

Towards Guaranteed Collision Avoidance for Multiple Autonomous Underactuated Unmanned Surface Vehicles in Restricted Waters

Erick J. Rodríguez-Seda ^a

*Department of Weapons, Robotics, and Control Engineering
United States Naval Academy, Annapolis, MD, U.S.A.*

Keywords: Artificial Potential Field, Collision Avoidance, Multi-Agent Systems, Unmanned Vehicles.

Abstract: As autonomous surface vessels increasingly operate in restricted and congested waters, the need for distributed, reactive collision avoidance algorithms becomes more crucial. Traditional avoidance control algorithms are typically conservative, opting for a worst-case scenario approach and restricting the total area where Unmanned Surface Vehicles (USVs) can navigate. This paper presents a distributed collision avoidance framework for USVs, based on the concepts of Artificial Potential Field (APF) and avoidance functions, that aims to reduce the minimum safe distance that vehicles need to keep from obstacles by explicitly considering their shape, relative position, and relative orientation. The proposed control framework is theoretically demonstrated and validated through simulations to ensure collision avoidance at all times and to facilitate the travel of vehicles in obstacle-dense environments.

1 INTRODUCTION

Collision avoidance is arguably one of the most critical challenges when operating autonomous USVs. Not only does the USV need to compensate for disturbances such as currents, waves, and wind, but it is typically subject to underactuation, which restricts the vehicle's maneuverability (Er et al., 2023). Furthermore, the environment in which these vehicles operate is often unknown and dynamic, requiring the implementation of reactive avoidance control strategies.

Several reactive collision avoidance methods for USVs have been proposed and studied (refer to (Vagale et al., 2021; Lyu et al., 2023) for reviews). One particular approach of interest due to their relative ease of analysis and implementation is the use of APF functions. APF-based methods use repulsive forces around obstacles to maneuver away from a collision (Xue et al., 2009). These forces can then be shown, via Lyapunov-based analysis, to guarantee the safety of a large number of vehicles (Stipanović et al., 2007). Examples in the literature for USVs vary based on the vehicle's maneuverability and the compliance with other restrictions and regulations (Li et al., 2021; Zhang et al., 2022; Li et al., 2025). Yet a common drawback of these APF-based strategies is the treatment of vehicles and obstacles as points or objects of

circular shape. This assumption simplifies the analysis and implementation of control algorithms, but artificially increases the minimum distance that agents need to keep from each other by assuming a worst-case scenario. One solution to reduce this conservatism is the modeling of obstacles and vehicles as a set of multiple smaller spheres, hence reducing the agent's footprint at the expense of increasing the number of obstacles and artificial potential field functions.

Alternatively, one can wrap the vehicles and obstacles with a convex envelope that considers not only their shape but also their relative position and orientation. For instance, the work in (Rodríguez-Seda, 2024b; Rodríguez-Seda, 2024a) defines the repulsive potential field as a function of the vehicles and obstacles' relative position, orientation, and shape. In contrast to the use of a constant distance, as in the case of agents of circular shape, the work in (Rodríguez-Seda, 2024b; Rodríguez-Seda, 2024a) uses a continuous, differentiable, non-constant distance function that is typically smaller than the radius of the minimum enclosing circle. Such an approach is shown to safely allow the travel of multiple nonholonomic, underactuated ground vehicles through narrow passages and highly occluded spaces.

In this paper, we apply the avoidance control concept developed in (Rodríguez-Seda, 2024b; Rodríguez-Seda, 2024a) for ground vehicles with no-

^a  <https://orcid.org/0000-0003-1108-4329>

slip constraints to underactuated USVs subject to sway and surge motion. We propose a distributed control law for an arbitrary number of vehicles and static obstacles and show, via Lyapunov-based analysis, that if the vehicles start from a safe state and their velocities are non-zero while they try to resolve the conflict, the vehicles are guaranteed to avoid collisions. A simulation example with four USVs traveling through a narrow passage illustrates the benefits of the proposed approach.

2 PROBLEM FORMULATION

We consider the task of safely coordinating the motion of N heterogeneous, underactuated USVs. Let $x_i(t)$ and $y_i(t)$ represent the position coordinates of the center of mass and $\psi_i(t)$ be the orientation in the Earth-fixed frame for the i th vehicle (refer to Figure 1). Assume that the vehicle is neutrally buoyant. Then, the nonlinear kinematic and dynamic equations of motion of the i th USV in still water are given by

$$\dot{x}_i(t) = u_i(t) \cos \psi_i(t) - v_i(t) \sin \psi_i(t) \quad (1a)$$

$$\dot{y}_i(t) = u_i(t) \sin \psi_i(t) + v_i(t) \cos \psi_i(t) \quad (1b)$$

$$\dot{\psi}_i(t) = \omega_i(t) \quad (1c)$$

$$\dot{u}_i(t) = -\frac{m_{2i}}{m_{1i}} v_i(t) \omega_i(t) - \frac{d_{1i}}{m_{1i}} u_i(t) + \frac{f_i(t)}{m_{1i}} \quad (1d)$$

$$\dot{v}_i(t) = -\frac{m_{1i}}{m_{2i}} u_i(t) \omega_i(t) - \frac{d_{2i}}{m_{2i}} v_i(t) \quad (1e)$$

$$\dot{\omega}_i(t) = \frac{m_{1i} - m_{2i}}{m_{3i}} u_i(t) v_i(t) - \frac{d_{3i}}{m_{3i}} \omega_i(t) + \frac{\tau_i(t)}{m_{3i}} \quad (1f)$$

where $u_i(t)$ and $v_i(t)$ are the surge and sway speeds, $\omega_i(t)$ is the yaw rate, $f_i(t)$ and $\tau_i(t)$ are the control force and torque inputs, and $m_{ji} > 0$ and $d_{ji} > 0$ are the mass and damping terms for $j \in \{1, 2, 3\}$ (Rehanoglu, 1996; Fossen, 2021). The control objective is to design¹ f_i and τ_i such that the i th vehicle is stabilized at a desired configuration while avoiding collisions with other USVs and obstacles.

It is well known that the position and orientation of (1) cannot be simultaneously stabilized at a desired value using a continuous (Brockett, 1983) or, even, a discontinuous (Pettersen and Egeland, 1996) state feedback control law. Therefore, this paper proposes the use of input-output feedback linearization control law, where the objective is to regulate the position of a *reference point* in front of (x_i, y_i) given by

$$z_{1i} = x_i + L_i \cos \psi_i, \quad z_{2i} = y_i + L_i \sin \psi_i \quad (2)$$

¹In what follows, we will omit the time argument of signals unless deemed necessary.

where $L_i > 0$ is a constant parameter and $\mathbf{z}_i = [z_{1i}, z_{2i}]^T$ are the Cartesian coordinates of the reference point (Rodríguez-Seda et al., 2014; Paliotta et al., 2018). Now, differentiating twice equation (2) and applying the following control force and torque

$$\begin{bmatrix} f_i \\ \tau_i \end{bmatrix} = \begin{bmatrix} \frac{m_{1i} \cos \psi_i}{L_i} & \frac{m_{1i} \sin \psi_i}{L_i} \\ -\frac{m_{3i} \sin \psi_i}{L_i} & \frac{m_{3i} \cos \psi_i}{L_i} \end{bmatrix} \begin{bmatrix} w_{1i} - F_i \\ w_{2i} - G_i \end{bmatrix} \quad (3)$$

$$\begin{aligned} F_i = & -\frac{m_{2i} \omega_i v_i + d_{1i} u_i}{m_{1i}} \cos \psi_i + \frac{d_{2i} v_i + m_{1i} u_i \omega_i}{m_{2i}} \sin \psi_i \\ & + \frac{(m_{2i} - m_{1i}) u_i v_i + d_{3i} \omega_i}{m_{3i}} L_i \sin \psi_i \\ & - u_i \omega_i \sin \psi_i + v_i \omega_i \cos \psi_i - L_i \omega_i^2 \cos \psi_i \\ G_i = & -\frac{m_{2i} \omega_i v_i + d_{1i} u_i}{m_{1i}} \sin \psi_i - \frac{d_{2i} v_i + m_{1i} u_i \omega_i}{m_{2i}} \cos \psi_i \\ & - \frac{(m_{2i} - m_{1i}) u_i v_i + d_{3i} \omega_i}{m_{3i}} L_i \cos \psi_i \\ & + u_i \omega_i \cos \psi_i - v_i \omega_i \sin \psi_i - L_i \omega_i^2 \sin \psi_i \end{aligned}$$

one can show that (1) reduces to

$$\dot{\mathbf{z}}_i = \mathbf{w}_i \quad (4a)$$

$$\dot{\psi}_i = \frac{1}{L_i} [-\sin \psi_i \quad \cos \psi_i] \mathbf{z}_i - \frac{v_i}{L_i} \quad (4b)$$

where $\mathbf{w}_i = [w_{1i}, w_{2i}]^T$ is the new control input for the linearized system. While the internal dynamics (4b) can only be shown to be Lagrange stable, the linear dynamics of the reference point (4a) are controllable. That is, for a any desired position $\mathbf{z}_{di} \in \mathbb{R}^2$, one can design a state feedback control law $\mathbf{w}_i$ such that $\mathbf{z}_i \rightarrow \mathbf{z}_{di}$ as $t \rightarrow \infty$.

To formulate the collision avoidance objective, consider the interaction of a pair of vehicles as illustrated in Figure 1. Note that the *minimum safe distance* (or envelope) between both vehicles, denoted as r_{ij} , is a function of their relative position and orientations

$$r_{ij} := r_{ij}(\mathbf{z}_i, \mathbf{z}_j, \psi_i, \psi_j) = r_{ji}(\mathbf{z}_j, \mathbf{z}_i, \psi_j, \psi_i). \quad (5)$$

That is, r_{ij} depends on the shape of the vehicles and on how the j th USV or obstacle is positioned and oriented with respect to the i th vehicle. A collision is said to take place if $\|\mathbf{z}_i - \mathbf{z}_j\| \leq r_{ij}$ for some time $t \geq 0$. It is assumed that one can find an envelop function r_{ij} that is continuously differentiable with bounded derivative and that the USVs can detect, either via communication or onboard sensors, the relative position and orientation of other agents within a bounded *detection radius* $R > \sup_{i,j \neq i} \{r_{ij} + \Delta_R\}$, where $\Delta_R > 0$ denotes the *reaction gap distance*. The *reaction distance*, $R_{ij} = R_{ji} = r_{ij} + \Delta_R$, defines the distance at which the vehicles start avoiding each other (see Figure 1).

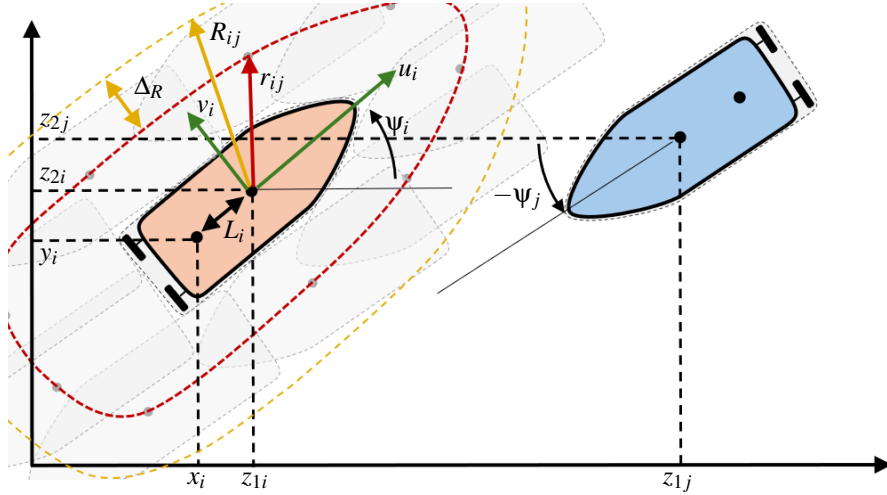


Figure 1: Minimum safe distance between two USVs considering their shape, relative positions, and relative orientations.

Having defined the minimum safe distance, one can formulate the control objective as follows. Design a control strategy $\mathbf{w}_i$ such that $\mathbf{z}_i \rightarrow \mathbf{z}_{di}$ as $t \rightarrow \infty$ and $\|\mathbf{z}_i - \mathbf{z}_j\| > r_{ij} \forall i, j \neq i, t \geq 0$, where $\mathbf{z}_{di}$ is the desired position.

3 CONTROL FRAMEWORK

To achieve the control objective, we propose a decentralized control law based on the concept of avoidance control (Leitmann and Skowronski, 1977)

$$\mathbf{w}_i = K_i(\mathbf{z}_{di} - \mathbf{z}_i) - D_i \dot{\mathbf{z}}_i + \mathbf{p}_i - \sum_{j \in \mathbb{N}_i} \mathbf{a}_{ij} \quad (6a)$$

$$\mathbb{N}_i = \{j \neq i \mid \|\mathbf{z}_i - \mathbf{z}_j\| \leq R_{ij}\} \quad (6b)$$

$$A_{ij} = \left(\min \left\{ 0, \frac{\|\mathbf{z}_i - \mathbf{z}_j\|^2 - R_{ij}^2}{\|\mathbf{z}_i - \mathbf{z}_j\|^2 - r_{ij}^2} \right\} \right)^2 \quad (6c)$$

$$D_i = \max \left\{ \underline{d}, \frac{1}{\|\dot{\mathbf{z}}_i\|^2} \sum_{j \in \mathbb{N}_i} \frac{\partial A_{ij}}{\partial r_{ij}} \frac{\partial r_{ij}}{\partial \psi_i} \frac{v_i}{L_i} \right\} \quad (6d)$$

$$\mathbf{p}_i = \frac{\mu_i}{\|\dot{\mathbf{z}}_i\|} [-\dot{z}_{2i} \ \dot{z}_{1i}]^T, \quad \text{if } \mathbb{N}_i \neq \emptyset \quad (6e)$$

$$\mathbf{a}_{ij} = \frac{\partial A_{ij}^T}{\partial \mathbf{z}_i} + \frac{\partial A_{ij}}{\partial r_{ij}} \frac{\partial r_{ij}}{\partial \mathbf{z}_i} + \frac{1}{L_i} \frac{\partial A_{ij}}{\partial r_{ij}} \frac{\partial r_{ij}}{\partial \psi_i} \begin{bmatrix} -\sin \psi_i \\ \cos \psi_i \end{bmatrix} \quad (6f)$$

where K_i is a positive definite constant matrix, $\mu_i \neq 0$ and $\underline{d} > 0$ are constants, $\mathbb{N}_i$ is the set of agents within the i th USV's reaction distance, and A_{ij} is the avoidance function (Stipanović et al., 2007; Rodríguez-Seda, 2024b). The first two terms in (6) represent the proportional derivative action aimed at stabilizing the system at the desired configuration, where D_i , the damping coefficient, is assumed to be a time-varying function lower-bounded by $\underline{d}$. The last term,

$\sum_{j \in \mathbb{N}_i} \mathbf{a}_{ij}$, is the collision avoidance control, while $\mathbf{p}_i$ is a perturbation to incentivize a non-zero velocity of the reference point while the avoidance control is active and to facilitate the avoidance of deadlocks. Note that, when there are no obstacles or other vehicles within the USV's reaction distance, $\mathbf{a}_{ij} = \mathbf{p}_i = \mathbf{0}$ and $D_i = \underline{d}$.

Assumption 1. The velocity of the i th vehicle's reference point is non-zero when the avoidance control is active. That is, $\|\dot{\mathbf{z}}_i\| \neq 0$ if $A_{ij} > 0$ for some j .

Assumption 1 may not be satisfied in some special cases, such as in environments with dead-ends or traps. To avoid such cases, the proposed framework incorporates a control perturbation (6e) that aims to keep the vessel in motion by making it rotate when the vehicle is almost stationary.

Theorem 1 (Collision Avoidance). Consider the system in (1) with control law (3) and (6). Assume that $\mathbf{z}_{di}$ is constant, that $\|\mathbf{z}_i(0) - \mathbf{z}_j(0)\| > r_{ij} \forall i, j \neq i$, and that Assumption 1 holds. Then, $\|\mathbf{z}_i(t) - \mathbf{z}_j(t)\| > r_{ij} \forall t \geq 0$.

Proof. Consider the following Lyapunov function

$$V = \frac{1}{2} \sum_{i=1}^N \left(K_i \|\mathbf{z}_i - \mathbf{z}_{di}\|^2 + \|\dot{\mathbf{z}}_i\|^2 + \sum_{j \in \mathbb{N}_i} A_{ij} \right) \quad (7)$$

Taking its time derivative yields

$$\begin{aligned}\dot{V} &= \sum_{i=1}^N (K_i(\mathbf{z}_i - \mathbf{z}_{di})^T \dot{\mathbf{z}}_i + \dot{\mathbf{z}}_i^T \mathbf{w}_i) \\ &+ \sum_{i=1}^N \sum_{j \in \mathbb{N}_i} \left(\underbrace{\frac{\partial A_{ij}}{\partial \mathbf{z}_i} \dot{\mathbf{z}}_i + \frac{\partial A_{ij}}{\partial r_{ij}} \frac{\partial r_{ij}}{\partial \mathbf{z}_i} \dot{\mathbf{z}}_i + \frac{\partial A_{ij}}{\partial r_{ij}} \frac{\partial r_{ij}}{\partial \psi_i} \dot{\psi}_i}_{= \frac{1}{2} \sum_{j \in \mathbb{N}_i} \left(\frac{\partial A_{ij}}{\partial \mathbf{z}_i} \dot{\mathbf{z}}_i + \frac{\partial A_{ij}}{\partial r_{ij}} \frac{\partial r_{ij}}{\partial \mathbf{z}_i} \dot{\mathbf{z}}_i + \frac{\partial A_{ij}}{\partial r_{ij}} \frac{\partial r_{ij}}{\partial \psi_i} \dot{\psi}_i \right)} \right) \\ &+ \frac{1}{2} \sum_{i=1}^N \sum_{j \in \mathbb{N}_i} \left(\frac{\partial A_{ij}}{\partial \mathbf{z}_j} \dot{\mathbf{z}}_j + \frac{\partial A_{ij}}{\partial r_{ij}} \frac{\partial r_{ij}}{\partial \mathbf{z}_j} \dot{\mathbf{z}}_j + \frac{\partial A_{ij}}{\partial r_{ij}} \frac{\partial r_{ij}}{\partial \psi_j} \dot{\psi}_j \right)\end{aligned}$$

Now, substituting (6) and (4b) into the above equation and canceling all applicable terms yields

$$\dot{V} = \sum_{i=1}^N \left(-D_i \|\dot{\mathbf{z}}_i\|^2 + \sum_{j \in \mathbb{N}_i} \frac{\partial A_{ij}}{\partial r_{ij}} \frac{\partial r_{ij}}{\partial \psi_i} \frac{v_i}{L_i} \right) \leq 0$$

where we used Assumption 1 and (6d). Since $\dot{V} \leq 0$, we have that V is non-increasing and bounded by $V(0)$ for all $t \geq 0$. Now, suppose that for a pair of agents $\|\mathbf{z}_i(t) - \mathbf{z}_j(t)\| \rightarrow r_{ij}$ for some t . The latter would imply that $A_{ij} \rightarrow \infty \Rightarrow V \rightarrow \infty$, which is a contradiction. Since the solutions of equation (4a) are continuous, one has that $\|\mathbf{z}_i(t) - \mathbf{z}_j(t)\| > r_{ij}$ for all $t \geq 0$ and the proof is complete. $\square$

Theorem 1 guarantees the safe transit of an arbitrary large number of USVs assuming they all start from a safe state and that they remain in motion while resolving a conflict.

Theorem 2 (Position Stabilization). *Assume $\exists T_0 \geq 0$ such that $A_{ij} = 0 \forall t \geq T_0, j \in \{1, \dots, N\}/i$. Then, $\mathbf{z}_i(t) \rightarrow \mathbf{z}_{di}$, $\dot{\mathbf{z}}_i(t) \rightarrow \mathbf{0}$, and $\psi_i(t) \rightarrow 0$.*

Proof. Define the error signals as $\mathbf{e}_i = \mathbf{z}_{di} - \mathbf{z}_i$ and consider the Lyapunov candidate function

$$W = K_i \|\mathbf{e}_i\|^2 + \frac{1}{2} \|\dot{\mathbf{e}}_i\|^2 + \frac{1}{2} \|\underline{d}\mathbf{e}_i + \dot{\mathbf{e}}_i\|^2.$$

Taking its time derivative and noting that $D_i = \underline{d}$ and $\mathbf{a}_{ij} = \mathbf{p}_i = \mathbf{0}$ for all $j \in \{1, \dots, N\}$ and $t \geq T_0$, one can show that

$$\dot{W} = -K_i \underline{d} \|\mathbf{e}_i\|^2 - \underline{d} \|\dot{\mathbf{e}}_i\|^2, \quad \forall t \geq T_0$$

which implies that $\mathbf{z}_{di} - \mathbf{z}_i$ and $\dot{\mathbf{z}}$ converge exponentially to zero as $t \rightarrow \infty$. Then, manipulating (1) and (4), one can obtain that $u_i = [\cos \psi_i \sin \psi_i] \dot{\mathbf{z}}_i$, which, in turns, implies that $u_i \rightarrow 0$ exponentially as $t \rightarrow \infty$. Similarly, substituting (4b) into (1e) yields that

$$\dot{v}_i = \frac{m_{1i} u_i}{m_{2i} L_i} [\sin \psi_i - \cos \psi_i] \dot{\mathbf{z}}_i + \frac{m_{1i} u_i}{m_{2i} L_i} v_i - \frac{d_{2i}}{m_{2i}} v_i. \quad (8)$$

Now, since $u_i \rightarrow 0$ exponentially, $\exists \tilde{T}_0 \geq T_0$ such that $|u_i| < u_{0i} = d_{2i} L_i / m_{1i} \forall t \geq \tilde{T}_0$. Returning to (8) one obtains that

$$\dot{v}_i = \frac{m_{1i} u_{0i}}{m_{2i} L_i} [\sin \psi_i - \cos \psi_i] \dot{\mathbf{z}}_i - \left| \frac{d_{2i}}{m_{2i}} - \frac{m_{1i} u_{0i}}{m_{2i} L_i} \right| v_i \quad (9)$$

for all $t \geq \tilde{T}_0$. Now, consider the following Lyapunov function $\mathcal{W} = \frac{1}{2} v_i^2$. Differentiating with respect to time yields

$$\begin{aligned}\dot{\mathcal{W}} &= \frac{m_{1i} u_{0i}}{m_{2i} L_i} [\sin \psi_i - \cos \psi_i] \dot{\mathbf{z}}_i v_i - \left| \frac{d_{2i}}{m_{2i}} - \frac{m_{1i} u_{0i}}{m_{2i} L_i} \right| v_i^2 \\ &\leq \frac{m_{1i} u_{0i}}{m_{2i} L_i} \|\dot{\mathbf{z}}_i\| |v_i| - \left| \frac{d_{2i}}{m_{2i}} - \frac{m_{1i} u_{0i}}{m_{2i} L_i} \right| v_i^2 \\ &\leq -\kappa v_i^2, \quad \forall |v_i| \geq \frac{m_{1i} u_{0i}}{m_{2i} L_i (1 - \kappa)} \|\dot{\mathbf{z}}_i\|\end{aligned}$$

where $\kappa \in (0, 1)$. Treating $\dot{\mathbf{z}}_i$ as the input to (9), one can conclude that (9) is input-to-state stable (Khalil, 2002). Therefore, if $\dot{\mathbf{z}}_i \rightarrow \mathbf{0}$, so does $v_i \rightarrow 0$, which in turns implies that $\psi_i \rightarrow 0$ and the proof is complete. $\square$

Theorem 2 establishes that, under no collision threat, the proposed control law is able to stabilize the reference point at a desired location in a fixed orientation. In general, we are more interested in the problem of driving the USV along a desired path or to visit a sequence of waypoints. For example, let $\{\chi_{i,1}, \chi_{i,2}, \dots, \chi_{i,m}\}$ be an ordered sequence of m waypoints for the i th vehicle and let $\bar{M} > 0$ be the switching distance threshold. Then, the desired configuration can be updated as $\mathbf{z}_{di}(t) = \chi_{i,k}$, $\forall t \geq 0$, where $k = 1$ for $t = 0$ and

$$k = \begin{cases} k, & \text{if } \|\mathbf{z}_i(t) - \chi_{i,k}^d\| > \bar{M} \\ k+1, & \text{otherwise} \end{cases}$$

for $k \in [2, m-1]$.

4 MINIMUM SAFE DISTANCE

One of the main contributions of the proposed approach is the use of a tighter minimum safe distance, r_{ij} , that takes into account not only the shape of the vehicles and obstacles but also their relative position and orientation. This is in contrast to conventional APF-based methods that assume objects are of spherical form, yielding always a conservative, worst-case scenario, minimum safe distance.

In the control framework (3) we assumed that a differentiable continuous function $r_{ij}(\mathbf{z}_i, \mathbf{z}_j, \psi_i, \psi_j)$ can be formulated in closed-form. Following the work of (Rodríguez-Seda, 2024b; Rodríguez-Seda, 2024a), here we present a closed-form of r_{ij} for vehicles and obstacles whose shape can be approximated by rectangles of different lengths and widths.

Assume the i th and j th vehicles (or obstacles) can be approximated by rectangles with length ℓ_i, ℓ_j and width λ_i, λ_j . Without loss of generality, let their

lengths be aligned with the x -axis and define the following orientation-dependent functions

$$\begin{aligned}\beta_{ij} &= \frac{\ell_i}{2} + \frac{\ell_j}{2} \sqrt{\epsilon^2 + \cos^2 \tilde{\psi}_{ij}} + \frac{\lambda_j}{2} \sqrt{\epsilon^2 + \sin^2 \tilde{\psi}_{ij}} \\ \gamma_{ij} &= \frac{w_i}{2} + \frac{\ell_j}{2} \sqrt{\epsilon^2 + \sin^2 \tilde{\psi}_{ij}} + \frac{\lambda_j}{2} \sqrt{\epsilon^2 + \cos^2 \tilde{\psi}_{ij}}\end{aligned}$$

where $\epsilon > 0$ is a small constant chosen for smoothness and $\tilde{\psi}_{ij} = \psi_i - \psi_j$ is the agents' relative orientation. Let $\theta_{ij} = \text{atan2}(z_{2j} - z_{2i}, z_{1j} - z_{1i})$ represent the angle between $\mathbf{z}_i$ and $\mathbf{z}_j$. Then, the minimum safe distance between both agents can be upper bounded by

$$r_{ij} = r_{ji} = \sqrt[2]{\frac{2}{\rho_{ij}^{-\delta} + \rho_{ji}^{-\delta}}} \quad (10)$$

where $\delta \geq 2$ is a constant parameter and

$$\begin{aligned}\rho_{pq} &= \frac{2\beta_{pq}\gamma_{pq}}{\zeta_{pq} + \eta_{pq} - 2\epsilon} \\ \zeta_{pq} &= \sqrt{\epsilon^2 + (\gamma_{pq} \cos(\theta_{pq} - \psi_p) + \beta_{pq} \sin(\theta_{pq} - \psi_p))^2} \\ \eta_{pq} &= \sqrt{\epsilon^2 + (\gamma_{pq} \cos(\theta_{pq} - \psi_p) - \beta_{pq} \sin(\theta_{pq} - \psi_p))^2}\end{aligned}$$

for $p, q \in \{i, j\}$. It is worth noting that choosing smaller $\epsilon \rightarrow 0$ and larger $\delta \rightarrow \infty$ yields tighter bounds on the minimum safe distance at the expense of larger changes in the r_{ij} , i.e., larger $\partial r_{ij} / \partial \mathbf{z}_i$ and $\partial r_{ij} / \partial \psi_i$ terms.

5 SIMULATIONS

We now present simulation results with the conventional APF approach of assuming obstacles of circular shape and the proposed avoidance control framework. We consider four USVs modeled according to (1) with parameters given as $m_{1i} = 200$ kg, $m_{2i} = 250$ kg, $m_{3i} = 80$ kg, $d_{1i} = 70$ kg/s, $d_{2i} = 100$ kg/s, and $d_{3i} = 50$ kg·m²/s (Reyhanoglu, 1996) for $i \in \{1, 2, 3, 4\}$. The vehicles are assumed to have length $\ell_i = 4$ m and width $\lambda_i = 0.5$ m, with the center of rotation located at 0.33 m from the vehicle's geometric center. The workspace, illustrated in the top diagram of Figure 2, consists of a 10 m long, 4 m wide rectangular passage and a 3 m diameter circular static obstacle. For the avoidance control, the structure creating the passage is modeled as a group of 5 m long, square-shaped obstacles. The vehicles are tasked with traveling through the passage by following a series of waypoints denoted by the small open square shapes, with a switching distance threshold of $\bar{M} = 1.5$ m.

We first simulated the case in which vehicles and obstacles are approximated by circular shapes. In this

scenario, the minimum safe distance is constant, i.e., $r_{ij} = \sqrt{(\frac{\ell_i}{2})^2 + (\frac{\lambda_i}{2})^2} + \sqrt{(\frac{\ell_j}{2})^2 + (\frac{\lambda_j}{2})^2}$, and the control law in (3) reduces to

$$\mathbf{w}_i = K_i(\mathbf{z}_{di} - \mathbf{z}_i) - \underline{d}\dot{\mathbf{z}}_i + \mathbf{p}_i - \sum_{j \in \mathbb{N}_i} \frac{\partial A_{ij}^T}{\partial \mathbf{z}_i}$$

where $\mathbf{p}_i$ has been kept to ease the avoidance of deadlocks. The control parameters are taken as $\Delta_R = 2$ m, $K_i = 1.25I_{2 \times 2}$, $\mu_i = (-1)^i$, and $\underline{d} = 6$. The geometric center of the vehicles is chosen as the reference point, i.e., $L = 0.67$ m. The simulation results are illustrated in Figure 2. Note that the first and second vehicles take nearly 15 s to resolve their first conflict, which was converging to the first waypoint (refer to the middle diagram in Figure 2). Eventually, all vehicles move toward the passage, but none of them is able to travel through. Instead, all four USVs remained in a deadlock.

We then simulated the multi-vehicle system with the proposed avoidance control (3) keeping same control parameters with the addition of $\epsilon = 0.01$ and $\delta = 6$ for the definition of r_{ij} . The results are shown in Figure 3. The vehicles are in close proximity of each other and the obstacles within the first 15 s. However, they are able to solve the conflicts and one by one is able to navigate safely through obstacles and the narrow passage, avoiding collisions at all times.

6 CONCLUDING REMARKS

This paper proposed a novel distributed, cooperative collision avoidance control framework for an arbitrarily large group of autonomous, underactuated USVs in restricted and congested waters. The control framework is built on the concepts of APF and avoidance functions to guarantee the collision-free transit of vessels under some mild assumptions. The novelty of the approach relies on the use of a non-constant minimum safe radius that takes into account the shape and the relative position and orientation of vehicles and obstacles. The result is a reduction in the minimum safe distance that vehicles must keep from each other and other obstacles, allowing them to maneuver in narrow and obstacle-dense waters. We mathematically show that the proposed control framework guarantees collision avoidance at all times as long as the vehicle maintains a non-zero velocity while in conflict, which can be enforced by forcing them to rotate. Simulation results with four vehicles in restricted waters demonstrated the advantage of the proposed approach over traditional APF-based methods.

Future research should investigate the ability of the vehicles to avoid collisions without the assumption

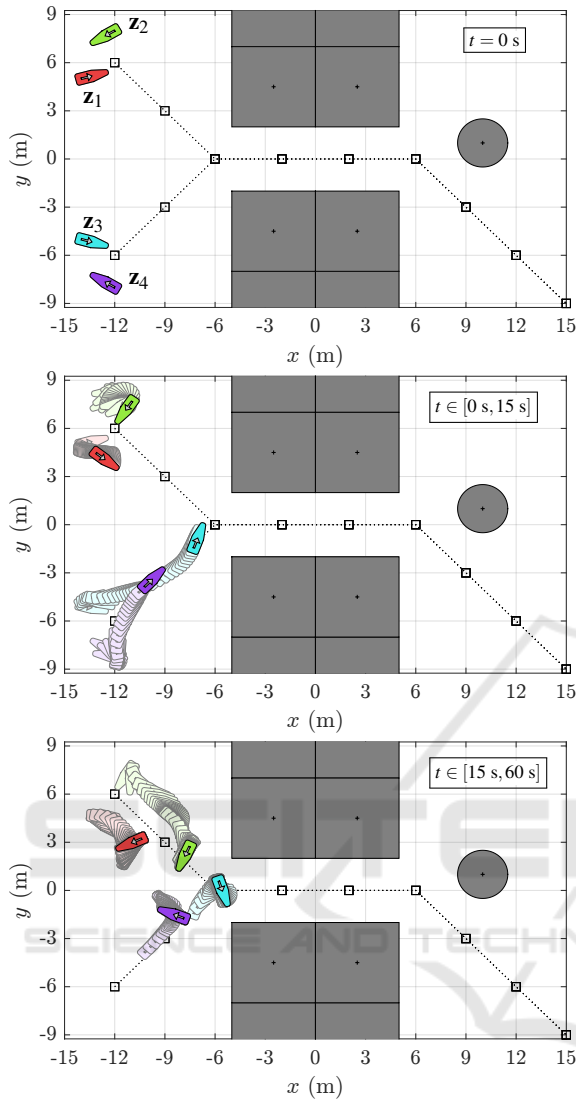


Figure 2: Sequential motion of the multi-vehicle system under the traditional approach of assuming a constant minimum safe distance. The top diagram illustrates the initial configuration. The two bottom plots illustrate the motion of the vehicles from $t = 0$ s to $t = 60$ s. Positions and orientations are superimposed every 0.5 s.

tion of non-zero velocity. In addition, we plan to formulate the proposed minimum safe distance and avoidance functions for vehicles and obstacles of other shapes and to include water disturbances, such as currents or waves, into the analysis.

REFERENCES

- Brockett, R. W. (1983). Asymptotic stability and feedback stabilization. In Brockett, R. W., Millman, R. S., and Sussmann, H. J., editors, *Differential Geometric Con-*

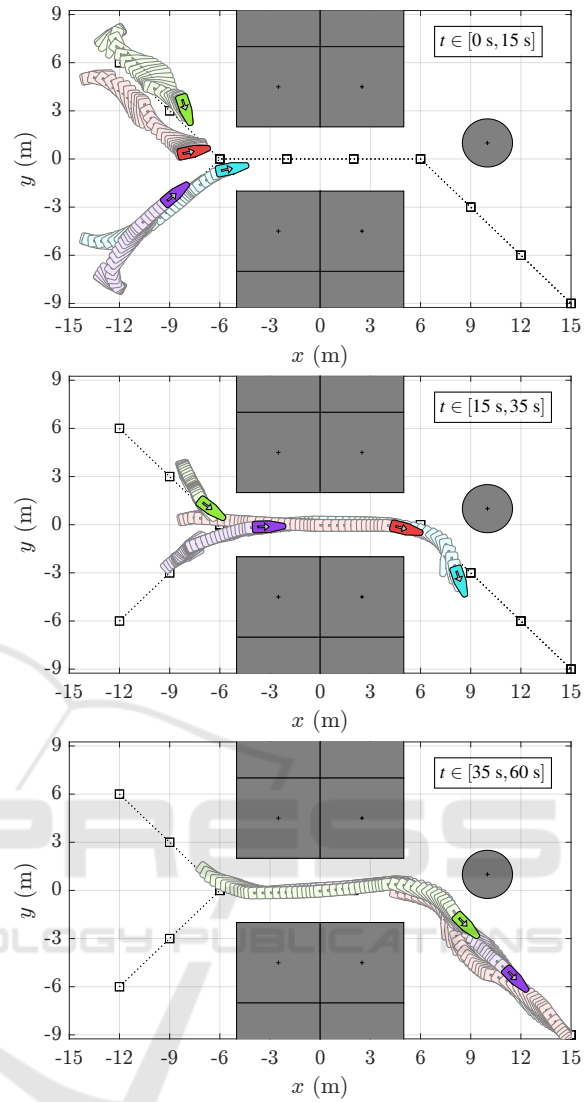


Figure 3: Sequential motion of the multi-vehicle system under the proposed approach, where the minimum safe distance is a function of their relative position and orientations. The plots illustrate the motion of the vehicles from $t = 0$ s to $t = 60$ s. Positions and orientations are superimposed every 0.5 s.

trol Theory, pages 181–191. Birkhauser, Boston.

- Er, M. J., Ma, C., et al. (2023). Intelligent motion control of unmanned surface vehicles: A critical review. *Ocean Engineering*, 280:114562.
- Fossen, T. I. (2021). *Handbook of marine craft hydrodynamics and motion control*. John Wiley & Sons, 2nd edition.
- Khalil, H. K. (2002). *Nonlinear Systems*. Prentice Hall, New Jersey.
- Leitmann, G. and Skowronski, J. (1977). Avoidance control. *J. Optim. Theory Appl.*, 23(4):581–591.
- Li, L., Wu, D., et al. (2021). A path planning strategy unified with a colregs collision avoidance function based

- on deep reinforcement learning and artificial potential field. *Applied Ocean Research*, 113:102759.
- Li, Y., Hou, P., et al. (2025). Research on collision avoidance methods for unmanned surface vehicles based on boundary potential field. *Journal of Marine Science and Engineering*, 13(1):88.
- Lyu, H., Hao, Z., et al. (2023). Ship autonomous collision-avoidance strategies—a comprehensive review. *Journal of Marine Science and Engineering*, 11(4):830.
- Paliotta, C., Lefeber, E., et al. (2018). Trajectory tracking and path following for underactuated marine vehicles. *IEEE Transactions on Control Systems Technology*, 27(4):1423–1437.
- Pettersen, K. Y. and Egeland, O. (1996). Exponential stabilization of an underactuated surface vessel. In *IEEE Conference on Decision and Control*, volume 1, pages 967–972.
- Reyhanoglu, M. (1996). Control and stabilization of an underactuated surface vessel. In *IEEE Conference on Decision and Control*, volume 3, pages 2371–2376.
- Rodríguez-Seda, E. J. (2024a). A decentralized guaranteed collision avoidance control framework for multi-vehicle systems in highly constrained spaces. In *Proc. International Conference on Informatics in Control, Automation and Robotics*, volume 1, pages 229–236.
- Rodríguez-Seda, E. J. (2024b). Reactive collision avoidance control for nonholonomic vehicles and obstacles of arbitrary shape. *ASME. Letters Dyn. Sys. Control*, 4(3):031005.
- Rodríguez-Seda, E. J., Tang, C., et al. (2014). Trajectory tracking with collision avoidance for nonholonomic vehicles with acceleration constraints and limited sensing. *Int. J. Robot. Res.*, 33(12):1569–1592.
- Stipanović, D. M., Hokayem, P. F., et al. (2007). Cooperative avoidance control for multiagent systems. *J. Dyn. Syst. Meas. Control*, 129:699–707.
- Vagale, A., Oucheikh, R., et al. (2021). Path planning and collision avoidance for autonomous surface vehicles I: A review. *Journal of Marine Science and Technology*, pages 1–15.
- Xue, Y., Lee, B., and Han, D. (2009). Automatic collision avoidance of ships. *Proc. of the Institution of Mechanical Engineers, Part M: Journal of Engineering for the Maritime Environment*, 223(1):33–46.
- Zhang, G., Han, J., et al. (2022). Apf-based intelligent navigation approach for usv in presence of mixed potential directions: Guidance and control design. *Ocean Engineering*, 260:111972.