Advanced Predictive Process Control for Industrial Thickeners

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Keywords: Advanced Process Control, Industrial Thickeners, Thickener Automation, Adaptive Model Predictive Control, Real-Time Parameter Estimation.

Abstract: Efficient control of industrial thickeners is crucial for optimizing solid-liquid separation processes, especially in fields like mining and wastewater treatment. Traditional model predictive control (MPC) strategies, even though useful in most applications, can face trouble trying to maintain their performance when faced with time-varying dynamics due to factors such as wear and tear of equipment or changes in feed properties. To address these limitations, this paper highlights an adaptive model predictive control (AMPC) strategy that uses real-time parameter identification to update the prediction model of the usual MPC algorithm. The results show that while AMPC improves the robustness of the controller significantly, keeping critical process parameters such as slurry density well within operational limits under changing conditions, it still faces a number of challenges. AMPC struggles to compensate for unknown disturbances or to optimize flocculant consumption, resulting in economic problems. These results suggest that, despite the improvements offered by AMPC, further research is required to develop advanced disturbance rejection mechanisms and incorporate flocculant optimization strategies for more efficient and cost-effective performances.

1 INTRODUCTION

Thickeners are vital elements in industrial processes, intended to enhance the effectiveness of solid-liquid separation. These units are essential in several industries, such as mining and wastewater treatment, improving the solid concentration of slurries to guarantee cleaner overflows and denser underflows. (Adewuyi et al., 2024; Wang et al., 2024). Thickeners utilize flocculants to consolidate tiny particles, thus enhancing settling rates and clarifying the overflow.(Farrow et al., 2000; Concha and Fernando, 2014). Although this task is straightforward, the variable nature of the input, which fluctuates in concentration, particle size, pH, and other parameters, presents ongoing challenges to control systems. (Betancourt et al., 2014; Juan and Aldo, 2019). In modern contexts, improved process control is essential for negotiating the intricate interactions of chemistry and physics within thickeners, assuring maximum performance. (Ojeda et al., 2014; Kosonen et al., 2017).The transition from manual to automated control systems in thickening operations illustrates this ongoing progress intended to address the difficulties through technological advances.(Xie et al., 2015).

Traditional Model Predictive Control (MPC) effectiveness accounts for its widespread use in many industrial applications. However, it faces constraints that necessitate the investigation of more adaptable control strategies.

The paper presents an Adaptive Model Predictive Control (AMPC) framework to improve the robustness and adaptability of thickening operations, ensuring process parameters remain within operational norms under variable conditions. It also explores the challenges faced by this method, particularly in addressing unknown dynamics and optimizing flocculant consumption, which are crucial for operational and economic efficiency in industrial thickening processes.

El Hamrani, M., Benjelloun, K., Kenné, J.-P., Maarouf, S. and Elkhouakhi, M.
Advanced Predictive Process Control for Industrial Thickeners.
DOI: 10.5220/0013508000003970
In *Proceedings of the 15th International Conference on Simulation and Modeling Methodologies, Technologies and Applications (SIMULTECH 2025)*, pages 262-269
ISBN: 978-989-758-759-7; ISSN: 2184-2841

2 THICKENER OPERATION: NAVIGATING THE COMPLEXITIES

Thickening is considered the most implemented technology for producing high-solids slurries in the mining industry.(Oulhiq et al., 2024; Wang et al., 2024). Thickening provides two main purposes: clear water recovery at the surface and thickened tailings by sedimentation.(Wang and Xiao, 2024). Flocculants and coagulants are then widely used to increase the settling velocity of the solids to provide fast clear-water recovery.(Zaki et al., 2023). Figure 1 illustrates a typical thickener commonly used in many mining operations. Thickeners are complex systems, using several components to achieve a suitably high density. (Nagai et al., 2018). Their mode of operation is based on the coexistence of certain fundamental elements (Juan and Aldo, 2019).

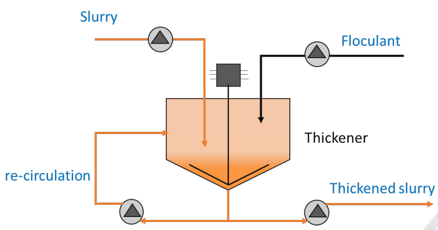

Figure 1: Typical Thickener used in the mining industry.

The operation of the thickener analyzed begins at the entry point, where the pulp is introduced into the thickener by a centrifugal pump with a density of between 1450 and 1530 kg/m³. The thickened sludge is transferred to the center of the thickener by means of slow-moving arms, also known as rake mechanisms, which play a key role in consolidating the sedimented particles, known as residues(Ojeda et al., 2014; Xu et al., 2017).

The pulp fed to the thickener is mixed with a flocculant solution using another pump. These flocculants, usually long-chain polymers, help to agglomerate small particles into large agglomerates, encouraging rapid settling and improving the separation of the solid and liquid phases.

Effective control of the tailings and overflow streams is essential to optimize the performance of the thickener (Ruuska et al., 2020; Oulhiq et al., 2024). Dense residues must be pumped out to avoid clogging, while clarified liquid is discharged from the top of the thickener. A recycle pump is used to reintroduce the pulp into the circuit, improving the overall process. This step is crucial for balancing the density of the lower flow and reducing the pressure around the thickener cone, which is essential for maintaining the mechanical integrity and operational stability of the thickener. The final stage of the thickening process produces a thickened slurry, ready to be processed or disposed of according to specific industrial needs (Serova et al., 2016; Oulhiq et al., 2021). Thickeners face challenges related to the interaction between operational variables and variable time constants. Downstream thickeners thus control variations in the flow characteristics of upstream operations, covering both physical and chemical aspects (Li and Gupta, 2023). These fluctuations, linked to feed rate and solids concentration, result from variations in upstream conditions (Pirouz et al., 2017). These defined problems result in inferior products, highlighting the need to improve operational practices and implement advanced process control systems.

3 ADVANCED PROCESS CONTROL OF THICKENERS

The thickening process requires elaborate control systems to ensure optimal performance. While the management of non-linearities and operational complexities is facilitated by intelligent control systems, notably fuzzy logic controllers and expert systems, their use is limited by the lack of historical or heuristic data.Thus, this paper will review and evaluate two predictive control solutions with the aim to assess their effectiveness, discern their limitations, and highlight opportunities for improvement.

3.1 Model Predictive Control (MPC)

A prominent advanced control strategy for thickening processes is Model Predictive Control has (MPC). It has been applied in a number of areas, including coal mining, in order to tackle the difficulties of time varying dynamics and complexity of measured parameters such as coal residue.(Tan et al., 2017) This predictive control strategy is based on an explicit dynamic model of the process to determine the consequence of future changes in the manipulated variables on the output. The control signal is obtained by minimizing a cost function and the controller performance depends highly on the accuracy of the input-output model.(Camacho and Bordons, 1999; Kouvaritakis and Cannon, 2016) The controlled variable of the thickener process is the output slurry density D_{out}, also a state variable. Additional state variables include bed level L_s, overflow turbidity T_d, rake torque T_r, and cone pressure P_c. Manipulated variables are feed slurry flow rate F_{in} and flocculant flow rate F_{floc}. Measured disturbances are inlet slurry density D_{in}, underflow slurry flow rate F_{out}, and circulation flow rate F_{circ}

To predict the output of the process at future times, we use the state space model proposed by (Oulhiq et al., 2024):

$$\begin{aligned} \hat{X}_{k|k-1} &= AX_{k-1} + BU_{k-1} + KD_{k-1} \\ \hat{y}_{k|k-1} &= C\hat{X}_{k|k-1} \end{aligned} \tag{1}$$

For a system with multiple delays, it is suggested that an extended state-space formulation be used. This approach amounts to rewriting the system in terms of additional state variables, which results in a standard delay-free system of increased order. The augmented state vector is defined as follows(Oulhiq et al., 2024):

$$\tilde{\mathbf{x}}_k = \begin{bmatrix} \mathbf{X}_k \\ \mathbf{U}_{k-1} \\ \mathbf{D}_{k-1} \end{bmatrix} \in \mathbb{R}^n, \quad \text{with } n = n_a m_x + \rho_u m_u + \rho_d m_d$$

Subsequently, an augmented state space representation of the system is obtained:

$$\begin{aligned} \tilde{\mathbf{X}}_{k|k-1} &= \tilde{A}\tilde{\mathbf{X}}_{k-1} + \tilde{B}\mathbf{u}(k-1) + \tilde{K}\mathbf{d}(k-1) \\ \tilde{\mathbf{y}}_{k|k-1} &= \tilde{C}\tilde{\mathbf{X}}_{k|k-1} \end{aligned} \tag{2}$$

The objective of the control is the thickener outlet slurry density under measured disturbances while considering the process constraints, namely, the bed level, the overflow turbidity, the rake torque, and the cone pressure. It also considers the process disturbances including the feed slurry density, the circulation flowrate, and the outlet slurry flowrates. The MPC problem is then formulated as:

$$\begin{aligned} & \min_S J(S) = \sum_{i=1}^{N} Q\left(\hat{x}_i - w_i\right)^2 + \sum_{i=1}^{M} Ru_i^2 \\ & \text{Subject to:} \\ & \text{for } k \in [1, N] \\ & \hat{x}(k+1) = Ax(k) + Bu(k) + Kd(k) \\ & \hat{y}(k+1) = C\hat{x}(k+1) \\ & x_{\min} \leq \hat{x}(k+1) \leq x_{\max} \\ & u_{\min} \leq u(k) \leq u_{\max} \\ & \text{for } i \in [M, N] \\ & u(i) = 0. \end{aligned} \tag{3}$$

Where w_i is the underflow slurry density reference at time step i, $S = \{u(1), \ldots, u(M)\}$ is the set of optimized manipulated inputs, M is the control horizon, N is the prediction horizon, R is the penalty for the manipulated variables, Q is the penalty for the quadratic error, $x_{\max}$ and $x_{\min}$ are the maximum and minimum values, respectively, of the state vector x, and $u_{\max}$ and $u_{\min}$ are the maximum and minimum values of the manipulated inputs, respectively.

Model Predictive Control (MPC) is fundamentally based on its capacity to resolve a time-dependent optimization problem during each iteration of the algorithm. Following the completion of the optimization, the system is transitioned to a new state through the execution of a segment of the computed optimal control sequence within the established application horizon. Nonetheless, the accuracy of the prediction model is a pivotal element in the efficacy of MPC, a feature that can be significantly influenced by the uncertainties and dynamic characteristics commonly present in real-world situations.

The integration of adaptive control strategies into the MPC framework presents a promising solution to this challenge. The predictive model's robustness and reliability are enhanced by adaptive control, which allows the system to respond dynamically to changing conditions, even in the presence of ambiguous or fluctuating parameters.

3.2 Adaptive Model Predictive Control

Adaptive Model Predictive Control (AMPC) is a version of Model Predictive Control (MPC) designed to handle the challenges of systems with time-varying dynamics. While traditional MPC relies on a determinist model of the process, AMPC continuously updates the model in real time based on new data.(Landau et al., 2024; Sanchez, 1981) This allows AMPC to compensate for changes in system behavior due to external disturbances, equipment wear, or changing process conditions, ensuring optimal performance over time.

3.2.1 Real-Time Model Adaptation

The fundamental principle of AMPC is real-time model adaptation. AMPC periodically updates the process model as new data becomes available.(Landau et al., 2024). This is crucial in processes where the system dynamics change over time, such as in industrial thickeners, where slurry composition, feed rates, or chemical properties often fluctuate.

Real-time model adaptation typically involves the following steps:

1. **Model Identification.** The system parameters are identified from process data using online estimation techniques, such as Recursive Least Squares (RLS) or Kalman Filtering. The identified parameters reflect the current operating conditions of the process.

2. **Model Update.** The process model used in the MPC optimization is updated at each control interval using the latest parameter estimates.
3. **MPC Optimization.** The MPC algorithm solves an optimization problem using the updated model to compute the optimal control inputs.
4. **Control Implementation.** The first control input in the optimized sequence is applied to the process.

these steps are represented in the framework illustrated in 2 This real-time adaptation allows AMPC

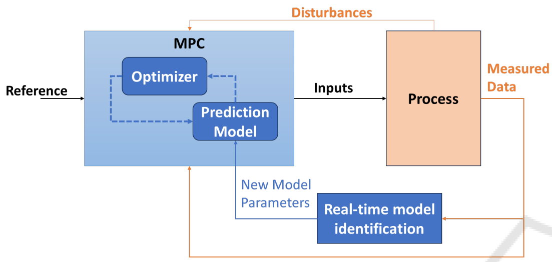

Figure 2: AMPC Framework.

to track changes in process dynamics and maintain high-performance control, even in the presence of uncertainties or disturbances.

3.2.2 Recursive Least Squares (RLS) for Parameter Identification

Recursive Least Squares (RLS) is a popular technique for online parameter estimation in adaptive control systems.(Mohseni et al., 2020; Salamati et al., 2017) It is particularly suited for AMPC because of its ability to efficiently update parameter estimates in real-time, with minimal computational overhead.(Nguyen et al., 2020) RLS is used to minimize the prediction error between the measured system output and the output predicted by the current model.(Gholaminejad et al., 2016) At each discrete time step, the RLS algorithm updates the parameter estimates based on the new input-output data, ensuring that the model accurately reflects the current system behavior. (Ziemann, 2023; Jin and Ding, 2023) The updated parameters are then used by the MPC to compute the control inputs for the next time step.

In the thickener control context, the system dynamics are governed by a set of state-space equations, where the system matrix A and input matrix B can change due to variations in feed characteristics, chemical properties of the slurry, or environmental conditions. The RLS algorithm is employed to update these matrices in real time based on process measurements.

At each time step, the AMPC algorithm solves the following optimization problem:

$$\min_{U(1),\dots,U(M)} \sum_{i=1}^{N} (\hat{x}(k+i|k)-x_{\text{ref}})^T Q(\hat{x}(k+i|k)-x_{\text{ref}}) + \sum_{i=1}^{M} U(i)^T RU(i) \tag{4}$$

Subject to:

$$\hat{x}(k+i+1|k) = A(k)\hat{x}(k+i|k)+B(k)U(i),$$
$$\hat{x}(k) = x(k),$$
$$U_{\min} \le U(i) \le U_{\max},$$

Where:

- $\hat{x}(k+i|k)$ is the predicted state at time $k+i$,
- x_{ref} is the reference state (setpoint),
- Q and R are the weighting matrices for state deviations and control effort, respectively,
- $A(k)$ and $B(k)$ are the updated system matrices obtained using RLS.

The AMPC controller uses the updated model $A(k), B(k)$ at each time step to compute the optimal control inputs that minimize the cost function while satisfying the system constraints.

3.3 Practical Application

To compare the control strategies, we utilize a model developed by (Oulhiq et al., 2024) using the Historical Data-Driven System Identification method. The state space model developed by (Oulhiq et al., 2024) is as follows:

$$\begin{aligned} \hat{x}_{k|k-1} &= A\hat{x}_{k-1} + BU_{k-1} + KD_{k-1} \\ \hat{y}_{k|k-1} &= C\hat{x}_{k|k-1} \end{aligned} \tag{5}$$

Where:

- $\hat{x}_{k|k-1}$ is the state estimate at time k, given the information up to time $k-1$,
- $\hat{y}_{k|k-1}$ is the output estimate at time k, given the state estimate $\hat{x}_{k|k-1}$,
- A is the state transition matrix,
- B is the input matrix,
- K is the gain matrix for the disturbance,
- C is the output matrix,
- U_{k-1} is the control input at time $k-1$,
- D_{k-1} is the disturbance input vector at time $k-1$,
- $\hat{E}_{k|k-1}$ is the prediction error for the state estimate.

The matrices are defined by (Oulhiq et al., 2024).

Table 1: MPC Parameters.

Parameter	Symbol	Value
Prediction horizon	N	20
Control horizon	M	2
Control weighting	R	0.01
Error weighting	Q	1

3.3.1 Application of MPC

Table 1 presents the MPC parameters used in our application.

Initially, the MPC, the model parameters, and the disturbance constraints specified by (Oulhiq et al., 2024) are established. At each sampling instance k, the current states $x(k)$ and disturbances $d(k)$ are measured, and the state prediction $\hat{X}_k$ is calculated utilizing the previous values of X, U, and D.
Subsequently, the MPC problem, as formulated in Equation (11), is addressed using an optimizer. Once the optimization concludes, the control inputs $u_1(k)$ and $u_2(k)$ are derived from the solution and conveyed to the processing system. At the termination of each sampling period, the variables X_{k-1}, $u(k-1)$, and $d(k-1)$ are updated to inform the subsequent iteration. The effectiveness of this approach is evidenced by the simulation results presented in Figure 3

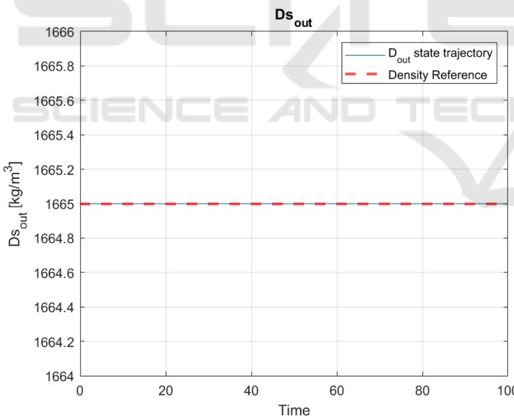

Figure 3: Underflow slurry density $D_{s,\text{out}}$ trajectory with MPC.

As depicted in Figure 3 and Figure 4, the performance of the Model Predictive Control (MPC) is mainly assessed by the controlled process variable and control inputs.

Figure3 illustrates the course of the underflow slurry density, referred to as Dsout. The trajectory closely corresponds with the reference(desired set point) value. This demonstrates the controller's proficiency in sustaining the slurry density near the intended set point.

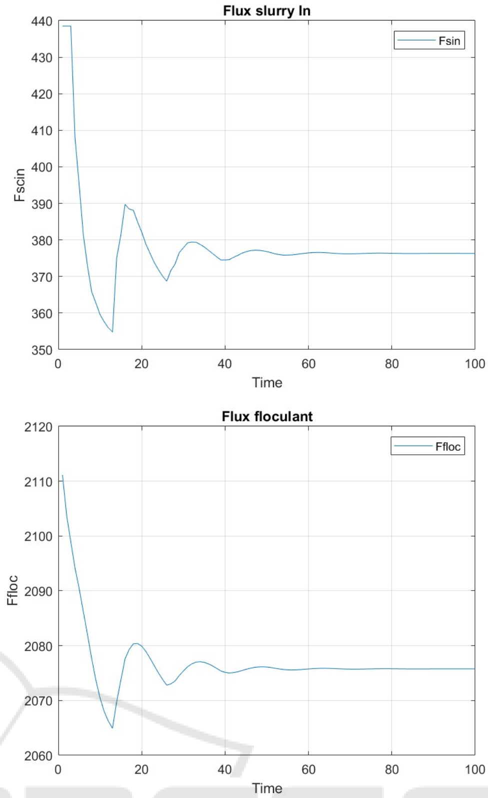

Figure 4: Control inputs.

Figure 4 illustrates the manipulated inputs: feed slurry flow rate (F_{sin}) and flocculant flow rate (F_{floc}). Each plot tracks the respective flow rate as a function of time during the MPC operation.

The feed slurry flow rate stabilizes within set limits, with smooth and decisive control input avoiding saturation. The rest of the parameters also transition smoothly and remain within operational constraints. This highlights the ability of the MPC to effectively regulate inputs without exceeding physical limits or causing reluctance in the command signal.

However, in real-world industrial applications, the dynamics of the system are not always constant. Gradual changes in parameters, such as those caused by wear and tear of pumps or partial clogging of pipes, can significantly alter the behavior of the process. These physical degradations manifest as shifts in the system matrices A and B, affecting the performance of the control strategy.

In our simulation, we introduced parameter variations to emulate these real-world challenges. Starting from $t = 30$ minutes, the matrices A and B were adjusted to simulate scenarios where equipment efficiency declines or flow characteristics are adversely affected. For instance, reduced pump efficiency can alter the gain of the system, while a partially clogged pipe may increase the resistance and impact fluid flow rates. Despite the Model Predictive Control (MPC) initially maintaining robust control, these parameter changes exposed its limitations.

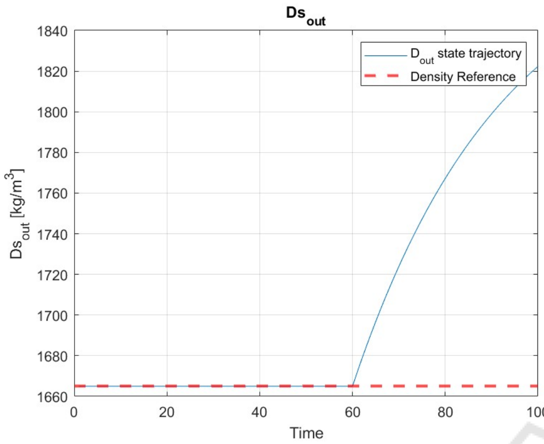

Figure 5: Results of MPC faced with a change in system parameters.

As shown in Figure 5, the underflow slurry density $D_{s,\text{out}}$ begins to deviate significantly from the reference trajectory when the system parameters shift beyond a certain threshold. This occurs because the MPC, relying on a static model, cannot fully compensate for these evolving dynamics. Consequently, the controller struggles to uphold performance, highlighting the need for adaptive control strategies that can adjust in real-time to changing process conditions.

3.3.2 Application of Adaptative MPC

To address the limitations of conventional Model Predictive Control (MPC) in handling time-varying system dynamics, we implement an Adaptive Model Predictive Control (AMPC) strategy. In the previous example of the thickener model, the physical degradations alter the dynamics of the thickening process, affecting critical parameters in the system matrices A and B. The adaptive approach involves identifying these changes and updating the system model in real-time. The identification process focuses on detecting variations in specific elements of the matrices A and B. The simulation of change in the matrices concerns the elements $A_{1,1}(k)$, $A_{3,3}(k)$, and $A_{4,4}(k)$ of matrix A and $B_{1,13}(k)$, $B_{2,3}(k)$, and $B_{4,7}(k)$ of matrix B which change from their initial value, reflecting the degradation or changes in the system dynamics.

The recursive update of the matrices parameters estimates is performed using the RLS algorithm, previously explained.

In our simulations, parameter changes were introduced between $t = 30\,\text{min}$ and $t = 80\,\text{min}$ to emulate the effects of equipment deterioration. The AMPC framework was then applied to adapt the control strategy in real time. Figures 6, 7, and 8 illustrate the performance of the AMPC in managing the system under these challenging conditions.

The trajectory of the underflow slurry density $D_{s,\text{out}}$ is shown in Figure 6. Unlike the standard MPC, the AMPC maintains the density close to the reference value despite the parameter variations, demonstrating improved robustness and adaptability. Figure 7 and 8 depict the behavior of the feed slurry flow rate $F_{\text{sin}}$and the flocculant flow rate F_{floc}. The AMPC provides a smooth and responsive adjustment, stabilizing the flow rates within their operational limits. However, it is important to note that F_{sin} is saturated at the upper constraint limits.

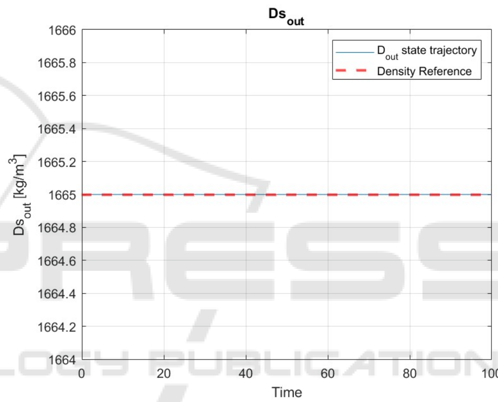

Figure 6: Underflow slurry density $D_{s,\text{out}}$ trajectory with AMPC.

The results demonstrate AMPC's ability to adapt to real-time parameter variations, ensuring thickener performance, but reveals a critical issue where feed slurry flow rate becomes saturated at upper constraints. Once saturation occurs, the AMPC loses its ability to respond effectively to further disturbances or changes in the process, potentially compromising stability and performance.

Additionally, it is important to note that the flocculant flow rate F_{floc} is not optimized in this control approach. As a result, the consumption of flocculant may become inefficient and economically challenging. Excessive use of flocculants increases operational costs and can lead to wastage, which is a significant concern in large-scale industrial processes.

Moreover While AMPC demonstrates robustness in adapting to parameter variations, its effectiveness is limited when faced with unknown disturbances. Figure 9 illustrates the response of the underflow slurry density $D_{s,\text{out}}$ when subjected to an unknown distur-

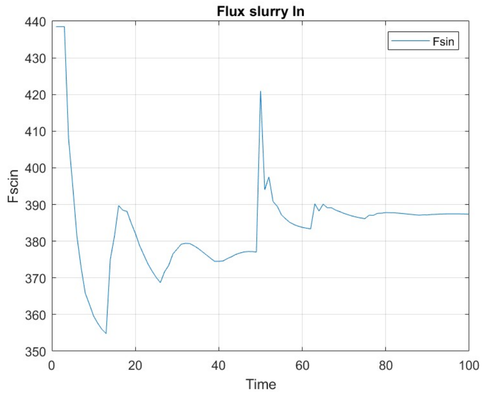

Figure 7: Feed slurry flow rate F_{sin} under AMPC.

bance modeled as white noise. The figure shows that the AMPC cannot fully compensate for these random variations, leading to deviations from the desired setpoint. This limitation underscores the inherent challenge of handling unpredictable physical phenomena, which may require more advanced strategies, such as disturbance observers or robust control techniques, to mitigate.

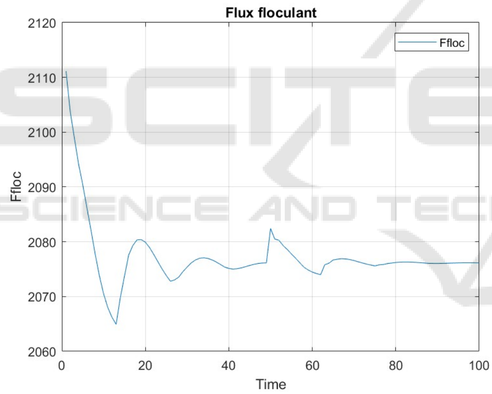

Figure 8: Flocculant flow rate F_{floc} controlled by AMPC.

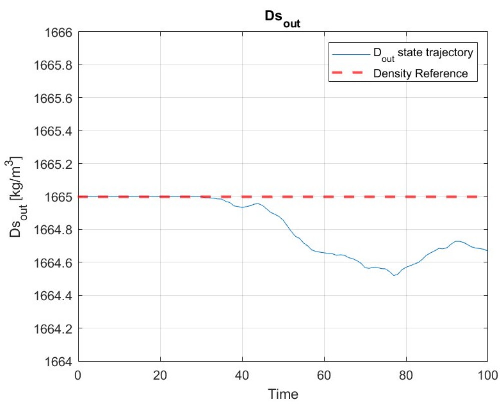

Figure 9: Underflow slurry density $D_{s,\text{out}}$ trajectory with unknown dynamics.

4 CONCLUSION

This paper details the deployment and assessment of Adaptive Model Predictive Control (AMPC) for industrial thickening operations. The AMPC framework enhances control performance by the continuous update of the system model via real-time parameter estimation methods. The findings indicate that AMPC markedly enhances the resilience of thickener operations, maintaining essential parameters within specified operational thresholds despite varying system conditions. The study underscores ongoing issues, notably the AMPC framework's failure to address unforeseen disturbances like white noise and the suboptimal use of flocculants, resulting in economic inefficiencies and elevated operational costs. Consequently, subsequent research should concentrate on incorporating intelligent control methodologies to augment AMPC's disturbance rejection proficiency, as well as implementing flocculant optimization methods to decrease operational costs and enhance thickening control efficiency.

ACKNOWLEDGEMENTS

The authors acknowledge having used artificial intelligence-based authoring tools such as Quillbot, ChatGPT, and Grammarly to refine the language, improve clarity, and increase the overall readability of this document. Despite the contribution of these tools to facilitating language improvements, it should be emphasized that the intellectual content, analysis, and conclusions set out in this document remain entirely the authors' responsibility.

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