KINEMATIC AND SINGULARITY ANALYSIS OF THE HYDRAULIC SHOULDER
A 3-DOF Redundant Parallel Manipulator

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Abstract: In this paper, kinematic modeling and singularity analysis of a three DOF redundant parallel manipulator has been elaborated in detail. It is known, that on the contrary to series manipulators, the forward kinematic map of parallel manipulators involves highly coupled nonlinear equations, whose closed-form solution derivation is a real challenge. This issue is of great importance noting that the forward kinematics solution is a key element in closed loop position control of parallel manipulators. Using the novel idea of kinematic chains recently developed for parallel manipulators, both inverse and forward kinematics of our parallel manipulator are fully developed, and a closed-form solution for the forward kinematic map of the parallel manipulator is derived. The closed form solution is also obtained in detail for the Jacobian of the mechanism and singularity analysis of the manipulator is performed based on the computed Jacobian.

1 INTRODUCTION

Over the last two decades, parallel manipulators have been among the most considerable research topics in the field of robotics. A parallel manipulator typically consists of a moving platform that is connected to a fixed base by several legs. The number of legs is at least equal to the number of degrees of freedom (DOF) of the moving platform so that each leg is driven by no more than one actuator, and all actuators can be mounted on or near the fixed base. These robots are now used in real-life applications such as force sensing robots, fine positioning devices, and medical applications (Merlet, 2002).

In the literature, mostly 6 DOF parallel mechanisms based on the Stewart-Gough platform are analyzed (Joshi et al., 2003). However, parallel manipulators with 3 DOF have been also implemented for applications where 6 DOF are not required, such as high-speed machine tools. Recently, 3 DOF parallel manipulators with more than three legs have been investigated, in which the additional legs separate the function of actuation from that of constraints at the cost of increased mechanical complexity (Joshi et al., 2003). Complete kinematic modeling and Jacobian analysis of such mechanisms have not received much attention so far and is still regarded as an interesting problem in parallel robotics research. It is known that unlike serial manipulators, inverse position kinematics for parallel robots is usually simple and straight-forward. In most cases joint variables may be computed independently using the given pose of the moving platform. The solution to this problem is in most cases uniquely determined. But forward kinematics of parallel manipulators is generally very complicated. Its solution usually involves systems of nonlinear equations which are highly coupled and in general have no closed form and unique solution. Different approaches are provided in literature to solve this problem either in general or in special cases. There are also several cases in which the solution to this problem is obtained for a special or novel architecture (Baron et al., 2000, Merlet, 96, Song et al., 2001, Bonev et al., 2001). Two such special 3 DOF constrained mechanisms have been studied in (Siciliano, 99, Fattah et al., 2000), where kinematics, Jacobian and dynamics have been considered for

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such manipulators. Joshi and Tsai (Joshi et al., 2003) performed a detailed comparison between a 3-UPU and the so called Tricept manipulator regarding the kinematic, workspace and stiffness properties of the mechanisms. In general, different solutions to the forward kinematics problem of parallel manipulators can be found using numerical or analytical approaches, or closed form solution for special architectures (Didrit et al., 98, Dasgupta et al., 2000).

In this paper, complete kinematic modeling has been performed and a closed-form forward kinematics solution is obtained for a three DOF actuator redundant hydraulic parallel manipulator. The mechanism is designed by Hayward (Hayward, 94), borrowing design ideas from biological manipulators particularly the biological shoulder. The interesting features of this mechanism and its similarity to human shoulder have made its design unique, which can serve as a basis for a good experimental setup for parallel robot research.

In a former study by the authors, different numerical approaches have been used to solve the forward kinematic map of this manipulator (Sadjadian et al., 2004). The numerical approaches are an alternative to estimate the forward kinematic solution, in case such solutions cannot be obtained in closed form. In this paper, however, the novel idea of kinematic chains developed for parallel manipulators structures (Siciliano, 99, Fattah et al., 2000), is applied for our manipulator, and it is observed that in a systematic manner the closed form solution for this manipulator can also be obtained in detail.

The paper is organized as following: Section 2 contains the mechanism description. Kinematic modeling of the manipulator is discussed in section 3, where inverse and forward kinematics is studied and the need for appropriate method to solve the forward kinematics is justified. In section 4, The Jacobian matrix of the manipulator is derived through a complete velocity analysis of the mechanism, and finally, in section 5, singularity analysis is performed using the configuration-dependent Jacobian.

2 MECHANISM ESCRIPTION

A schematic of the mechanism, which is currently under experimental studies in ARAS Robotics Lab, is shown in Fig. 1. The mobile platform is constrained to spherical motions. Four high performance hydraulic piston actuators are used to give three degrees of freedom in the mobile platform. Each actuator includes a position sensor of LVDT type and an embedded Hall Effect force sensor. The four limbs share an identical kinematic structure. A passive leg connects the fixed base to the moving platform by a spherical joint, which suppresses the pure translations of the moving platform. Simple elements like spherical and universal joints are used in the structure. A complete analysis of such a careful design will provide us with required characteristics regarding the structure itself, its performance, and the control algorithms.

From the structural point of view, the shoulder mechanism which, from now on, we call it "the Hydraulic Shoulder" falls into an important class of robotic mechanisms called parallel robots. In these robots, the end effector is connected to the base through several closed kinematic chains. The motivation behind using these types of robot manipulators was to compensate for the shortcomings of the conventional serial manipulators such as low precision, stiffness and load carrying capability. However, they have their own disadvantages, which are mainly smaller workspace and many singular configurations. The hydraulic shoulder, having a parallel structure, has the general features of these structures. It can be considered as a shoulder for a light weighed seven DOF robotic arm, which can carry loads several times, its own weight. Simple elements, used in this design, add to its lightness and simplicity. The workspace of such a mechanism can be considered as part of a spherical surface. The orientation angles are limited to vary between $-\pi/6$ and $\pi/6$. No sensors are available for measuring the orientation angles of the moving platform which justifies the importance of the forward kinematic map as a key element in feedback position control of the shoulder with the LVDT position sensors used as the output of such a control scheme.

3 KINEMATICS

Fig. 2 depicts a geometric model for the hydraulic shoulder manipulator which will be used for its kinematics derivation.

The parameters used in kinematics can be defined as:

$$l_b = \left\|\overrightarrow{CA_i}\right\|, \; l_p = \left\|\overrightarrow{CP}\right\| \quad l_d = \left\|\overrightarrow{PP_i}\right\|_{y4} \quad l_k = \left\|\overrightarrow{PP_i}\right\|_{z4}$$

α : The angle between CA_4 and y_0

C : Center of the reference frame

P : Center of the moving plate

l_i : Actuator lengths i=1, 2, 3, 4

P_i : Moving endpoints of the actuators

A_i : Fixed endpoints of the actuators

Figure 1: The hydraulic shoulder in movement

Two coordinate frames are defined for the purpose of analysis. The base coordinate frame {A}: $x_0y_0z_0$ is attached to the fixed base at point C (rotation center) with its z_0-axis perpendicular to the plane defined by the actuator base points $A_1A_2A_3A_4$ and an x_0-axis parallel to the bisector of angle $\angle A_1CA_4$. The second coordinate frame {B}: $x_4y_4z_4$ is attached to the center of the moving platform P with its z-axis perpendicular to the line defined by the actuators moving end points (P_1P_2) along the passive leg. Note that we have assumed that the actuator fixed endpoints lie on the same plane as the rotation center C. The position of the moving platform center P is defined by:

$$^{A}p = [p_x, p_y, p_z]^T \tag{1}$$

Also, a rotation matrix $^{A}R_B$ is used to define the orientation of the moving platform with respect to the base frame:

$$^{A}R_B = R_z(\theta_z)R_y(\theta_y)R_x(\theta_x)$$
$$= \begin{bmatrix} c\theta_z c\theta_y & c\theta_z s\theta_y s\theta_x - s\theta_z c\theta_x & c\theta_z s\theta_y c\theta_x + s\theta_z s\theta_x \\ s\theta_z c\theta_y & s\theta_z s\theta_y s\theta_x + c\theta_z c\theta_x & s\theta_z s\theta_y c\theta_x - c\theta_z s\theta_x \\ -s\theta_y & c\theta_y s\theta_x & c\theta_y c\theta_x \end{bmatrix} \tag{2}$$

where $\theta_x, \theta_y, \theta_z$ are the orientation angles of the moving platform denoting rotations of the moving frame about the fixed $x, y,$ and z axes respectively. Also $c\theta$ and $s\theta$ denote $\cos(\theta)$ and $\sin(\theta)$ respectively.

With the above definitions, the 4×4 transformation matrix $^{A}T_B$ is easily found to be:

$$^{A}T_B = \begin{bmatrix} ^{A}R_B & ^{A}p \\ \mathbf{0} & 1 \end{bmatrix} \tag{3}$$

Hence, the position and orientation of the moving platform are completely defined by six variables, from which, only three orientation angles $\theta_x, \theta_y, \theta_z$ are independently specified as the task space variables of the hydraulic shoulder.

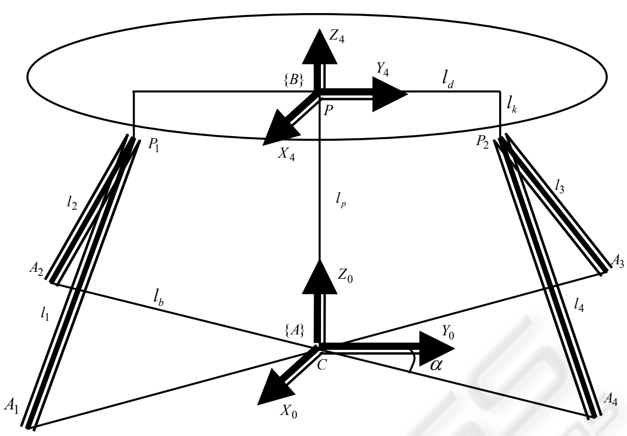

Figure 2: A geometric model for the hydraulic shoulder manipulator

3.1 Inverse Kinematics

In modeling the inverse kinematics of the hydraulic shoulder we must determine actuator lengths (l_i) as the actuator space variables given the task space variables $\theta_x, \theta_y, \theta_z$ as the orientation angles of the moving platform. First, note that the passive leg connecting the center of the rotation to the moving platform can be viewed as a 3-DOF open-loop chain by defining three joint variables $\theta_1, \theta_2,$ and θ_3 as the joint space variables of the hydraulic shoulder. Hence, applying the Denavit-Hartenberg (D-H) convention, the transformation $^{A}T_B$ can also be written as:

$$^{A}T_B = {}^{A}T_1(\theta_1).{}^{1}T_2(\theta_2).{}^{2}T_3(\theta_3).{}^{3}T_B \tag{4}$$

The D-H transformation matrices $^{i}T_j$ are computed using the coordinate systems for the passive leg in Fig. 3, according to the D-H convention. As shown in Fig. 3, the x_0 axis of frame {A} points along the first joint axis of the passive leg; the first link frame is attached to the first moving link with its x_1 axis pointing along the second joint axis of the passive leg; the second link frame is attached to the second moving link with its x_2 axis pointing along the third joint axis of the passive leg; and the third link frame is attached to the moving platform in accordance with the D-H convention. Using the above frames, the D-H parameters of the passive leg are found as in Table (1).

Table 1: D-H Parameters for the passive supporting leg

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	90°	0	0	θ_1
2	90°	0	0	θ_2
3	90°	0	0	θ_3
B	0	0	l_p	0

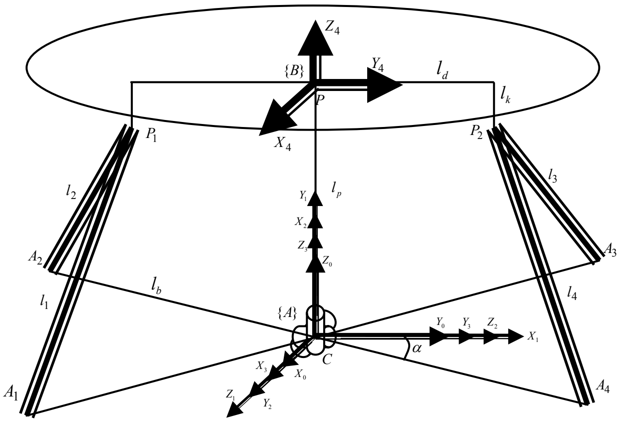

Figure 3: D-H Frame attachments for the passive supporting leg

Using the D-H parameters in Table (1), the D-H transformation matrices in (4) can be found as:

$$
{}^{1}T_2 = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ s\theta_2 & c\theta_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^{A}T_1 = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},
$$
$$
{}^{2}T_3 = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ s\theta_3 & c\theta_3 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad {}^{3}T_B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & l_p \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (5)
$$

Substituting (5) into (4) yields:

$$
{}^{A}T_B = \begin{bmatrix} {}^{A}R_B & {}^{A}p \\ \mathbf{0} & 1 \end{bmatrix} \qquad (6)
$$

Where:

$$
{}^{A}R_B = \begin{bmatrix} c\theta_1 c\theta_2 c\theta_3 + s\theta_1 s\theta_3 & -c\theta_1 c\theta_2 s\theta_3 + s\theta_1 c\theta_3 & c\theta_1 s\theta_2 \\ -s\theta_2 c\theta_3 & s\theta_2 s\theta_3 & c\theta_2 \\ s\theta_1 c\theta_2 c\theta_3 - c\theta_1 s\theta_3 & -s\theta_1 c\theta_2 s\theta_3 - c\theta_1 c\theta_3 & s\theta_1 s\theta_2 \end{bmatrix} \qquad (7)
$$

and:

$$
{}^{A}p = \begin{bmatrix} l_p c\theta_1 s\theta_2 & l_p c\theta_2 & l_p s\theta_1 s\theta_2 \end{bmatrix}^T \qquad (8)
$$

For the inverse kinematics, the three independent orientation angles in (2) are given. Hence, equating (2) to (7) yields:

$$
\theta_2 = \cos^{-1}({}^{A}R_{B(2,3)}) = \cos^{-1}(s\theta_z s\theta_y c\theta_x - c\theta_z s\theta_x) \qquad (9)
$$

Once θ_2 is known, we can solve for θ_1 and θ_3 as:

$$
\theta_1 = A\tan 2\left(\frac{{}^{A}R_{B(3,3)}}{s\theta_2}, \frac{{}^{A}R_{B(1,3)}}{s\theta_2}\right) \qquad (10)
$$

And

$$
\theta_3 = A\tan 2\left(\frac{{}^{A}R_{B(2,2)}}{s\theta_2}, \frac{-{}^{A}R_{B(2,1)}}{s\theta_2}\right) \qquad (11)
$$

Provided that $s\theta_2 \neq 0$ .Having the joint space variables θ_1, θ_2, and θ_3 in hand, we can easily solve for the position of the moving platform using (8).

Now, in order to find the actuator lengths, we write a kinematic vector-loop equation for each actuated leg as:

$$
L_i = l_i . s_i = {}^{A}p + {}^{A}R_B {}^{B}p_i - a_i \qquad (12)
$$

where l_i is the length of the i^{th} actuated leg and s_i is a unit vector pointing along the direction of the i^{th} actuated leg. Also, ${}^{A}p$ is the position vector of the moving platform and ${}^{A}R_B$ is its rotation matrix. Vectors a_i and ${}^{B}p_i$ denote the fixed end points of the actuators (A_i) in the base frame and the moving end points of the actuators respectively, written as:

$$
\begin{aligned}
a_1 &= {}^{A}A_1 = \begin{pmatrix} l_b \sin\alpha & -l_b \cos\alpha & 0 \end{pmatrix}^T, \\
a_2 &= {}^{A}A_2 = \begin{pmatrix} -l_b \sin\alpha & -l_b \cos\alpha & 0 \end{pmatrix}^T, \\
a_3 &= {}^{A}A_3 = \begin{pmatrix} -l_b \sin\alpha & l_b \cos\alpha & 0 \end{pmatrix}^T, \\
a_4 &= {}^{A}A_4 = \begin{pmatrix} l_b \sin\alpha & l_b \cos\alpha & 0 \end{pmatrix}^T,
\end{aligned} \qquad (13)
$$

and

$$
\begin{aligned}
{}^{B}p_1 &= \begin{pmatrix} 0 & -l_d & -l_k \end{pmatrix}^T, \\
{}^{B}p_2 &= \begin{pmatrix} 0 & l_d & -l_k \end{pmatrix}^T,
\end{aligned} \qquad (14)
$$

Hence, the actuator lengths l_i can be easily computed by dot-multiplying (12) with itself to yield:

$$
L_i^T . L_i = l_i^2 = [{}^{A}p + {}^{A}R_B {}^{B}p_i - a_i]^T [{}^{A}p + {}^{A}R_B {}^{B}p_i - a_i] \qquad (15)
$$

Writing (15) four times with the corresponding parameters given in (7),(8),(13) and (14), and simplifying the results yields:

$$
l_1^2 = k_1 + k_2 c\theta_2 + k_3 c\theta_1 s\theta_2 + k_4(s\theta_1 c\theta_3 - c\theta_1 c\theta_2 s\theta_3) + k_5 s\theta_2 s\theta_3 \qquad \text{(16-a)}
$$

$$
l_2^2 = k_1 + k_2 c\theta_2 - k_3 c\theta_1 s\theta_2 - k_4(s\theta_1 c\theta_3 - c\theta_1 c\theta_2 s\theta_3) + k_5 s\theta_2 s\theta_3 \qquad \text{(16-b)}
$$

$$
l_3^2 = k_1 - k_2 c\theta_2 - k_3 c\theta_1 s\theta_2 + k_4(s\theta_1 c\theta_3 - c\theta_1 c\theta_2 s\theta_3) + k_5 s\theta_2 s\theta_3 \qquad \text{(16-c)}
$$

$$
l_4^2 = k_1 - k_2 c\theta_2 + k_3 c\theta_1 s\theta_2 - k_4(s\theta_1 c\theta_3 - c\theta_1 c\theta_2 s\theta_3) + k_5 s\theta_2 s\theta_3 \qquad \text{(16-d)}
$$

where:

$$
\begin{aligned}
k_1 &= l_b^2 + l_d^2 + (l_p - l_k)^2 \\
k_2 &= 2 l_b (l_p - l_k) \cos(\alpha) \\
k_3 &= -2 l_b (l_p - l_k) \sin(\alpha) \\
k_4 &= 2 l_b l_d \sin(\alpha) \\
k_5 &= -2 l_b l_d \cos(\alpha)
\end{aligned} \qquad (17)
$$

Finally, the actuator lengths are given by the square roots of (16), yielding actuator space variables as the unknowns of the inverse kinematics problem.

3.2 Forward Kinematics

Forward kinematics is undoubtedly a basic element in modeling and control of the manipulator. In forward kinematic analysis of the hydraulic shoulder, we shall find all the possible orientations of the moving platform for a given set of actuated leg lengths. Equation (16) can also be used for the forward kinematics of the hydraulic shoulder but with the actuator lengths as the input variables. In fact, we have four nonlinear equations to solve for three unknowns. First, we try to express the moving platform position and orientation in terms of the joint variables θ_1, θ_2, and θ_3 using (7)-(8). As it is obvious from (16), the only unknowns are the joint variables θ_1, θ_2, and θ_3, since actuator lengths are given and all other parameters are determined by the geometry of the manipulator. Hence, we must solve the equations for six unknowns from which only three are independent. Summing (16-a) and (16-b) we get:

$$l_1^2 + l_2^2 = 2k_1 + 2k_2 c\theta_2 + 2k_5 s\theta_2 s\theta_3 \tag{18}$$

Similarly adding (16-c) and (16-d) yields:

$$l_3^2 + l_4^2 = 2k_1 - 2k_2 c\theta_2 + 2k_5 s\theta_2 s\theta_3 \tag{19}$$

Subtracting (19) from (18), we can solve for $c\theta_2$ as:

$$c\theta_2 = \frac{l_1^2 + l_2^2 - l_3^2 - l_4^2}{4k_2} \tag{20}$$

Substituting (20) into the trigonometric identity $s\theta_2^2 + c\theta_2^2 = 1$, we get:

$$s\theta_2 = \pm\sqrt{1 - c\theta_2^2} \tag{21}$$

Having $s\theta_2$ and $c\theta_2$ in hand, we can solve for $s\theta_3$ from (18) as:

$$s\theta_3 = \frac{l_1^2 + l_2^2 - 2k_1 - 2k_2 c\theta_2}{2k_5 s\theta_2} \tag{22}$$

Similarly:

$$c\theta_3 = \pm\sqrt{1 - s\theta_3^2} \tag{23}$$

To solve for the remaining unknowns, $c\theta_1$ and $s\theta_1$, we sum (16-b) and (16-c) to get:

$$l_2^2 + l_3^2 = 2k_1 - 2k_3 c\theta_1 s\theta_2 + 2k_5 s\theta_2 s\theta_3 \tag{24}$$

Having computed $s\theta_2$ and $s\theta_3$, we obtain:

$$c\theta_1 = \frac{2k_1 + 2k_5 s\theta_2 s\theta_3 - l_2^2 - l_3^2}{2k_3 s\theta_2} \tag{25}$$

And finally:

$$s\theta_1 = \pm\sqrt{1 - c\theta_1^2} \tag{26}$$

Hence, the joint space variables are given by:

$$\begin{aligned} \theta_1 &= a\tan 2(s\theta_1, c\theta_1) \\ \theta_2 &= a\tan 2(s\theta_2, c\theta_2) \\ \theta_3 &= a\tan 2(s\theta_3, c\theta_3) \end{aligned} \tag{27}$$

Also, the moving platform position ${}^A p$ and orientation ${}^A R_B$ are found using (7)-(8). The final step is to solve for the orientation angles θ_x, θ_y and θ_z using (3) which completes the solution process to the forward kinematics of the hydraulic shoulder. It should be noted that there are some additional erroneous solutions to the forward kinematics as stated above due to several square roots involved in the process. These solutions must be identified and omitted. Another important assumption made in our solution procedure was that all four actuator fixed endpoints are coplanar, just as the actuator moving endpoints.

4 JACOBIAN ANALYSIS

The Jacobian matrix of a 3-DOF parallel manipulator relates the task space linear or angular velocity to the vector of actuated joint rates in a way that it corresponds to the inverse Jacobian of a serial manipulator. In this section, we derive the Jacobian for the Hydraulic shoulder as a key element in singularity analysis and position control of this manipulator. For the Jacobian analysis of the Hydraulic shoulder, we must find a relationship between the angular velocity of the moving platform, $\boldsymbol{\omega}$, and the vector of leg rates as the actuator space variables, $\dot{\mathbf{l}} = [\dot{l}_1 \quad \dot{l}_2 \quad \dot{l}_3 \quad \dot{l}_4]^T$, so that:

$$\dot{\mathbf{l}} = J\boldsymbol{\omega} \tag{28}$$

From the above definition, it is easily observed that the Jacobian for the Hydraulic shoulder will be a 4×3 rectangular matrix as expected, regarding the mechanism as an actuator redundant manipulator. Using the same idea of mapping between actuator, joint and task space, we find that the Jacobian depends on the actuated legs as well as the passive supporting leg. Therefore, we first derive a 4×6 Jacobian, J_l, relating the six-dimensional velocity of the moving platform, $\mathbf{v}$, to the vector of actuated leg rates, $\dot{\mathbf{l}}$. Then, we find the 6×3 Jacobian of the passive supporting leg, J_p The Jacobian of the Hydraulic shoulder will be finally derived as:

$$J = J_l J_p \tag{29}$$

4.1 Jacobian of the actuated legs

The Jacobian of the actuated legs, J_l, relates the six-dimensional velocity of the moving platform, $\mathbf{v}$, to the vector of actuated leg rates, $\dot{\mathbf{l}}$, such that:

$$\dot{\mathbf{l}} = J_l \mathbf{v} \tag{30}$$

We can write the six-dimensional moving platform velocity as:

$$\mathbf{v} = \begin{bmatrix} {}^A\dot{p} \\ \boldsymbol{\omega} \end{bmatrix} = [\dot{p}_x \quad \dot{p}_y \quad \dot{p}_z \quad \omega_x \quad \omega_y \quad \omega_z]^T \tag{31}$$

where ${}^A\dot{p}$ is the velocity of the moving platform center and $\boldsymbol{\omega}$ is the angular velocity of the moving platform. Differentiating the kinematic vector-loop equation (12) with respect to time we get:

$$\dot{l}_i s_i + (\omega_i \times s_i) l_i = {}^A\dot{p} + \boldsymbol{\omega} \times {}^A R_B {}^B p_i \tag{32}$$

where ω_i is the angular velocity of the i[th] leg written in the base frame. Dot multiplying (32) by s_i we have:

$$\dot{l}_i = s_i^T {}^A\dot{p} + ({}^A R_B {}^B p_i \times s_i)^T \boldsymbol{\omega} \tag{33}$$

writing the above equation four times for each actuated leg and comparing the result to (30) gives the actuated legs Jacobian as:

$$J_l = \begin{bmatrix} s_1^T & ({}^A R_B {}^B p_1 \times s_1)^T \\ s_2^T & ({}^A R_B {}^B p_2 \times s_2)^T \\ s_3^T & ({}^A R_B {}^B p_1 \times s_3)^T \\ s_4^T & ({}^A R_B {}^B p_2 \times s_4)^T \end{bmatrix}_{4\times 6} \qquad (34)$$

4.2 Jacobian of the passive leg

In order to find the manipulator Jacobian, we need to find a relationship between the six-dimensional velocity vector of the moving platform, $\mathbf{v}$, and the angular velocity of the moving platform, $\boldsymbol{\omega}$. First, by differentiating (8) with respect to time we get:

$${}^A\dot{p} = \begin{bmatrix} -l_p s\theta_1 s\theta_2 & l_p c\theta_1 c\theta_2 & 0 \\ 0 & -l_p s\theta_2 & 0 \\ l_p c\theta_1 s\theta_2 & l_p s\theta_1 c\theta_2 & 0 \end{bmatrix} \dot{\boldsymbol{\theta}} \qquad (35)$$

where $\dot{\boldsymbol{\theta}} = \begin{bmatrix} \dot{\theta}_1 & \dot{\theta}_2 & \dot{\theta}_3 \end{bmatrix}^T$ is the vector of passive joint rates. The angular velocity of the moving platform can also be expressed as:

$$\boldsymbol{\omega} = {}^A\dot{R}_B \, {}^A R_B^{-1} \qquad (36)$$

Substituting ${}^A R_B$ from (7) and computing (36), we have:

$$\boldsymbol{\omega} = \begin{bmatrix} 0 & s\theta_1 & c\theta_1 s\theta_2 \\ -1 & 0 & c\theta_2 \\ 0 & -c\theta_1 & s\theta_1 s\theta_2 \end{bmatrix} \dot{\boldsymbol{\theta}} \qquad (37)$$

Solving (37) for $\dot{\boldsymbol{\theta}}$ and substituting it in (35) yields:

$${}^A\dot{p} = \begin{bmatrix} 0 & lps\theta_1 s\theta_2 & -lpc\theta_2 \\ -lps\theta_1 s\theta_2 & 0 & lpc\theta_1 s\theta_2 \\ lpc\theta_2 & -lpc\theta_1 s\theta_2 & 0 \end{bmatrix} \boldsymbol{\omega} \qquad (38)$$

Complementing the above equation with the identity map $\boldsymbol{\omega} = I_3 \boldsymbol{\omega}$, we finally obtain:

$$\mathbf{v} = \begin{bmatrix} {}^A\dot{p} \\ \boldsymbol{\omega} \end{bmatrix} = J_p \boldsymbol{\omega} \qquad (39)$$

where:

$$J_p = \begin{bmatrix} 0 & l_p s\theta_1 s\theta_2 & -l_p c\theta_2 \\ -l_p s\theta_1 s\theta_2 & 0 & l_p c\theta_1 s\theta_2 \\ l_p c\theta_2 & -l_p c\theta_1 s\theta_2 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{6\times 3} \qquad (40)$$

Having J_l and J_p in hand, the Hydraulic shoulder Jacobian, $J_{4\times 3}$ will be easily found using (29).

5 SINGULARITY ANALYSIS

As shown in the previous section, the linear velocities of the actuators $\dot{\mathbf{l}}$ are related to the angular velocity of the moving platform $\boldsymbol{\omega}$ by (28), in which J was the 4×3 Jacobian matrix of the hydraulic shoulder. Singularities will occur if the Jacobian rank is lower than three, the number of DOF of the moving platform, or equivalently if:

$$\det(J^T J) = 0 \qquad (41)$$

Such a case occurs only if the determinants of all 3×3 minors of J are identically zero. These square minors correspond to the Jacobian matrices of the hydraulic shoulder with one of the actuating legs removed. Therefore, the redundant manipulator will be in a singular configuration only if all the non-redundant structures resulted by suppressing one of the actuating legs are in a singular configuration. Such a case will not occur in the workspace of the hydraulic shoulder owing to the specific and careful design of the mechanism (Hayward, 94). In fact, one of the remarkable features of adding the fourth actuator is the elimination of the loci of singularities. Fig.(5), shows the determinants of the four minor Jacobian matrices, computed in the workspace of the manipulator where DM1-DM4 denote the non-redundant minor determinants. This can be of interest in cases where one of the actuators malfunctions. It can be seen that the possibility of getting into a singular configuration is increased when either of the redundant actuators are removed.

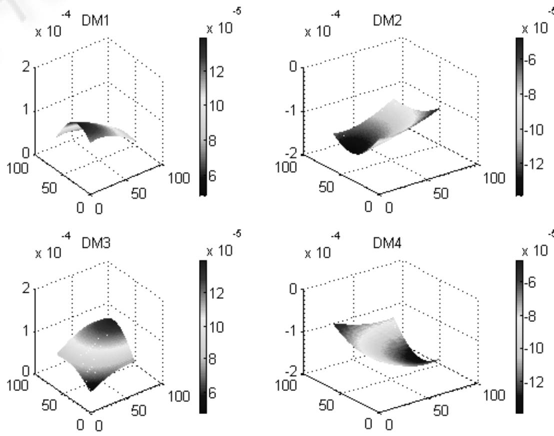

Figure 4: Minor Determinants for the non-redundant structures

6 CONCLUSIONS

In this paper, kinematic modeling and singularity analysis of a 3-DOF actuator redundant parallel manipulator has been studied in detail. The closed form solution to the forward kinematic is obtained using a vector approach by considering the individual kinematic chains inherent in such parallel mechanisms. It is proposed to consider suitable mapping between actuator, joint and task spaces in both kinematic and Jacobian modeling of the manipulator. The proposed method paves the way for the feedback position control of the manipulator, using a closed-form solution to the forward kinematics and leaving out the approximation errors inherent in numerical identification methods. It is also shown that the forward kinematics map provides us with some extra solutions which should be regarded properly. Singularity analysis was also performed using the analytic Jacobian obtained for the mechanism. The manipulator workspace was shown to be free of singularities due to the redundancy in actuation. Future work will consider Dynamic analysis of the hydraulic shoulder manipulator.

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