ADAPTIVE STACK FILTERS IN SPECKLED IMAGERY

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Abstract: Stack filters are a special case of non-linear filters. They have a good performance for filtering images with different types of noise while preserving edges and details. A stack filter decomposes an input image into several binary images according to a set of thresholds. Each binary image is filtered by using a boolean function. Adaptive stack filters are optimized filters that compute a boolean function by using a corrupted image and ideal image without noise. In this work the behaviour of an adaptive stack filter is evaluated for the classification of synthetic apreture radar (SAR) images, which are affected by speckle noise. With this aim it is carried out a Monte Carlo experiment in which simulated images are generated and then filtered with a stack filter trained with one of them. The results of their maximum likelihood classification are evaluated and then are compared with the results of classifying the images without previous filtering.

1 INTRODUCTION

Stack filters are a special case of non-linear filters. They have a good performance for filtering images with different types of noise while preserving edges and details. These filters consist of a decomposition by thresholds of an input signal obtaining a binary signal for each threshold. Each binary signal is filtered using a sliding window. Stack filters can be generated using an adaptive algorithm, in such a way that the so-called stacking property holds. The stack filter design method used in this work is based on an algorithm proposed by Yoo *et al.* (Yoo et al., 1999). In this paper we study the application of this type of filter to Synthetic Aperture Radar (SAR) images. SAR images (Goodman, 1976) and (Oliver and Quegan, 1998) are generated by a coherent illumination system and are affected by the coherent interference of the signal backscatter by the elements on the terrain. This interference causes fluctuations of the detected intensity which varies from pixel to pixel. This effect is called speckle noise. Speckle noise, unlike noise in optical images, is neither Gaussian nor additive; it follows other distributions and is multiplicative. Due to all of this it is not possible to treat these images using the classical techniques appropiate for optical image processing. The analysis of this type of images has been treated in the literature using several statistical methods, see for example (Frery et al., 1999), (Mejail et al., 2001), (Mejail et al., 2003) and (Mejail, 1999). Under the multiplicative model, the returned image Z can be thought as two independent random variables: the random variable X that represents the backscatter and the random variable Y that represent the speckle noise. Different statistical distributions have been proposed in the literature. In this work we use the Gamma distribution, Γ, for the speckle, the reciprocal of Gamma distribution, Γ^{-1}, for the backscatter, which results in the $\mathcal{G}^0$ (Frery et al., 1996) distribution for the return. These distributions depend on three parameters: α that is a roughness parameter, γ a scale parameter, and n the equivalent number of looks. In this work, we classify an image into different regions according to their homogeneity degree, which will be refered to section 5. After filtering, the image data have undergone changes in their statistical distribution functions. A study of the kurtosis and the skewness coefficients obtained after filtering show that the image data follows a more gaussian distribution. Then, we classify the image by using the maximum likelihood method and consider the normal distribution with different parameters for each region. The structure of this paper is as follows: section 2 gives an introduction to stack filters, sec-

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tion 3 describes the filter design method used in this work. In section 4 we summarises the $\mathcal{G}^0$ distribution for SAR images. In section 5 we exhibit the modification undergone by data after applying the filter, and also report on the results of the classification. Finally, we present the conclusions in section 6.

2 STACK FILTERS: DEFINITIONS AND DESIGNING

This section is dedicated to a brief synthesis of stack filter definitions and design. For more details on this subject, see (Lin and Kim, 1994), (Wendt et al., 1986), (Astola and Kuosmanen, 1997), (Yoo et al., 1999), (Coyle and Lin, 1988), (Coyle et al., 1989), (J.Lin et al., 1990). In the first place the necessary definitions are presented to explain this type of filters.

Definition 1

The threshold operator is given by $T^m : \{0,1,\ldots,M\} \rightarrow \{0,1\}$

$$T^m(x) = \begin{cases} 1 & \textit{si} \quad x \geq m \\ 0 & \textit{si} \quad x < m \end{cases}, \tag{1}$$

$$X^m = T^m(x). \tag{2}$$

According to this definition, the value of a non-negative integer number $x \in \{0,1,\ldots,M\}$ can be reconstructed making the summation of its thresholded values between 0 and M. The formula corresponding to this operation is

$$x = \sum_{m=1}^{M} X^m. \tag{3}$$

Figure 1 shows a diagram of the threshold decomposition of a unidimensional signal. The threshold operator can be extended to bi-dimensional signals.

X= [2,1,4,5,3,2,4,3]
X^1= [1,1,1,1,1,1,1,1]
X^2= [1,0,1,1,1,1,1,1]
X^3= [0,0,1,1,1,0,1,1]
X^4= [0,0,1,1,0,0,1,0]
X^5 =[0,0,0,1,0,0,0,0]

Figure 1: Example of threshold decomposition.

Definition 2

Let $X = (x_0,\ldots,x_{n-1})$ *and* $Y = (y_0,\ldots,y_{n-1})$ *be binary vectors of length* n*, then let us define a relation* $\leq$ *given by*

$$X \leq Y \quad \textit{if and only if} \quad \forall i,\ x_i \leq y_i. \tag{4}$$

This relation is reflexive, anti-symmetric and transitive, generating therefore a partial ordering on the set of binary vectors of fixed length.

Definition 3

A boolean function $f : \{0,1\}^n \rightarrow \{0,1\}$*, where* n *is the length of the input vectors, has the stacking property if and only if*

$$\forall X,Y \in \{0,1\}^n,\ X \leq Y \Rightarrow f(X) \leq f(Y). \tag{5}$$

Definition 4

We say that f *is a positive boolean function if and only if it can be written by means of an expression that contains only non-complemented input variables. That is,*

$$f(x_1,x_2,\ldots,x_n) = \bigvee_{i=1}^{K} \bigwedge_{j \in P_i} x_j, \tag{6}$$

where n *is the number of arguments of the function,* K *is the number of terms of the expression and the* P_i *are subsets of the interval* $\{1,\ldots,N\}$ *.* $\bigvee$ *and* $\bigwedge$ *are Boolean operators AND and OR. It is possible to proof that this type of functions has the stacking property.*

If the function f used to filter an image X fulfills the stacking property, then from (4) and (5) it is deduced that, for two binary images X^i and X^j, obtained from X as the result of the application of the thresholds T^i and T^j respectively, the following implication is valid

$$i \geq j \Rightarrow X^i \leq X^j \Rightarrow f\left(X^i\right) \leq f\left(X^j\right) \tag{7}$$

A stack filter is defined by the function $S_f : \{0,\ldots,M\}^n \rightarrow \{0,\ldots,M\}$, corresponding to the Positive Boolean function $f(x_1,x_2,\ldots,x_n)$ expressed in the given form by (6). The function S_f can be expressed by means of

$$S_f(X) = \sum_{m=1}^{M} f(T^m(X)) \tag{8}$$

In Figure 2 it can be observed a scheme of aplication of the filter to an unidimensional signal. S_f represents the boolean function which filters each binary thresholded signal and whose outputs are added together to finally obtain the filtered signal.

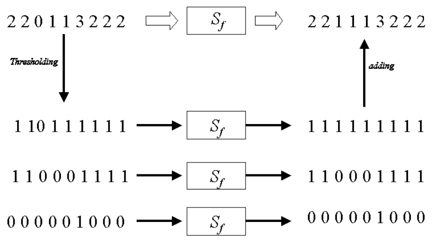


Figure 2: Scheme of the stack filter applied to an unidimensional signal. S_f represents the boolean function applied to each level.

3 ADAPTIVE ALGORITHMS FOR STACK FILTERS DESIGN

In this work we applied the stack filter generated with the fast algorithm described in (Yoo et al., 1999). This algorithm, arises as a result of studies on the methods proposed in (Lin and Kim, 1994) and (J.Lin et al., 1990). To construct a stack filter following any of these methods, a training process that generates a positive boolean function that preserves the stacking property, represented by the so-called decision vector, is carried out. In what follows, the generation and behaviour of the decision vector is explained. An image is defined as the set given by: $\{(s, v) : s \in S \subseteq \mathbb{Z}^2, v \in [0, \ldots, M], M \in \mathbb{Z}\}$, where s is position and v is the value of a pixel. Let E and R be two images, where R is the noisy version of E. A $W(s)$ window is a subimage of R of size b=r x r centered at position s. Let us define V as a 2^b dimension vector, and call it the decision vector. In the training phase of the filter design, for each $s \in S$, the data in $W(s)$ are decomposed in M thresholds obtaining M windows $w_i(s)$, $i = 1, \ldots, M$, where $w_i(s)$ is the ith thresholded version of $W(s)$. Then, for each $s \in S$ in the image E ,a threshold $e_i(s)$ is defined (notice that here $e_i(s)$ has dimension 1). If $e_i(s) = 1$, then the position on V given by $w_i(s)$, considered as a binary number, is increased by one, if $e_i(s)$= 0, then the same position is decremented by 1. Periodically, V is checked to see wether the stacking property holds. If not, the decision vector V is suitable modified. Finally, the decision vector is transformed into a binary vector that is an implementation of the positive boolean function f sought. In Figure 3 a filter training example for a $1D$ signal, is shown . Let $W(s)$ be a window of length 3 on the noisy signal, centered at position s, and let $E(s)$ be the value of the noiseless signal at the same position. In this case, $W(s) = [2, 1, 4]$, $E(s) = [3]$ and thresholds varying between 1 and 5 are considered. The decision vector V is of size $2^3 = 8$ and is represented by the table that contains at the first column indices expressed in decimal notation, at the second column indices expressed in binary notation, at the third column the vector values after n iterations and at the fourth column the vector values after the $n + 1$ iteration.

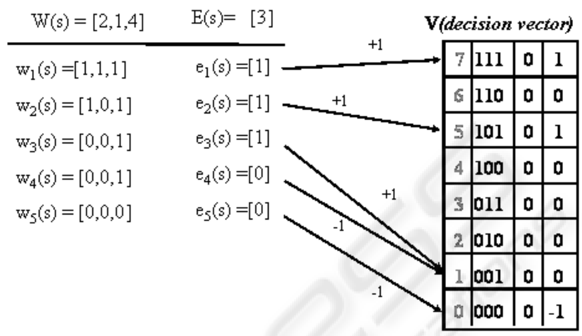


Figure 3: Scheme of the generation of the stack filter.

The training of this stack filter consists of the alternate application of two different stages: a stage in which the decision vector is modified according to the scheme indicated in Figure 3, and a stage in which the stacking property is checked and enforced on the decision vector.

4 SAR IMAGES. THE MULTIPLICATIVE MODEL

In this section we introduce the statistical laws commonly used under the multiplicative model for Synthetic Aperture Radar (SAR) images. The multiplicative model considers the image returned by the SAR, named $\mathbf{Z}$, as a product of two independent random variables, one corresponding to the backscattter $\mathbf{X}$ and the other one corresponding to the speckle noise $\mathbf{Y}$, so

$$\mathbf{Z} = \mathbf{X}\mathbf{Y}. \tag{9}$$

Where we suppose independence among the random variables corresponding to each image pixel. We can write formula (9) for each pixel (i, j) of an image of size $M \times N$ as:

$$Z_{i,j} = X_{i,j}Y_{i,j}, \; 0 \le i \le M-1, \; 0 \le j \le N-1. \tag{10}$$

The format of the SAR image (complex, amplitude or intensity) determines the distribution followed by the speckle noise random variables $Y_{i,j}$. These variables are i.i.d and the equivalent number of looks n is their only statistical parameter. On the other hand, the type of target each pixel belongs to (forest, pasture, crops, city) determines de most appropiate distribution for each of the backscatter random variables $X_{i,j}$.

4.1 Speckle Distribution

The speckle noise comes from the coherent addition of individual returns produced by elements present in each resolution cell. So, for example, in an image corresponding to a scene of land covered by vegetation, the returns from the elements of the plants and the ground are added taking into account the phase, yielding as a result a complex number. In an amplitude SAR image, the gray level of each pixel is the module of the this complex number. In an intensity SAR image, the gray level of each pixel is the square of this magnitude.

For every pixel, the model for the speckle noise is the $\Gamma(n, 2n)$ distribution, where n is the equivalent number of looks. Then, within this model the density function for the speckle noise Y is given by

$$f_Y(y) = \frac{n^n}{\Gamma(n)} y^{n-1} e^{-ny}, \;\; y \geq 0. \quad (11)$$

In SAR images the minimun value for n is 1. This value corresponds to images generated without making the average of several looks. Images generated in this manner are noiser than those generated with more number of looks, but they have better azimuth resolution and, therefore, potencially more information. We can suppose that the parameter n is known or that it can be estimated at an initial stage of the image analysis. Therefore, although in theory it would have to be an integer number, in practice it is necessary to consider it as a real number for the case in which it is estimated from the data. Figure 4 shows the curves corresponding to the speckle distribution for different values of n. The moments of the speckle distribution are given by:

$$E[Y^r] = \frac{1}{n^r} \frac{\Gamma(n+r)}{\Gamma(n)} \quad (12)$$

where r is the moment order and $n \geq 1$ is the number of looks.

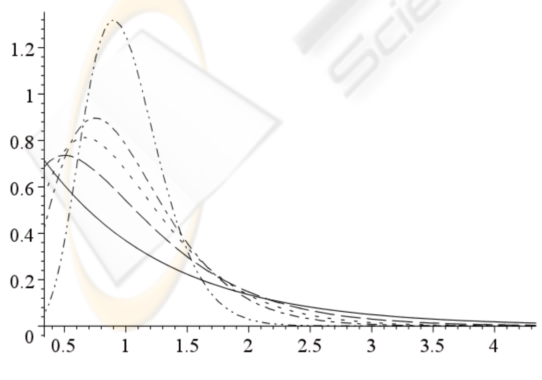


Figure 4: Curves of the Γ distribution corresponding to speckle for number of looks equal to: 1 (solid), 2 (dashes), 3(dots), 4 (dot-dash) and 10 (dot-dot-dash).

4.2 Backscatter Distribution

There are several models for the backscatter, that is, different statistical distributions exist for the random variables $X_{i,j}$. From the results presented in (Frery et al., 1996) it is possible to consider the Generalised Inverse Gaussian distribution as a general model for the backscatter. This distribution is very general and allows us to describe many different targets, but from an analitical and numerical point of view the estimation of its parameters is very complex and unstable. This distribution has various particular cases, one of which: the Inverse Gamma distribution, is of special interest to this work. This distribution is proposed as a universal model for SAR data and it leads to the $\mathcal{G}^0$ distribution for the return. The Inverse Gamma distribution, called Γ^{-1}, is characterised by the density function given by

$$f_X(x) = \frac{2^\alpha}{\gamma^\alpha \Gamma(-\alpha)} x^{\alpha-1} \exp\left(-\frac{\gamma}{2x}\right), \quad (13)$$

and its moments are expressed as

$$E[X^r] = \left(\frac{\gamma}{2}\right)^r \frac{\Gamma(-\alpha-r)}{\Gamma(-\alpha)}, \quad (14)$$

where $\alpha < 0$ and $|\alpha| > r$. In Figure 5 we can see the curves corresponding to a Γ^{-1} density function as we vary the α parameter keeping the mean value equal to one.

4.3 Distributions for the Return

The distribution corresponding to the return Z, is fixed by the distribution of the backscatter X and the distribution of the speckle Y. Given that $Z = XY$ and that these random variables are independent, $f_Z(z)$ can be calculated as

$$f_Z(z) = \int_{\mathbb{R}_+} f_{Z|Y=y}(z) f_Y(y)\, dy, \quad (15)$$

where $f_{Z|Y=y}$ is the density for the return Z considering $X = x$ constant and f_Y the density function of the speckle Y. For the random variable corresponding to the return (intensity format) we have that $Z \sim \mathcal{G}^0(\alpha, \gamma, n)$, and the density function is given by

$$f_Z(z) = \frac{n^n \Gamma(n-\alpha)}{\left(\frac{\gamma}{2}\right)^\alpha \Gamma(-\alpha)\Gamma(n)} z^{n-1} \left(\frac{\gamma}{2} + nz\right)^{\alpha-n}, \quad (16)$$

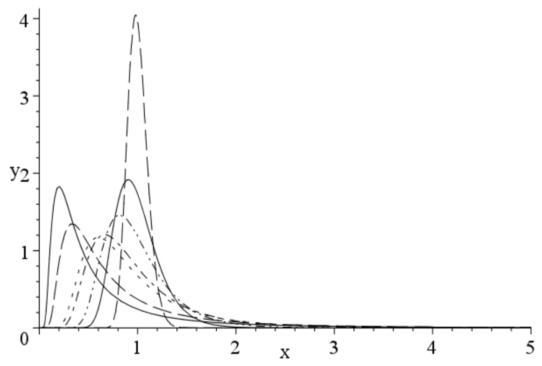


Figure 5: Curves of Γ^{-1} distribution with mean value equal to one and α equal to: -1.5 (solid), -2 (dash), -4 (dots), -5 (dot-dash), -10 (dot-dot-dash), -20 (solid) y -100(dash).

with $\alpha < 0$, $\gamma > 0$ and $n \geq 1$. Given the independence between the backscatter X and the speckle Y, the moments of the return Z are the product of the moments of X and the moments of Y (equations (14) and (12)) yielding

$$\mathbb{E}\left[Z^r\right] = \left(\frac{\gamma}{2n}\right)^r \frac{\Gamma\left(-\alpha - r\right)}{\Gamma\left(-\alpha\right)} \frac{\Gamma(n+r)}{\Gamma(n)}, \qquad (17)$$

recalling that this moments are finite for $-\alpha > r$. The variation coeficient is given by

$$C_V = \frac{\sigma}{\mu} = -\frac{\sqrt{\alpha\left(\alpha + n + 1\right)}}{\alpha}, \alpha < -\left(n+1\right),$$

where

$$\sigma^2 = \frac{\gamma^2}{4}\alpha\left(\alpha + n + 1\right), \mu = \frac{-\alpha\gamma}{2}.$$

This distribution was proposed in (Frery et al., 1996) as a model for extremely heterogeneous data, but its utility for description of a great variety of natural and artificial targets was verified, which resulted in its being proposed as a universal model for SAR data.

5 RESULTS

This section is dedicated to show the results of applying a stack filter to synthetic SAR images. These images are generated in such a way that their data have different degrees of homogeneity. We consider different values of the α parameter and we compute the γ parameter so that the mean value of the data is unitary. The α parameter corresponds to image roughness (or heterogenity). It adopts negative values, varying from $-\infty$ to 0. If α is near 0, then the image data are extremely heterogeneous (for example: urban areas), and if α is far of the origin then the data correspond to a homogeneous region (for example: pasture areas), the values for forests and crops lay in-between. The parameter γ is a scale parameter. Finally, in order to evalute the behaviour of the fiters we carried out a maximun likelihood classification. The results are analysed in the classified images, within this images we compare the filtered and the non-filtered images.

5.1 Statistical Analysis

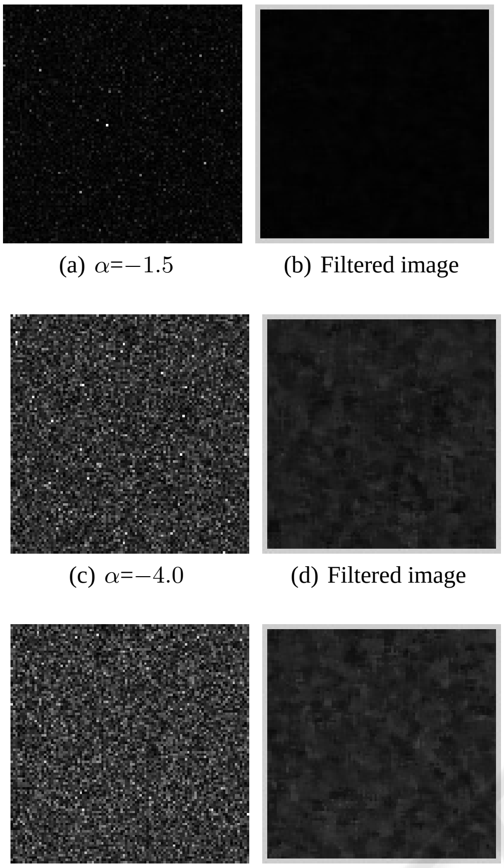
An important task in statistical analysis is the characterization of the mean value and the variability of a data set. To this end, the behaviour of some statistics for filtered and non-filtered images, are compared. Fifteen 100×100 images of $\mathcal{G}^0$ distributed data were generated using the values of α and γ given in Table 1. In order to design the stack filter, an image formed with the mean values of each region, as described in section 3, was used.

Table 1: Values of α and γ used in the experiment.

α	γ
-1.5	1.00000
-2.0	1.62114
-2.5	2.25000
-3.0	2.88202
-3.5	3.51562
-4.0	4.15012
-4.5	4.78516
-5.0	5.42056
-5.5	6.05621
-6.0	6.69205
-6.5	7.32802
-7.0	7.96409
-7.5	8.60024
-8.0	9.23646
-8.5	9.87273

Examples of these images are shown in Figure 6. This figure shows the results of applying a 5×5 window size filter to a group of images. Figures 6(a), 6(c) and 6(e) correspond to the original speckled images and figures 6(b), 6(d) and 6(f) correspond to the filtered images.

A statistical tool for characterizing the signal to noise ratio is the variation coefficient, defined as the quotient between the standard deviation and the mean value: C_V=σ/μ. The values of this coefficient for filtered and non-filtered images are given in Table 2. The values corresponding to the filtered images are lower than the values corresponding to the non-filtered images, indicating that the effect of filtering was a decrease in the speckle noise. It can also be seen that the value of the variation coefficient C_V is



(a) α=-1.5 (b) Filtered image

(c) α=-4.0 (d) Filtered image

(e) α=-8.5 (f) Filtered image

Figure 6: Synthetic images (left) and their corresponding filtered images (right).

lower when the α parameter is lower for both, filtered and non-filtered images. In order to assess the normality of the data, the skewness and kurtosis statistics were used. They are defined by

$$SK = \frac{\sum_{i=1}^{N}(Y_i - \bar{Y})^3}{(N-1)s^3}$$
$$K = \frac{\sum_{i=1}^{N}(Y_i - \bar{Y})^4}{(N-1)s^4}$$

where, $Y_1, Y_2, \ldots, Y_N$ are univariate data, $\bar{Y}$ is the mean value, s is the standard deviation, and N is the sample size. For a normal distribution, the skewness SK is zero (symmetric density) and the kurtosis K is 3.

The distribution of the data is modified when they are filtered. If we consider the skewness and the kurtosis as a measure of asymmetry and peakedness between the data and the density, we can state that the data are more Gaussian after filtering. This can be seen in Tables 3 and 4 where the skewness and the kurtosis for each α value are shown.

Table 2: Variation coefficients.

α	C_V non-filtered images	C_V filtered images
-1.5	1.020400	0.327207
-2.0	0.803039	0.251616
-2.5	0.738860	0.256685
-3.0	0.685162	0.240635
-3.5	0.668856	0.239666
-4.0	0.637395	0.230422
-4.5	0.612268	0.223510
-5.0	0.613924	0.227741
-5.5	0.598406	0.224422
-6.0	0.591566	0.221399
-6.5	0.588123	0.217759
-7.0	0.571000	0.215808
-7.5	0.585607	0.216695
-8.0	0.577546	0.221254
-8.5	0.577740	0.216642

It can be noticed that, the more heterogeneous the non-filtered data are, that is α closer to 0, the higher the values for the skewness and kurtosis.

Table 3: Skewness.

α	Skewness non-filtered images	Skewness filtered images
-1.5	5.79204	0.542360
-2.0	2.71243	0.452572
-2.5	2.17279	0.354620
-3.0	2.06542	0.404203
-3.5	1.79158	0.396256
-4.0	1.56213	0.287269
-4.5	1.30829	0.283221
-5.0	1.18022	0.339163
-5.5	1.15137	0.323194
-6.0	1.15225	0.261475
-6.5	1.03733	0.322413
-7.0	0.972302	0.381224
-7.5	0.990236	0.317580
-8.0	0.964581	0.381402
-8.5	0.904661	0.286144

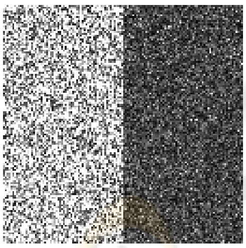
5.2 Maximum Likelihood Classification

Finally, ten 128×128 images were generated with two regions with $\alpha = -1.5$, $\gamma = 1$ for the left side, $\alpha = -10$, $\gamma = 1$ for the right side and $n = 1$ for both sides. The influence of stack filtering on maximum likelihood classification performance was studied. Figure 7 shows an example of a classified image with and without filtering.

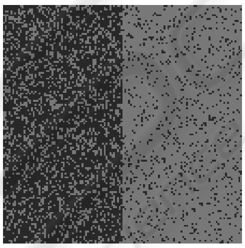
Table 4: Kurtosis.

α	Kurtosis non-filtered images	Kurtosis filtered images
-1.5	81.5878	1.05008
-2.0	14.7974	0.797948
-2.5	9.90102	0.330502
-3.0	10.2707	0.229317
-3.5	7.36229	0.852436
-4.0	5.14167	0.117459
-4.5	3.10693	0.318509
-5.0	2.43030	0.145160
-5.5	2.35810	0.123555
-6.0	2.79451	0.297602
-6.5	1.93492	0.233789
-7.0	1.39388	0.328363
-7.5	1.66122	0.238216
-8.0	1.36633	0.299916
-8.5	0.998106	0.0621805

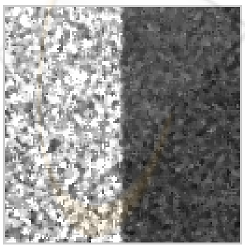
Table 5 shows the corresponding confusion matrix: odd lines correspond to non-filtered images and even lines correspond to filtered images. Data must be read as follows: R_i/R_j means the percentage of pixels that belong to region R_j but were classified into region R_i. I_k and F_k correspond to the non-filtered image and to the filtered imge, respectively. From these values it can be seen that the classification performance was better for filtered images than for non- filtered images.



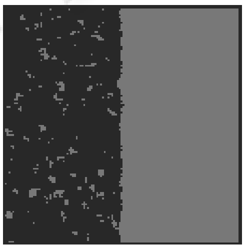
(a) Original image



(b) Image 7(a) classified



(c) Filtered image



(d) Image 7(c) classified

Figure 7: Synthetic, filtered and classified images.

Table 5: Confusion matrix.

Imagen	R_1/R_1	R_2/R_1	R_2/R_2	R_1/R_2
I_0	71.46	28.54	89.27	10.73
F_0	93.57	6.43	94.60	5.41
I_1	70.69	29.31	89.55	10.45
F_1	93.35	6.65	94.60	5.39
I_2	72.51	27.49	89.59	10.41
F_2	93.48	6.52	94.57	5.43
I_3	71.94	28.05	89.66	10.34
F_3	92.43	7.57	94.60	5.41
I_4	71.58	28.42	89.05	10.95
F_4	92.21	7.79	94.58	5.42
I_5	71.67	28.33	89.72	10.28
F_5	93.10	6.90	94.48	5.52
I_6	71.09	28.91	89.46	10.53
F_6	93.35	6.65	94.59	5.41
I_7	70.76	29.24	88.49	11.51
F_7	92.87	7.13	94.57	5.43
I_8	71.53	28.47	89.53	10.47
F_8	91.99	8.01	94.59	5.41
I_9	71.81	28.19	89.37	10.63
F_9	91.76	8.24	94.52	5.48

6 CONCLUSION

In this work, a study on the behaviour of adaptive stack filters, applied to synthetic aperture radar (SAR) images, was performed. The fundamentals of stack filters were presented. These filters were applied to simulated SAR images generated using the $\mathcal{G}^0$ law and were based on the algorithm proposed by (Yoo et al., 1999). The statistical distribution of the filtered data was studied, and it was observed that they no longer followed the above mentioned distribution. We could infer that it turned out to be more similar to the Gaussian distribution. Filtered and non-filtered images were classified according to the maximum likelihood criterion. The classification performance was better for filtered images than for non-filtered images.

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