob}(t)) = \begin{bmatrix} k_1 & k_2 & 1 \end{bmatrix} z_{ob}(t) \quad (34a)$$

$$\sigma(z(t)) = \begin{bmatrix} k_1 & k_2 & 1 \end{bmatrix} Tz(t) = sz(t) \quad (34b)$$

In this case, $u_2(t)$ is chosen to be $-\alpha sgn(\sigma(x(t)))$. Figure 5. shows the performance of the above sliding mode design. The figure shows simulation results, but on-line control is similar.

Figure 6. shows the values of $\sigma(t)$ And Figure 7. shows the state trajectory:

It is shown that the system follows the ideal trajectory. The switching law works while sliding condition is not met. Equivalent control regulates the state to move along the sliding surface until the equilibrium point is achieved.

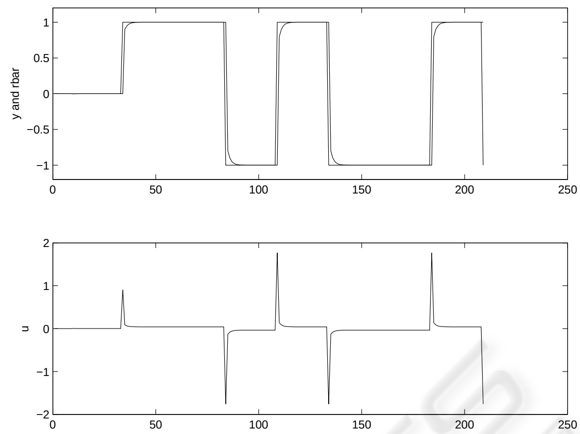

Figure 5: Performance with a linear model.

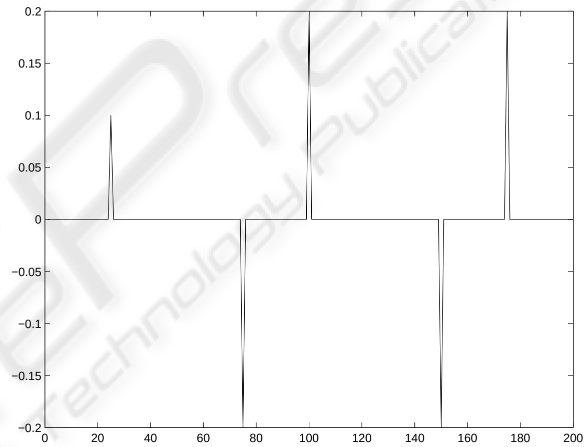

Figure 6: $\sigma(t)$ defined in equation (13).

5 CONCLUSION

In this paper, discrete sliding mode control is applied to a Hammerstein model which results from nonlinear system identification. The nonlinearity is separated during the sliding surface design, so that the switching surface is actually a linear one. The surface is derived by an MPC approach. The nonlinearity is re-

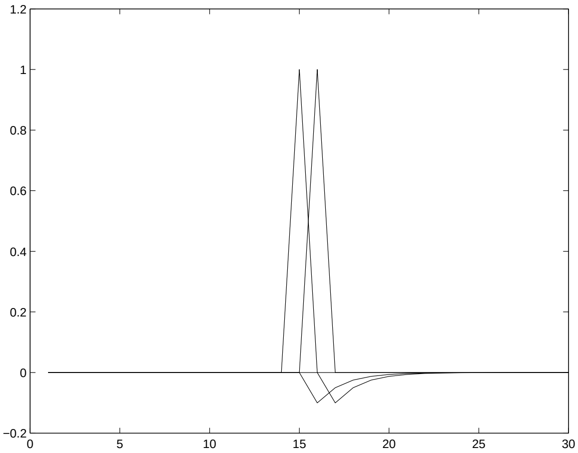

Figure 7: State variables.

considered in design of the reaching control. The ball and beam system is used as an example and simulation results show satisfying performance.

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