TRACKING MULTIPLE OBJECTS USING THE VITERBI ALGORITHM

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Abstract: Tracking multiple targets is a great challenge for most tracking algorithms, since these algorithms tend to loose some of the targets when they get close to each other. Hence, several algorithms like the MHT, the JPDAF and the PMHT have been developed for this task. However, these algorithms are specialized on punctiform targets, whereas in mobile robotics one has to deal with extended targets. Therefore, in this paper an algorithm is proposed that can solve this problem. It uses the Viterbi algorithm and some geometrical characteristics of the problem. The proposed algorithm was tested with real world data.

1 INTRODUCTION AND RELATED WORK

For many real world applications it is essential that a robot is able to interact with its environment. This is true for multi–robot systems where a group of robots has to solve a given task or where robots are supposed to support people. For such situations, the awareness of the position of people and other robots is a fundamental ability for a mobile unit to be able to interact with its environment in an appropriate way.

This problem can be analyzed under the superordinate concept of tracking. Tracking denotes the estimation of the position of an object based on consecutive sensor measurements. It is well studied in the field of aerial surveillance with radar devices (Bar-Shalom and Fortmann, 1988). In the area of mobile robots tracking is also a well established research topic (Prassler et al., 1999; Schulz et al., 2001; Fod et al., 2002; Fuerstenberg et al., 2002). In mobile robotics laser range scanners are one of the preferred sensor devices. A Sick laser range scanner for example can measure the distance to the next reflecting obstacle with a high angular resolution of e.g. 0.25 degree. Lasers have rapidly gained popularity for mobile robotic applications such as collision avoidance, navigation, localization and map building (Thrun, 1998).

The problem of tracking people and other objects in densely populated environments with a robot–borne laser scanner can be characterized in the following way: most of the readings are from obstacles like walls or other objects and only a few measurements come from the tracked object itself. The problem of allocation of data obtained from the presently accounted target is called the data association problem (Bar-Shalom and Fortmann, 1988). As a solution to this problem, a tracking algorithm might use a validation gate which separates the signals belonging to the current target from other signals. A second characteristic of tracking people with laser range scanners is the occurrence of several measurements from the same object. In contrast to common radar based tracking sensors the Sick laser scanner has a much higher resolution and refresh rate. This leads to the fact that the tracked object generates several measurements. Therefore, we have to deal with what we call extended objects instead of punctiform objects like in the common radar tracking literature. Thereby, punctiform targets are those ones, which are the origin of just one measurement. A third characteristic of tracking in the field of mobile robotics is the occurrence of crossing targets. This means that two or more targets get very close to each other, so that they cannot be separated by common tracking algorithms (Fortmann et al., 1983), (Kräußling et al., 2005). This situation can appear e.g. when two humans meet, talk to each other and split again and is a well known problem in mobile robotics (Prassler et al., 1999).

There are several methods for tracking punctiform

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crossing targets in clutter:

1. the MHT (Multi Hypothesis Tracker) introduced by Reid in 1979 (Reid, 1979).
2. the JPDAF (Joint Probabilistic Data Association Filter) introduced by Fortmann, Bar–Shalom and Scheffe in 1983 (Fortmann et al., 1983).
3. the PMHT (Probabilistic Multi Hypothesis Tracker) introduced by Streit and Luginbuhl in 1994 (Streit and Luginbuhl, 1994).

Of course, an extension of these algorithms for tracking extended crossing objects is straightforward. But there are several reasons, why such approaches might fail:

- in most cases there are several measurements from the same target.
- the crossing can last for a longer time period.
- one of the objects might be occluded by the other object for some time.
- the objects can accomplish very abrupt maneuvers during the crossing or especially at the end of the crossing.

As an example we give the results for an EM based method (Stannus et al., 2004), which is an extension of the PMHT (Streit and Luginbuhl, 1994) to extended targets. Figure 1 shows the results when applying this algorithm to real data. It shows the estimates for the position of the objects by use of ellipses. Thereby the estimated position is the centre of the ellipse, whereas the shape of the ellipse represents the actual geometry of the tracked object. The objects start in the left and move to the right. Obviously, the algorithm looses the one of the targets, which moves to the upper right after the crossing. See also figure 2. Moreover, the computational burden for applying these algorithms is very high when applied to extended targets, since these objects can be the origin of up to ten measurements. These difficulties are well known in the mobile robotics community:

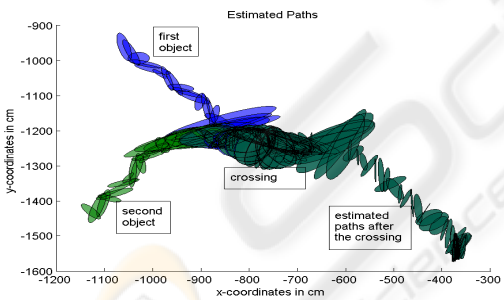


Figure 1: Crossing of two objects, real data, EM based method.

- Tracking moving objects whose trajectories cross each other is a very general problem ... Problems of this type cannot be eliminated even by more sophisticated methods ... (Prassler et al., 1999).
- Tracks are lost when people walk too closely together ... (Schumitch et al., 2006).

With respect to these circumstances our research group has developed a new method that solves the problem of tracking two extended crossing targets (Kräußling et al., 2004b; Kräußling et al., 2005). Unfortunately this algorithm is not able to deal with more than two crossing objects in its current state. Therefore, recently our research group has developed a more general algorithm, which is able to deal with an arbitrary number of crossing objects, i.e. objects that get very close to each other. This algorithm is the main contribution of this work.

The remainder of this paper is organized as follows. In section 2 the mathematical background including the used model, the validation gate, and two basic tracking algorithms are given. In section 3 the new algorithm for tracking multiple interacting objects is introduced. Furthermore, in section 4 the experimental results are presented. Finally, section 5 contains the conclusions.

2 THE MATHEMATICAL BACKGROUND OF THE ALGORITHMS

2.1 The Model

The dynamics of the objects to be observed and the observation process itself are modeled by a hidden Gauß–Markov chain with the equations

$$x_k = Ax_{k-1} + w_{k-1} \tag{1}$$

and

$$z_k = Bx_k + v_k. \tag{2}$$

x_k is the object state vector at time k, A is the state transition matrix, z_k is the observation vector at time k and B is the observation matrix. Furthermore, w_k and v_k are supposed to be uncorrelated zero mean white Gaussian noises with covariances Q and R.

Since the motion of a target in the plane has to be described a two dimensional kinematic model is used. Therefore, it is

$$x_k = \begin{pmatrix} x_{k1} & x_{k2} & \dot{x}_{k1} & \dot{x}_{k2} \end{pmatrix}^\top \tag{3}$$

with x_{k1} and x_{k2} the Cartesian coordinates of the target and $\dot{x}_{k1}$ and $\dot{x}_{k2}$ the corresponding velocities. z_k just gives the Cartesian coordinates of the target. For the coordinates the equation of a movement with constant velocity is holding, i.e. it is

$$x_{k+1,j} = x_{kj} + \Delta T \dot{x}_{kj}. \quad (4)$$

ΔT is the time interval between two consecutive measurements. For the progression of the velocities we use the equation

$$\dot{x}_{k+1,j} = e^{-\Delta T/\Theta}\dot{x}_{kj} + \Sigma\sqrt{1-e^{-2\Delta T/\Theta}}u(k) \quad (5)$$

from (van Keuk, 1971) with the zero mean white Gaussian noise $u(k)$ with $E[u(m)u(n)^\top] = \delta_{mn}$. Thus, the velocity is supposed to decline exponentially. The term

$$\Sigma\sqrt{1-e^{-2\Delta T/\Theta}}u(k) \quad (6)$$

models the process noise and the accelerations.

2.2 The Validation Gate

The validation gate is realized using the Kalman filter. The Kalman filter calculates a prediction $y(k+1|k)$ for the measurements $z_{k+1,l}$ from the actually handled target at time step $k+1$ via the formula

$$y(k+1|k) = B \cdot A \cdot x(k|k). \quad (7)$$

$x(k|k)$ is the estimate for the position of the target at time step k. For every sensor reading $z_{k+1,l}$ of the time step $k+1$ ($l = 1, \ldots 360$) the Mahalanobis distance λ (Mahalanobis, 1936) with

$$\lambda = (z_{k+1,l} - y(k+1|k))^\top \cdot [S(k+1)]^{-1} \cdot$$
$$\cdot (z_{k+1,l} - y(k+1|k)) \quad (8)$$

is computed. Then all measurements with $\lambda > \lambda_{max}$ with a given threshold λ_{max} are excluded. See (Bar-Shalom and Fortmann, 1988) for further details. This procedure results in a set $\{\hat{z}_{k+1,i}\}_{i=1}^{m_{k+1}}$ of m_{k+1} selected measurements $\hat{z}_{k+1,i}$. The matrix $S(k+1)$ is the innovations covariance from the Kalman filter. In common filter applications this matrix is calculated from the predictions covariance $P(k+1|k)$ with the equation

$$S(k+1) = BP(k+1|k)B^\top + R \quad (9)$$

with the given covariance matrix R of the measurement noise. But for tracking extended objects this approach is not sufficient, since there is an additional influence of the extendedness of the object to the deviation of the measurements from the prediction $y(k+1|k)$. To take care of this feature an accessory positive definite matrix E should be added in equation 9. Otherwise a lot of the measurements from the target would be excluded by the gating process. Because the lateral dimension of people usually shows a radius in the range of $30\,cm$, the entries of E should be in the range of 900. Thus, after some optimization process we used

$$E = \begin{pmatrix} 780 & 0 \\ 0 & 780 \end{pmatrix} \quad (10)$$

and

$$S(k+1) = BP(k+1|k)B^\top + R + E. \quad (11)$$

The values of the entries of the matrix E vastly exceed the values of the entries of the matrix R, so that the main contribution in equation 11 comes from the matrix E.

2.3 The Kalman Filter Algorithm with Equal Weights

This algorithm first calculates an unweighted mean z_{k+1} of the m_{k+1} measurements $\{\hat{z}_{k+1,l}\}_{l=1}^{m_{k+1}}$, that have been selected by the gate, i.e. it is

$$z_{k+1} = \frac{1}{m_{k+1}} \sum_{l=1}^{m_{k+1}} \hat{z}_{k+1,l}. \quad (12)$$

This mean is used as the input for the update equation of the Kalman filter, i.e. it is

$$x(k+1|k+1) = x(k+1|k) + K_{k+1}(z_{k+1} - y(k+1|k)) \quad (13)$$

with the predictions $x(k+1|k)$ and $y(k+1|k)$ and the Kalman gain K_{k+1} derived from the Kalman filter. Finally, the estimates $x(k+1|k+1)$ are further improved by the use of the Kalman smoother (Shumway and Stoffer, 2000). The corresponding algorithm is called Kalman filter algorithm (KFA). As has been shown in (Kräußling et al., 2005) it is very fast and gives good information about the position of the target, but cannot reproduce multimodal probability distributions. Thus it is not able to handle multiple interacting targets.

2.4 The Viterbi Based Algorithm

The Viterbi algorithm has been introduced in (Viterbi, 1967). A good description is also given in (Forney Jr., 1973). It has been recommended for tracking punctiform targets in clutter in (Quach and Farooq, 1994) and for tracking extended targets in (Kräußling et al., 2004a). It calculates for each selected measurement $\hat{z}_{k+1,i}$ a separate estimate $x(k+1|k+1)_i$. For the calculation of the estimates $x(k+1|k+1)_i$ in the update equation 13 the measurement $\hat{z}_{k+1,i}$ and the predictions $x(k+1|k)_j$ and $y(k+1|k)_j$ from the predecessor j are used. When tracking punctiform targets in clutter, the predecessor is determined by minimizing the length of the paths ending in $\hat{z}_{k+1,i}$. When regarding extended targets in most cases all measurements in the validation gate are from the target, so that it is not meaningful to consider the lengths of the paths ending in the possible predecessors $\hat{z}_{k,j}$ when determining the predecessor. Therefore, a better choice for the predecessor is that one for which the Mahalanobis distance (Mahalanobis, 1936)

$$\nu_{k+1,j,i}^{\top}[S(k+1)]^{-1}\nu_{k+1,j,i} \qquad (14)$$

is kept to a minimum. There, $\nu_{k+1,j,i}$ is the innovation

$$\nu_{k+1,j,i} = z_{k+1,i} - y(k+1|k)_j \qquad (15)$$

and $S(k+1)$ is the innovations covariance. This procedure is similar to a nearest neighbour algorithm. When applying the Viterbi algorithm the application of the validation gate is performed in the following way. At first for every selected measurement $\hat{z}_{k,j}$ the gate is applied to the measurements at time $k+1$. That results in the sets $Z_{k+1,j}$ of measurements which have passed the particular gate for the measurement $\hat{z}_{k,j}$ successfully. The set of all measurements $\hat{z}_{k+1,i}$, that are associated with the target, is then just the union of these sets. By this procedure it is ensured, that the corresponding tracking algorithms can deal with multimodal probability distributions to some extend, which is a major improvement when dealing with multiple interacting targets.

The estimates delivered by the Viterbi algorithm are used as follows. One of these estimates is chosen as an estimate of the position of a target. This estimate is the one with index one. Again, different from tracking a punctiform target in clutter, it is not meaningful to make use of the lengths of the paths corresponding to the estimates and to choose the estimate with the shortest corresponding path. The corresponding algorithm is called Viterbi based algorithm (VBA) and has been introduced in (Kräußling et al., 2004a). The disadvantages of this algorithm are a great computational burden and a poor information about the position of the tracked objects (Kräußling et al., 2005).

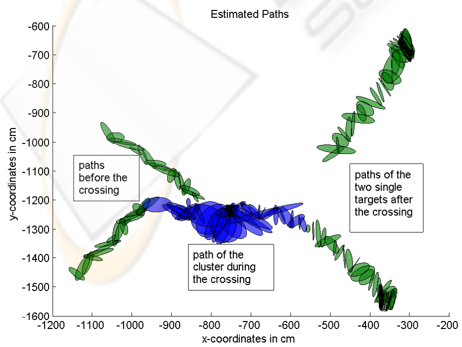
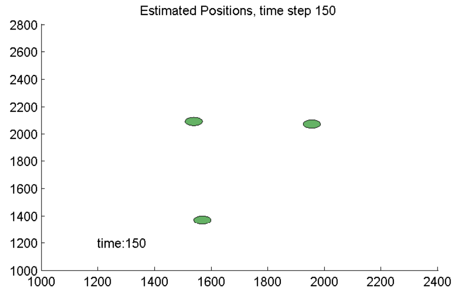
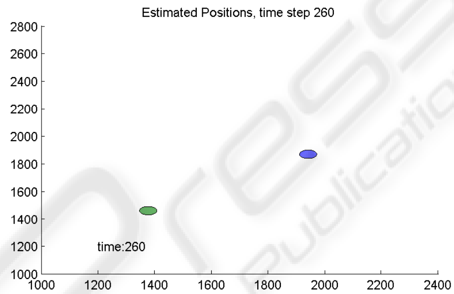


Figure 2: Two crossing targets.



(a) The algorithm tracks three distinct, moving targets.



(b) Two of the targets get in close proximity of each other, and are therefore represented by one cluster.

Figure 3: First Experiment.

3 AN ALGORITHM FOR TRACKING MULTIPLE INTERACTING OBJECTS

Our new algorithm uses two classes of objects:

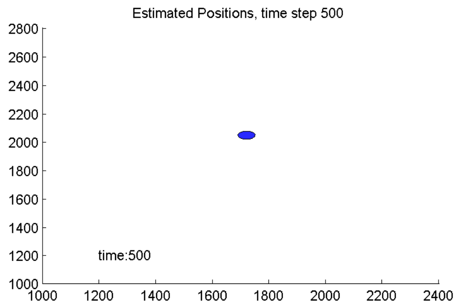
- single targets.
- clusters, which represent at least two interacting objects, i.e. objects that are moving very close to each other.

Single targets are tracked with the KFA, since there is no need for representing multimodal probability distributions. Furthermore, this algorithm is very fast and gives very accurate information about the position of the tracked object.

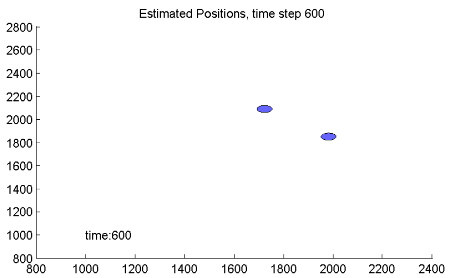
Clusters are tracked with the VBA, since multimodal distributions have to be represented. This approach guarantees that none of the objects that are associated with the cluster is lost. This fact is important especially when the objects split and start to move separately again.

Three different events have to be regarded when tracking multiple interacting objects:

1. The merging of two single targets. This means that two single targets get very close to each other. This is the case, if at least one measurement lies in the

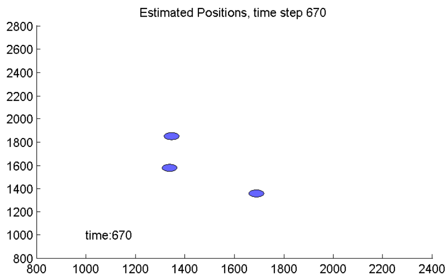


(a) The third target joins the other two and is also associated with the cluster.

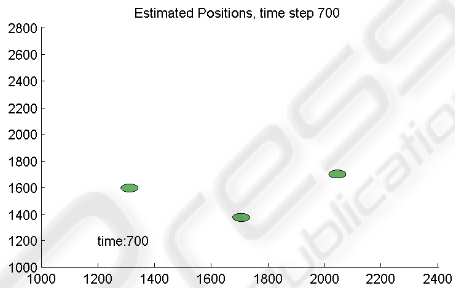


(b) One of the targets separates from the others, thus we can track subclusters.

Figure 4: First Experiment.



(a) The other two targets also separate and therefore three subclusters are tracked.



(b) Once these subclusters leave each others proximity, the single targets can be tracked.

Figure 5: First Experiment.

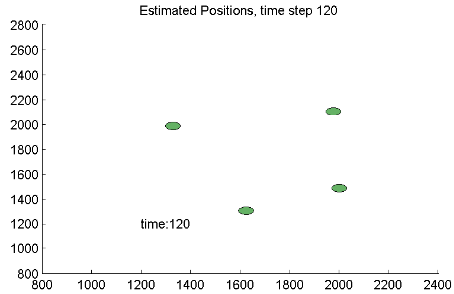
validation gates of both targets. Then the algorithm stops to track the two single targets with the KFA and starts tracking a cluster, which contains both targets, using the VBA. Therefore, it uses the measurements lying in the validation gates of both targets.

2. The merging of a single target and a cluster. This means that a single target and a cluster get very close to each other. This happens, if at least one measurement lies in the validation gates of the target and the cluster. In this case the algorithm stops to track the single target and the cluster separately. Instead it starts tracking a combined cluster. Therefore, it uses the measurements lying in the validation gates of both the single target and the previously considered cluster.
3. The merging of two clusters. This means that two clusters get very close to each other. This is the case, if at least one measurement lies in the validation gates of both clusters. If this is true, the algorithm stops to track the two clusters and starts tracking a combined cluster. Therefore, it uses the measurements lying in the validation gates of both previously considered clusters.

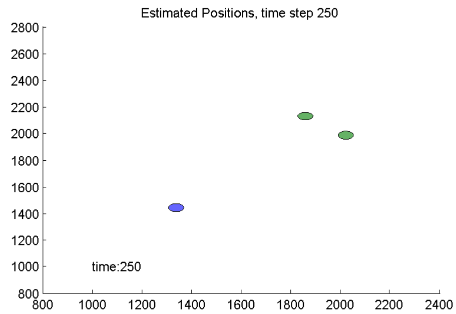
Note, that whenever a merging takes place, the algorithm remembers the single targets which correspond to the new combined cluster.

For each tracked cluster, we have to detect if it disperses into its single targets. For this, we define three conditions, that are checked by the algorithm:

1. The position estimates corresponding to the measurements in the validation gates are separated into subclusters. For this purpose, we select the first estimate, which then is associated with the first subcluster. For all other estimates associated with the cluster, the Euclidian distance to the first estimate is calculated. If this distance is below a certain threshold, the estimate is associated with the first subcluster. In our experiments, we set the threshold to $150\,cm$, which corresponds to the maximum distance between the legs of a walking person. We then have to consider the estimates, for which the Euclidean distance to the first subcluster exceeds this manually chosen threshold. Using the same procedure we applied for building the first subcluster, we now construct subclusters until all estimates are associated with one of these smaller clusters. If the number of subclusters equals the number of targets that were merged into this cluster, the first condition for the dispersion of the cluster is fulfilled. Then, we proceed with step 2.
2. We now check the distance between the subclusters pairwise. If the distance is above a manually chosen bound, we regard these clusters as separated.

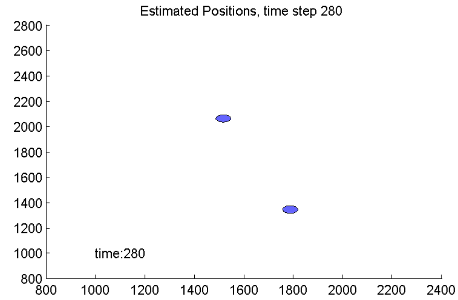


(a) The algorithm tracks four distinct, moving targets.

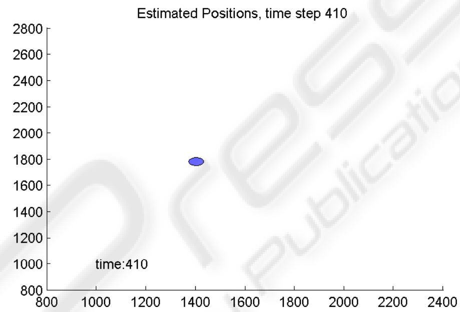


(b) Two of the targets get in close proximity of each other, and are therefore represented by one cluster.

Figure 6: Second Experiment.



(a) The third and fourth target get close to each other and are also represented by a cluster.



(b) The two clusters join and thus all four targets are associated with one cluster.

Figure 7: Second Experiment.

We chose the value of that bound to be $300\,cm$. The second condition is fulfilled, if the number of pairs of separated subclusters equals $\frac{n(n-1)}{2}$. Thereby, n is the number of single targets associated with the cluster. Hence, we are checking if all subclusters are pairwise separated.

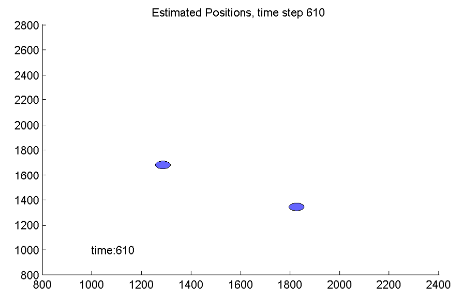
3. Above this, we can separate single subclusters from the cluster. This follows the same logic as in step 1. Note, the algorithm is not able to determine, how many targets are represented by a single subcluster.

If conditions 1 and 2 are met, the n subclusters are associated with the n single targets, that are therefrom tracked with the KFA. When separating targets from clusters, we cannot guarantee if the target association is the same as before merging the targets into the cluster. We believe that there is no solution to this problem, if we exclusively use anonymous sensors like laser range finders.

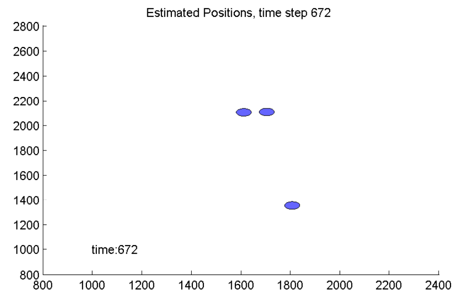
4 EXPERIMENTS

The presented algorithm was tested with real world data, recorded in our laboratory. In this section we show the data of two experiments. Before this, figure 2 illustrates how our algorithm works in the case of two targets using the data from figure 1. In the figures in this section single targets are indicated by a green ellipse, whereas clusters are indicated by a blue ellipse. As soon as the above mentioned conditions are met, we are able to track the two targets separately again.

In the first experiment, three persons moved around the observer in counterclockwise direction. At first, these three persons are tracked separately and thus, the algorithm uses the KFA (figure 3(a)). Then, two of the single targets get into close proximity of each other and are therefore merged into one cluster, which is tracked using the VBA (figure 3(b)). In the next figure, the third person joins the group and is also associated with the cluster. This combined cluster is still tracked by the VBA (figure 4(a)). Next, the combined cluster split after a while into two subclusters which are indicated in figure 4(b). Then, the cluster split into three subclusters, which are indicated in figure 5(a). At this stage of the experiment, condition one is met, but not condition two. Finally, the three persons split up fulfilling condition two and the persons are tracked as single targets again (figure 5(b)). Since we are not able to tell how many targets are associated with a subcluster in general, only the total number of targets

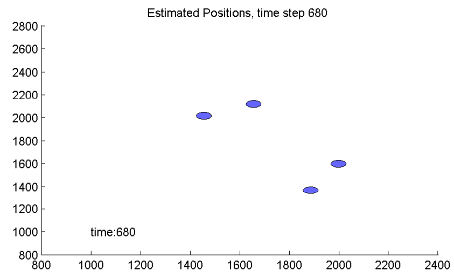


(a) The combined cluster splits after a while into two subclusters, each of them consisting of an unknown number of targets.

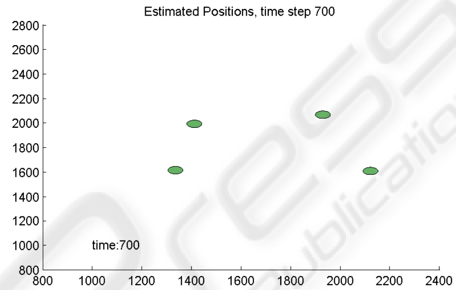


(b) One of the subclusters splits into two subclusters.

Figure 8: Second Experiment.



(a) The targets split after a while and are therefrom tracked as subclusters.



(b) Then, the targets can be tracked individually.

Figure 9: Second Experiment.

is known. Thus, we have to wait until all targets are separated according to the defined conditions before the algorithm tracks the individual targets using the KFA.

In the second experiment, we let the algorithm track four people. All four persons started walking alone. Consequently, they are tracked independently using the KFA (figure 6(a)). Then two of them are walking together and are therefore tracked in a single cluster using the VBA (figure 6(b)), whereas the other two people are still walking separately. In the next step, the other two persons are also walking together. The result is that these two are also tracked using a single cluster and thus, the algorithm tracks two distinct clusters, each representing two persons (figure 7(a)). Then, in figure 7(b) all four of them are walking together, what results in the merging of the two clusters to one combined cluster, representing all four persons. Next, this combined cluster splits into two subclusters, each consisting of an unknown number of targets (figure 8(a)). In figures 8(b) and 9(a) these subclusters split up into four subclusters. Finally, condition two is met and therefore the four targets are tracked individually again (figure 9(b)).

In the experiments, we showed that our algorithm is able to keep track of all targets for all situations that can occur when tracking multiple targets:

- the merging of two single targets to a cluster.
- the merging of a single target and a cluster.
- the merging of two clusters to a combined cluster.
- the splitting of a cluster into subclusters which are indicated separately in the graphics.
- the splitting of several targets tracked in a cluster to single targets.

5 CONCLUSIONS

In this work we presented a novel algorithm for tracking multiple extended interacting objects. It consists of a Kalman filter tracking procedure for tracking single targets with high accuracy and low computational effort, and a Viterbi based method for tracking objects that are close to each other. The latter part facilitates clustering to be able to separate the targets correctly if they split up.

In experiments we showed this algorithm to be capable of tracking multiple crossing targets without any restriction. Of course, since we only use laser range finders, the target association after a crossing may be interchanged. There are several approaches that might be investigated to eliminate this drawback:

- different colours of the pairs of trousers people wear might result in different intensities of the reflected laser beams.

- people could wear badges sending out for instance infrared or ultrasound signals which can be received by specific sensors to identify the tracked people (Schulz et al., 2003).
- usually people wear clothing with different colours. This information could be exploited for the identification of the people using a camera network as has been proposed in (Schumitch et al., 2006).

Since, up to our knowledge, our method is the first to be able to track several interacting objects without loss of track, it might be challenging to combine our approach with one of these attempts.

So far, we only used a non moving observer. In principle it is possible to extend the method to be suitable for moving observers. This is part of the ongoing research and will be presented in following publications.

REFERENCES

Bar-Shalom, Y. and Fortmann, T. (1988). *Tracking and Data Association*. Academic Press.

Fod, A., Howard, A., and Mataric, M. J. (2002). Laser–based people tracking. In *Proceedings of the IEEE Intl. Conf. on Robotics and Automation*, pages 3024–3029.

Forney Jr., G.-D. (1973). The viterbi algorithm. *Proceedings of the IEEE*, 61(3):268–278.

Fortmann, T. E., Bar-Shalom, Y., and Scheffe, M. (1983). Sonar tracking of multiple targets using joint probabilistic data association. *IEEE Journal of Oceanic Engineering*, OE–8(3).

Fuerstenberg, K. C., Linzmeier, D. T., and Dietmayer, K. C. J. (2002). Pedestrian recognition and tracking of vehicles using a vehicle based multilayer laserscanner. In *Proceedings of IV 2002, Intelligent Vehicles Symposium*, volume 1, pages 31–35.

Kräußling, A., Schneider, F. E., and Wildermuth, D. (2004a). Tracking expanded objects using the viterbi algorithm. In *Proceedings of the IEEE Conference on Intelligent Systems, Varna, Bulgaria*.

Kräußling, A., Schneider, F. E., and Wildermuth, D. (2004b). Tracking of extended crossing objects using the viterbi algorithm. In *Proceedings of the 1st International Conference on Informatics in Control, Automation and Robotics (ICINCO)*.

Kräußling, A., Schneider, F. E., and Wildermuth, D. (2005). A switching algorithm for tracking extended targets. In *Proceedings of the 2nd International Conference on Informatics in Control, Automation and Robotics (ICINCO), also to be published in the Springer book of best papers of ICINCO 2005*.

Mahalanobis, P. C. (1936). On the generalized distance in statistics. *Proceedings of the National Institute of Science*, 12:49–55.

Prassler, E., Scholz, J., and Elfes, E. (1999). Tracking people in a railway station during rush–hour. In Christensen, H. I., editor, *Computer Vision Systems*, volume 1542, pages 162–179. Springer, lecture notes in computer science edition.

Quach, T. and Farooq, M. (1994). Maximum likelihood track formation with the viterbi algorithm. In *Proceedings of the 33rd Conference on Decision and Control, Lake Buena Vista, Florida*.

Reid, D. B. (1979). An algorithm for tracking multiple targets. *IEEE Trans. Automatic Control*, AC–24:843–854.

Schulz, D., Burgard, W., Fox, D., and Cremers, A. B. (2001). Tracking multiple moving objects with a mobile robot. In *Proceedings of the 2001 IEEE Computer Society Conference on Computer Vision and Pattern Recognition (CVPR 2001)*.

Schulz, D., Fox, D., and Hightower, J. (2003). People tracking with anonymous and id–sensors using rao–blackwellised particle filters. In *Proceedings of the 18th International Joint Conference on Artificial Intelligence (IJCAI 2003), Acapulco, Mexico*.

Schumitch, B., Thrun, S., Bradski, G., and Olukotun, K. (2006). The information–form data association filter. In *Proceedings of the 2005 Conference on Neural Information Processing Systems (NIPS), MIT Press.*

Shumway, R. H. and Stoffer, D. S. (2000). *Time Series Analysis and Its Applications*. Springer.

Stannus, W., Koch, W., and Kräußling, A. (2004). On robot–borne extended object tracking using the em algorithm. In *Proceedings of the 5th Symposium on Intelligent Autonomous Vehicles, Lisbon, Portugal*.

Streit, R. L. and Luginbuhl, T. E. (1994). Maximum likelihood method for multi–hypothesis tracking. *Signal and Data Processing of Small Targets, SPIE*, 2335.

Thrun, S. (1998). Learning metric-topological maps for indoor mobile robot navigation. *Artificial Intelligence*, 99(1):21–71.

van Keuk, G. (1971). Zielverfolgung nach kalman–anwendung auf elektronisches radar. Technical Report 173, Forschungsinstitut für Funk und Mathematik, Wachtberg–Werthhoven, Germany.

Viterbi, A. J. (1967). Error bounds for convolutional codes and an asymptotically optimum decoding algorithm. *IEEE Transactions On Information Theory*, IT–13(2).