DISTRIBUTED GRADIENT FOR MULTI-ROBOT MOTION PLANNING

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Abstract: Distributed stochastic search is proposed for cooperative behavior in multi-robot systems. Distributed gradient is examined. This method consists of multiple stochastic search algorithms that start from different points in the solutions space and interact to each other while moving towards the goal position. Distributed gradient is shown to be efficient when the motion of the robots towards the goal position is described by a quadratic cost function. The algorithm's performance is evaluated through simulation tests.

1 INTRODUCTION

In the recent years there has been growing interest in multi-robot systems (Guo and Parker, 2002). As the cost of robots goes down and as robots become more compact the number of military and industrial applications of multi-robot systems increases. Possible industrial applications of multi-robot systems include hazardous inspection, underwater or space exploration, assembling and transportation. Some examples of military applications are guarding, escorting, patrolling and strategic behaviors, such as stalking and attacking.

Of primary importance in the design of multi-robot systems is motion planning through obstacles. To solve this problem *distributed gradient* algorithms are proposed. These are an extension of the potential fields methods which have have been previously used for robot path-planning (Khatib, 1986)-(Reif and Wang, 1999). The potential of each robot consists of two terms: (i) the cost V^i due to the distance of the i-th robot from the goal state, (ii) the cost due to the interaction with the other $M-1$ robots. Moreover, a repulsive field, generated by the proximity to obstacles, is taken into account. The differentiation of the aggregate potential provides the kinematic model for each robot. It is proved that the velocity update equation is equivalent to a distributed gradient algorithm. The convergence to the goal state is studied with the use of Lyapunov stability theory. It is shown that in the case of a quadratic cost function V^i the mean position of the multi-robot system converges to the goal state x^* while each robot stays in a bounded area close to x^*. These results are also of interest for research in the area of particle systems where similar problems of cooperative behavior are studied (Levine and Rappel, 2000).

The structure of the paper is as follows: In Section 2 elements of stochastic search algorithms are summarized and distributed gradient is proposed for multi-robot motion planning. Stability analysis of the distributed gradient algorithms is performed with the use of Lyapunov theory. In Section 3 the performance of the distributed gradient algorithm in the problem of multi-robot motion planning is tested through simulation tests. Finally, in Section 4 concluding remarks are stated.

2 DISTRIBUTED STOCHASTIC SEARCH

Motion planning of multi-robot systems can be solved with the use of *distributed stochastic search* algorithms. These can be multiple gradient algorithms that start from different points in the solutions space and interact to each other while moving towards the goal position. Distributed gradient algorithms, stem

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from stochastic search algorithms treated in (Duflo, 1996) if an interaction term is added:

$$x^i(t+1) = x^i(t) + \gamma^i(t)[h(x^i(t)) + e^i(t)] + \sum_{j=1, j\neq i}^{M} g(x^i - x^j),\ i = 1, 2, \cdots, M \quad (1)$$

The term $h(x(t)^i) = -\nabla_{x^i} V^i(x^i)$ indicates a local gradient algorithm, i.e. motion in the direction of decrease of the cost function $V^i(x^i) = \frac{1}{2} e^i(t)^T e^i(t)$. The term $\gamma^i(t)$ is the algorithm's step while the stochastic disturbance $e^i(t)$ enables the algorithm to escape from local minima. The term $\sum_{j=1, j\neq i}^{M} g(x^i - x^j)$ describes the interaction between the i-th and the rest $M-1$ stochastic search algorithms. Convergence analysis based on the Lyapunov stability theory can be stated in the case of distributed gradient algorithms. This is important for the problem of multi-robot motion planning.

2.1 Kinematic model of the multi-robot system

The objective is to lead a swarm of M mobile robots, with different initial positions on the 2-D plane, to a desirable final position. The position of each robot in the 2-D space is described by the vector $x^i \in R^2$. The motion of the robots is synchronous, without time delays, and it is assumed that at every time instant each robot i is aware about the position and the velocity of the other $M-1$ robots. The cost function that describes the motion of the i-th robot towards the goal state is denoted as $V(x^i): R^n \to R$. The value of $V(x^i)$ is high on hills, small in valleys, while it holds $\nabla_{x^i} V(x^i) = 0$ at the goal position and at local optima. The following conditions must hold: (i) The cohesion of the swarm should be maintained, i.e. the norm $||x^i - x^j||$ should remain upper bounded $||x^i - x^j|| < \epsilon^h$, (ii) Collisions between the robots should be avoided, i.e. $||x^i - x^j|| > \epsilon^l$, (iii) Convergence to the goal state should be succeeded for each robot through the negative definiteness of the associated Lyapunov function $\dot{V}^i(x^i) = \dot{e}^i(t)^T e^i(t) < 0$. (Rigatos, et al., 2001). The interaction between the i-th and the j-th robot is taken to be

$$g(x^i - x^j) = -(x^i - x^j)[g_a(||x^i - x^j||) - g_r(||x^i - x^j||)] \quad (2)$$

where $g_a()$ denotes the attraction term and is dominant for large values of $||x^i - x^j||$, while $g_r()$ denotes the repulsion term and is dominant for small values of $||x^i - x^j||$. Function $g_a()$ can be associated with an attraction potential, i.e. $\nabla_{x_i} V_a(||x^i - x^j||) = (x^i - x^j) g_a(||x^i - x^j||)$. Function $g_r()$ can be associated with a repulsion potential, i.e. $\nabla_{x_i} V_r(||x^i - x^j||) = (x^i - x^j) g_r(||x^i - x^j||)$. A suitable function $g()$ that describes the interaction between the robots is given by (Gazi and Passino, 2004)

$$g(x^i - x^j) = -(x^i - x^j)(a - b e^{\frac{-||x^i - x^j||^2}{\sigma^2}}) \quad (3)$$

where the parameters a, b and c are suitably tuned. It holds that $g_a(x^i - x^j) = -a$, i.e. attraction has a linear behavior (spring-mass system) $||x^i - x^j|| g_a(x^i - x^j)$. Moreover, $g_r(x^i - x^j) = b e^{\frac{-||x^i - x^j||^2}{\sigma^2}}$ which means that $g_r(x^i - x^j)||x^i - x^j|| \leq b$ is bounded. Applying Newton's laws to the i-th robot yields

$$\begin{aligned} \dot{x}^i &= v^i \\ m^i \dot{v}^i &= U^i \end{aligned} \quad (4)$$

where the aggregate force is $U^i = f^i + F^i$. The term $f^i = -K_v v^i$ denotes friction, while the term F^i is the propulsion. Assuming zero acceleration $\dot{v}^i = 0$ one gets $F^i = K_v v^i$, which for $K_v = 1$ and $m^i = 1$ gives $F^i = v^i$. Thus an approximate kinematic model is

$$\dot{x}^i = F^i \quad (5)$$

According to the Euler-Langrange principle, the propulsion F^i is equal to the derivative of the total potential of each robot, i.e.

$F^i = -\nabla_{x^i}\{V^i(x^i) + \frac{1}{2}\sum_{i=1}^{M}\sum_{j=1, j\neq i}^{M}[V_a(||x^i - x^j|| - V_r(||x^i - x^j||)]\} \Rightarrow F^i = -\nabla_{x^i}\{V^i(x^i)\} - \sum_{j=1, j\neq i}^{M}[\nabla_{x^i} V_a(||x^i - x^j||) - \nabla_{x^i} V_r(||x^i - x^j||)] \Rightarrow F^i = -\nabla_{x^i}\{V^i(x^i)\} + \sum_{j=1, j\neq i}^{M}[-(x^i - x^j) g_a(||x^i - x^j||) + (x^i - x^j) g_r(||x^i - x^j||)] \Rightarrow F^i = -\nabla_{x^i}\{V^i(x^i)\} + \sum_{j=1, j\neq i}^{M} g(x^i - x^j)$.

Substituting in Eq. (5) one gets Eq. (1), i.e. $x^i(t+1) = x^i(t) + \gamma^i(t)[-\nabla_{x^i} V^i(x^i) + e^i(t+1)] + \sum_{j=1, j\neq i}^{M} g(x^i - x^j),\ i = 1, 2, \cdots, M$, with $\gamma^i(t) = 1$, which verifies that the kinematic model of a multi-robot system is equivalent to a distributed gradient search algorithm.

2.2 Stability of the multi-robot system

The behaviour of the multi-robot system is determined by the behaviour of its center (mean of the vectors x^i) and of the position of each robot with respect to this center. The center of the multi-robot system is given by $\bar{x} = E(x^i) = \frac{1}{M}\sum_{i=1}^{M} x^i$, therefore $\dot{\bar{x}} = \frac{1}{M}\sum_{i=1}^{M} \dot{x}^i \Rightarrow \dot{\bar{x}} = \frac{1}{M}\sum_{i=1}^{M}[-\nabla_{x^i} V^i(x^i) -$

$\sum_{j=1,j\neq i}^{M}(g(x^i-x^j))]$.

From Eq. (3) it can be seen that $g(x^i-x^j)=-g(x^j-x^i)$, i.e. $g()$ is an odd function. Therefore, it holds that $\frac{1}{M}(\sum_{j=1,j\neq i}^{M}g(x^i-x^j))=0$, and

$$\dot{\bar{x}}=\frac{1}{M}\sum_{i=1}^{M}[-\nabla_{x^i}V^i(x^i)] \quad (6)$$

Denoting the goal position by x^* , and the distance between the i-th robot and the mean position of the multi-robot system by $e^i(t)=x^i(t)-\bar{x}$ the objective of distributed gradient for robot motion planning can be summarized as follows: (i) $lim_{t\to\infty}\bar{x}=x^*$, i.e. the center of the multi-robot system converges to the goal position, (ii) $lim_{t\to\infty}x^i=\bar{x}$, i.e. the i-th robot converges to the center of the multi-robot system, (iii) $lim_{t\to\infty}\dot{\bar{x}}=0$, i.e. the center of the multi-robot system stabilizes at the goal position. If conditions (i) and (ii) hold then $lim_{t\to\infty}x^i=x^*$. Furthermore, if condition (iii) also holds then all robots will stabilize close to the goal position.

It is known that the stability of local gradient algorithms can be proved with the use of Lyapunov theory. A similar approach can be followed in the case of the distributed gradient algorithms given by Eq. (1) (Gazi and Passino, 2004). The following simple Lyapunov function is considered for each gradient algorithm:

$$V_i=\frac{1}{2}{e^i}^Te^i\Rightarrow V_i=\frac{1}{2}||e_i||^2 \quad (7)$$

Thus, one gets $\dot{V}^i={e^i}^T\dot{e}^i\Rightarrow\dot{V}^i=(\dot{x}^i-\dot{\bar{x}})e^i\Rightarrow\dot{V}^i=[-\nabla_{x^i}V^i(x^i)-\sum_{j=1,j\neq i}^{M}g(x^i-x^j)+\frac{1}{M}\sum_{j=1}^{M}\nabla_{x^j}V^j(x^j)]e^i$. Substituting $g(x^i-x^j)$ from Eq. (3) yields $\dot{V}_i=[-\nabla_{x^i}V^i(x^i)-\sum_{j=1,j\neq i}^{M}(x^i-x^j)a+\sum_{j=1,j\neq i}^{M}(x^i-x^j)g_r(||x^i-x^j||)+\frac{1}{M}\sum_{j=1}^{M}\nabla_{x^j}V^j(x^j)]e^i\Rightarrow\dot{V}_i=-a[\sum_{j=1,j\neq i}^{M}(x^i-x^j)]e^i+\sum_{j=1,j\neq i}^{M}g_r(||x^i-x^j||)(x^i-x^j)^Te^i-[\nabla_{x^i}V^i(x^i)-\frac{1}{M}\sum_{j=1}^{M}\nabla_{x^j}V^j(x^j)]^Te^i$.

It holds that $\sum_{j=1}^{M}(x^i-x^j)=Mx^i-M\frac{1}{M}\sum_{j=1}^{M}x^j=Mx^i-M\bar{x}=M(x^i-\bar{x})=Me^i$, therefore

$$\begin{array}{c}\dot{V}_i=-aM||e^i||^2+\sum_{j=1,j\neq i}^{M}g_r(||x^i-x^j||)(x^i-x^j)^Te^i\\ -[\nabla_{x^i}V^i(x^i)-\frac{1}{M}\sum_{j=1}^{M}\nabla_{x^j}V^j(x^j)]^Te^i\end{array} \quad (8)$$

It assumed that for all x^i there is a constant $\bar{\sigma}$ such that

$$||\nabla_{x^i}V^i(x^i)||\leq\bar{\sigma} \quad (9)$$

Eq. (9) is reasonable since for a robot moving on a 2-D plane, the gradient of the cost function $\nabla_{x^i}V^i(x^i)$ is expected to be bounded. Moreover it is known that the following inequality holds:

$$\sum_{j=1,j\neq i}^{M}g_r(x^i-x^j)^Te^i\leq\sum_{j=1,j\neq i}^{M}be^i\leq\sum_{j=1,j\neq i}^{M}b||e^i||$$

Thus the application of Eq. (8) gives $\dot{V}^i\leq aM||e^i||^2+\sum_{j=1,j\neq i}^{M}g_r(||x^i-x^j||)||x^i-x^j||\cdot||e^i||+||\nabla_{x^i}V^i(x^i)-\frac{1}{M}\sum_{j=1}^{M}\nabla_{x^j}V^j(x^j)||||e^i||\Rightarrow\dot{V}^i\leq aM||e^i||^2+b(M-1)||e^i||+2\bar{\sigma}||e^i||$ where it has been taken into account that

$$\sum_{j=1,j\neq i}^{M}g_r(||x^i-x^j||)^T||e^i||\leq\sum_{j=1,j\neq i}^{M}b||e^i||=b(M-1)||e^i||$$

and from Eq. (9)

$$\begin{array}{c}||\nabla_{x^i}V^i(x^i)-\frac{1}{M}\sum_{j=1}^{M}\nabla_{x^i}V^j(x^j)||\leq\\ ||\nabla_{x^i}V^i(x^i)||+\frac{1}{M}||\sum_{j=1}^{M}\nabla_{x^i}V^j(x^j)||\leq\\ \bar{\sigma}+\frac{1}{M}M\bar{\sigma}\leq2\bar{\sigma}\end{array}$$

Thus, one gets

$$\dot{V}^i\leq aM||e^i||\cdot[||e^i||-\frac{b(M-1)}{aM}-2\frac{\bar{\sigma}}{aM}] \quad (10)$$

The following bound ϵ is defined:

$$\epsilon=\frac{b(M-1)}{aM}+\frac{2\bar{\sigma}}{aM}=\frac{1}{aM}(b(M-1)+2\bar{\sigma}) \quad (11)$$

Thus, when $||e^i||>\epsilon$, $\dot{V}_i$ will become negative and consequently the error $e^i=x^i-\bar{x}$ will decrease. Therefore the error e^i will remain in an area of radius ϵ i.e. the position x^i of the i-th robot will stay in the cycle with center $\bar{x}$ and radius ϵ.

2.3 Stability in the case of a quadratic cost function

The case of a convex quadratic cost function is examined, for instance

$$V^i(x^i)=\frac{A}{2}||x^i-x^*||^2=\frac{A}{2}(x^i-x^*)^T(x^i-x^*) \quad (12)$$

where $x^*=[0,0]$ is a minimum point $V^i(x^i=x^*)=0$. The distributed gradient algorithm is expected to converge to x^*. The robotic vehicles will follow different different trajectories on the 2-D plane and will end at the goal position.

Using Eq.(12) yields $\nabla_{x^i}V^i(x^i) = A(x^i - x^*)$. Moreover, the assumption $\nabla_{x^i}V^i(x^i) \leq \bar{\sigma}$ can be used, since the gradient of the cost function remains bounded. The robotic vehicles will concentrate round $\bar{x}$ and will stay in a radius ϵ given by Eq. (11). The motion of the mean position $\bar{x}$ of the vehicles is $\dot{\bar{x}} = -\frac{1}{M}\sum_{i=1}^{M}\nabla_{x^i}V^i(x^i) \Rightarrow \dot{\bar{x}} = -\frac{A}{M}(x^i - x^*) \Rightarrow \dot{\bar{x}} - \dot{x}^* = -\frac{A}{M}x^i + \frac{A}{M}x^* \Rightarrow \dot{\bar{x}} - \dot{x}^* = -A(\bar{x} - x^*)$ The variable $e_\sigma = \bar{x} - x^*$ is defined, and consequently

$$\dot{e}_\sigma = -Ae_\sigma \Rightarrow \epsilon_\sigma(t) = c_1 e^{-At} + c_2 \tag{13}$$

with $c_1 + c_2 = e_\sigma(0)$. Eq. (13) is an homogeneous differential equation, which for $A > 0$ results into $lim_{t\to\infty}e_\sigma(t) = 0$, thus $lim_{t\to\infty}\bar{x}(t) = x^*$. It is left to make more precise the position to which each robot converges.

2.4 Convergence analysis using La Salle's theorem

It has been shown that $lim_{t\to\infty}\bar{x}(t) = x^*$ and from Eq. (10) that each robot will stay in a cycle C of center $\bar{x}$ and radius ϵ given by Eq. (11). The Lyapunov function given by Eq. (7) is negative semi-definite, therefore asymptotic stability cannot be guaranteed. It remains to make precise the area of convergence of each robot in the cycle C of center $\bar{x}$ and radius ϵ. To this end, La Salle's theorem can be employed (Gazi and Passino, 2004), (Khalil, 1996).

La Salle's Theorem: Assume the autonomous system $\dot{x} = f(x)$ where $f : D \to R^n$. Assume $C \subset D$ a compact set which is positively invariant with respect to $\dot{x} = f(x)$, i.e. if $x(0) \in C \Rightarrow x(t) \in C \ \forall\ t$. Assume that $V(x) : D \to R$ is a continuous and differentiable Lyapunov function such that $\dot{V}(x) \leq 0$ for $x \in C$, i.e. $V(x)$ is negative semi-definite in C. Denote by E the set of all points in C such that $\dot{V}(x) = 0$. Denote by M the largest invariant set in E and its boundary by L^+, i.e. for $x(t) \in E : lim_{t\to\infty}x(t) = L^+$, or in other words L^+ is the positive limit set of E. Then every solution $x(t) \in C$ will converge to M as $t \to \infty$.

La Salle's theorem in applicable in the case of the multi-robot system and helps to describe more precisely the area round $\bar{x}$ to which the robot trajectories x^i will converge. A generalized Lyapunov function is introduced which is expected to verify the stability analysis based on Eq. (10). It holds that:

$V(x) = \sum_{i=1}^{M}V^i(x^i) + \frac{1}{2}\sum_{i=1}^{M}\sum_{j=1,j\neq i}^{M}\{V_a(||x^i - x^j|| - V_r(||x^i - x^j||)\} \Rightarrow V(x) = \sum_{i=1}^{M}V^i(x^i) + \frac{1}{2}\sum_{i=1}^{M}\sum_{j=1,j\neq i}^{M}\{a||x^i - x^j||^2 - V_r(||x^i - x^j||)$, and

$\nabla_{x^i}V(x) = [\sum_{i=1}^{M}\nabla_{x^i}V^i(x^i)] + \frac{1}{2}\sum_{i=1}^{M}\sum_{j=1,j\neq i}^{M}\nabla_{x^i}\{a||x^i - x^j||^2 - V_r(||x^i - x^j||)\} \Rightarrow \nabla_{x^i}V(x) = [\sum_{i=1}^{M}\nabla_{x^i}V^i(x^i)] + \sum_{j=1,j\neq i}^{M}(x^i - x^j)\{g_a(||x^i - x^j||) - g_r(||x^i - x^j||)\} \Rightarrow \nabla_{x^i}V(x) = [\sum_{i=1}^{M}\nabla_{x^i}V^i(x^i)] - \sum_{j=1,j\neq i}^{M}g(||x^i - x^j||)$

and using Eq. (1) with $\gamma^i(t) = 1$ yields $\nabla_{x^i}V(x) = -\dot{x}^i$, and

$$\begin{gathered}\dot{V}(x) = \nabla_x V(x)^T\dot{x} = \sum_{i=1}^{M}\nabla_{x^i}V(x)^T\dot{x}^i \Rightarrow \\ \dot{V}(x) = -\sum_{i=1}^{M}||\dot{x}^i||^2 \leq 0\end{gathered} \tag{14}$$

Therefore, in the case of a quadratic cost function it holds $V(x) > 0$ and $\dot{V}(x) \leq 0$ and the set $C = \{x : V(x(t)) \leq V(x(0))\}$ is compact and positively invariant. Thus, by applying La Salle's theorem one can show the convergence of $x(t)$ to the set $M \subset C$, $M = \{x : \dot{V}(x) = 0\} \Rightarrow M = \{x : \dot{x} = 0\}$.

2.5 Stability in the case of a cost function with local minima

The following multi-modal Gaussian cost function is considered

$$V^i(x^i) = -\sum_{j=1}^{N}\frac{A^j}{2}e^{\frac{-||x^i - c^j||^2}{{\sigma^j}^2}} \tag{15}$$

where $c^j \in R^n$ is the center of the i-th Gaussian, $\sigma^j \in R^n$ is the variance of the i-th Gaussian, and $A^j \in R$ defines if the Gaussian stands for hill ($A^j > 0$) or a valley ($A^i < 0$). The gradient of the multi-modal Gaussian function in given by

$$\nabla_{x^i}V^i(x^i) = \sum_{j=1}^{N}\frac{A^j}{{\sigma^j}^2}(x^i - c^j)e^{\frac{-||x^i - c^j||^2}{{\sigma^j}^2}} \tag{16}$$

The velocity of the i-th robot is given by Eq.(1)

$$\begin{gathered}\dot{x}^i = -\nabla_{x^i}V^i(x^i) - \\ -\sum_{j=1,j\neq i}^{M}(x^i - x^j)[g_a(||x^i - x^j||) - g_r(||(x^i - x^j)||)]\end{gathered} \tag{17}$$

while the velocity of the center of the multi-robot system is given by Eq. (6)

$$\begin{aligned} \dot{\bar{x}} &= -\frac{1}{M}\nabla_{x^i}V^i(x^i) \Rightarrow \\ \dot{\bar{x}} &= -\frac{1}{M}\sum_{j=1}^{N}\frac{A^j}{{\sigma^j}^2}(x^i-c^j)e^{\frac{-||x^i-c^j||^2}{{\sigma^j}^2}} \end{aligned} \tag{18}$$

Eq. (18) does not give any information about the direction of the center $\bar{x}$ of the multi-robot system. Under specific assumptions convergence to local minima (valleys) or divergence from local maxima (hills) can be shown. To this end , it is assumed that at $t = 0$ holds,

$$\begin{aligned} ||x^i(0)-c^k|| &\leq \sigma^k, \quad k:\ 1\leq k\leq N \\ ||x^i(0)-c^j|| &\geq \sigma^j, \quad k:\ 1\leq j\leq N \end{aligned}$$

This means that the multi-robot system goes close to the Gaussian with center c^k and stays far from the Gaussian with center c^j. The following error definition is given

$$e^k = \bar{x} - c^k \tag{19}$$

and the Lyapunov function $V^k = \frac{1}{2}{e^k}^T e^k$ is considered. It holds that

$$\begin{aligned} \dot{V}^k &= (\dot{e}^k)^T e^k \Rightarrow \dot{V}^k = \dot{\bar{x}}^T e^k \Rightarrow \\ \dot{V}^k &= [-\frac{1}{M}\sum_{j=1}^{N}\frac{A^j}{{\sigma^j}^2}(x^i-c^j)e^{\frac{-||x^i-c^j||^2}{{\sigma^j}^2}}]e^k \end{aligned}$$

Assuming that $(x^i-c^j) < (e^k)^T$, i.e. $x^i-c^j < \bar{x}-c^k$, i.e. the mean of the multi-robot system is closer to the valley k yields

$$\dot{V}^k < -\frac{1}{M}\sum_{j=1}^{N}\frac{A^j}{{\sigma^j}^2}e^{\frac{-||x^i-c^j||^2}{{\sigma^j}^2}}||e^k||^2.$$

Therefore, for $A^j < 0$ it holds that $\dot{V}^k < 0$ and the multi-robot system will converge to the k-th Gaussian center c^k.

3 SIMULATION TESTS

In the conducted simulation tests the multi-robot system consisted of 10 robots which were randomly initialized in the 2-D field. Two cases were distinguished: (i) motion in an obstacle-free environment (Fig. 1 - Fig. 2 and (ii) motion in an environment with obstacles (Fig. 5 - Fig. 6). The objective was to lead the robot swarm to the origin $[x,y] = [0,0]$. To avoid obstacles, apart from the motion equations given in Sections 2 repulsive forces between the obstacles and the robots had to be taken into account. The reactive robot behavior for obstacle avoidance prevailed locally the motion laws which were derived using potential fields theory. This means that the collision avoidance was set to higher priority than maintenance of the cohesion of the robots swarm.

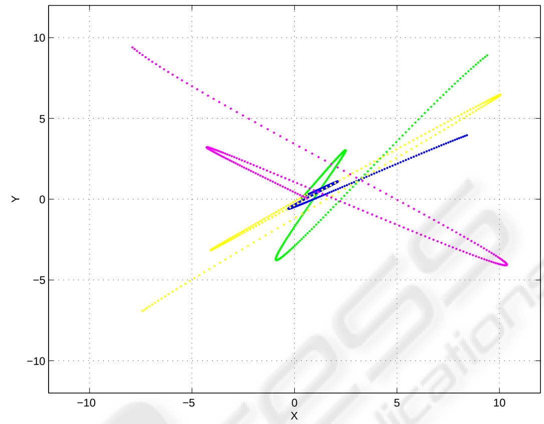

Figure 1: Motion of the individual robots in an obstacles-free environment, considering a quadratic cost function.

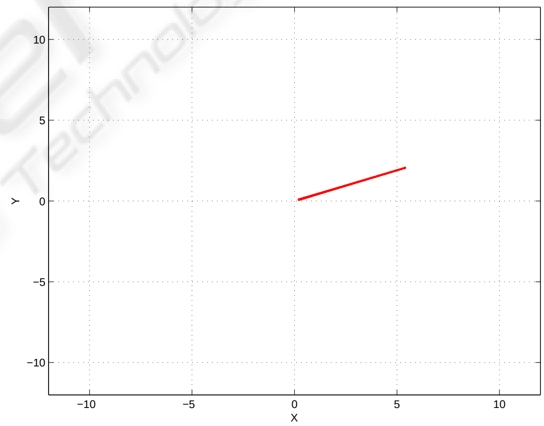

Figure 2: Motion of the mean of the multi-robot system in an obstacles-free environment, considering a quadratic cost function.

For the motion in an obstacle-free environment, the evolution of the aggregate Lyapunov function of the multi-robot system, as well as of the Lyapunov functions of the individual robots, is depicted in Fig. 3 and Fig. 4.

When the multi-robot system evolved in an environment with obstacles, the interaction between the individual robots (attractive and repulsive forces) had to be loose, so as to give priority to obstacles avoidance. Therefore coefficients a and b in Eq.

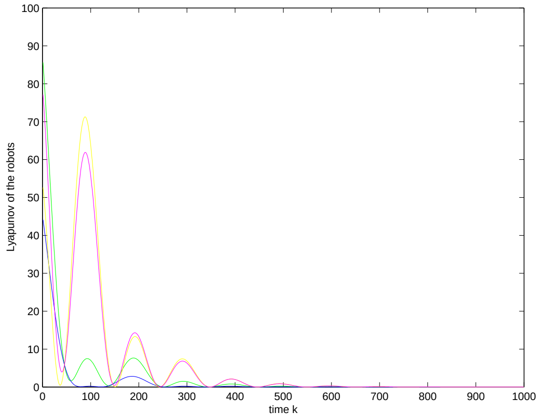

Figure 3: Lyapunov function of the individual robots in an obstacles-free environment.

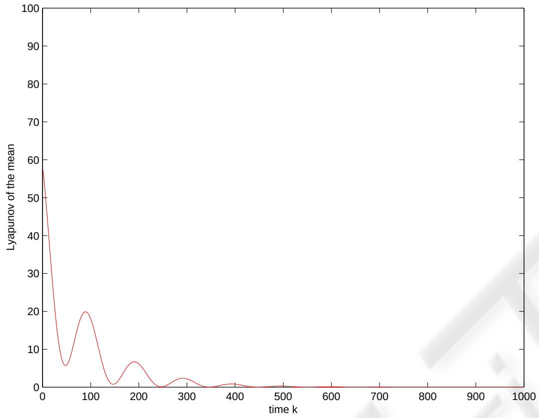

Figure 4: Lyapunov function of the mean of the multi-robot system in an obstacles-free environment.

(3) $g(x^i - x^j) = -(x^i - x^j)(a - be^{\frac{||x^i-x^j||^2}{\sigma^2}})$ were set to small values. The repulsive potential due to the obstacles was calculated by $g(x^i - x^j) = -(x^i - x_o^j)(a - be^{\frac{||x^i-x_o^j||^2}{\sigma^2}})$, where x_o^j was the center of the j-th obstacle.

The relative values of the parameters a and b that appear in the attractive and repulsive potential respectively, affected the performance of the algorithm. For $a > b$ the cohesion of the robotic swarm was maintained and abrupt displacements of the individual robots were avoided.
For the motion in an environment with obstacles, the evolution of the aggregate Lyapunov function of the multi-robot system, as well as of the Lyapunov functions of the individual robots, is depicted in Fig. 7 and Fig. 8 . Comparing to Fig. 3 and Fig. 4 the differences are due to the smaller value of the attractive force coefficient a.

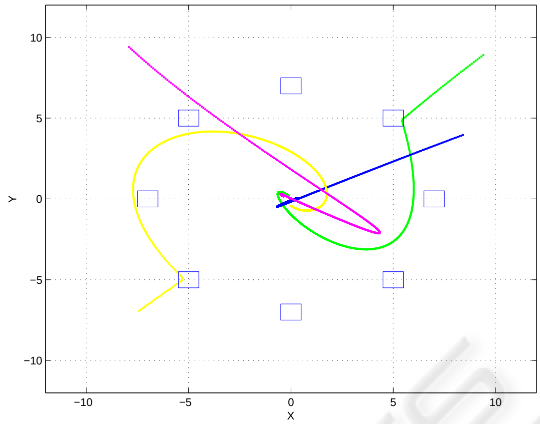

Figure 5: Motion of the individual robots in an environment with obstacles, considering a quadratic cost function.

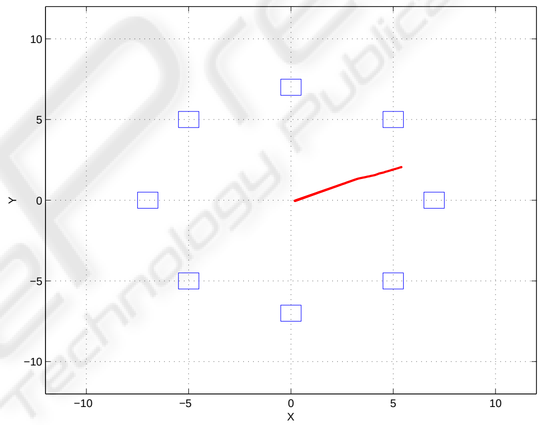

Figure 6: Motion of the mean of the multi-robot system in an environment with obstacles, considering a quadratic cost function.

4 CONCLUSIONS

In this paper the problem of distributed multi-robot motion planning was studied. A M-robot swarm was considered and the objective was to lead the swarm to a goal position. The kinematic model of the robots was derived using the potential fields theory. The potential of each robot consisted of two terms: (i) the cost V^i due to the distance of the i-th robot from the goal state, (ii) the cost due to the interaction with the other $M - 1$ robots. The differentiation of the potential provided the kinematic model for each robot. It was proved that the velocity update equation is equivalent to a *distributed gradient* algorithm. The convergence to the goal state was studied with the use of Lyapunov stability theory. It was shown that in the case of a quadratic cost function V^i the mean position of the multi-robot system converges to the goal state

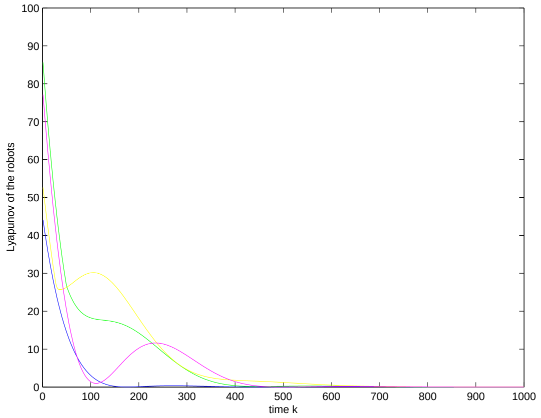

Figure 7: Lyapunov function of the individual robots in an environment with obstacles.

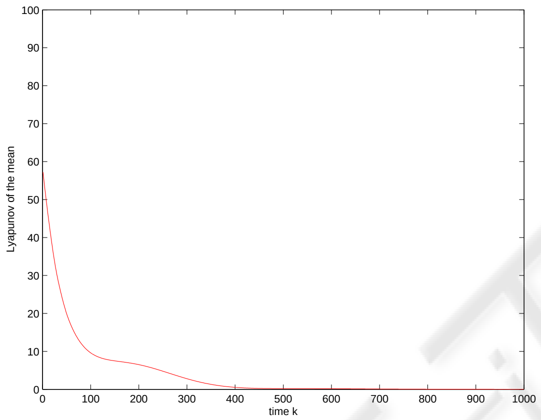

Figure 8: Lyapunov function of the mean of the multi-robot system in an environment with obstacles.

x^* while each robot stays in a bounded area close to x^*.

Distributed gradient for multi-robot motion planning was evaluated through simulation tests. It was observed that when the multi-robot system was evolving in an environment with obstacles, the interaction between the individual robots (attractive and repulsive forces) had to be loose, so as to give priority to obstacles avoidance. The performance of the method was satisfactory. The algorithm succeeded cooperative behavior of the robots without requirement for explicit coordination or communication.

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