

Empowering Mathematics Educators: Integrating ChatGPT as a Tool for Innovative Teaching Practices

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Abstract: This study investigates the potential of a customized ChatGPT model as a tool for enhancing mathematics teaching, specifically focusing on the concept of continuity for real-valued functions. Using the Knowledge for Teaching Mathematics with Technology (KTMT) framework as a theoretical basis, the research examines how personalized AI tools can improve task design, balance among mathematical representations, and the interplay between experimentation and justification. The experimentation involved in-service mathematics teachers who explored both a default and a customized ChatGPT model to create instructional resources. Qualitative analysis revealed that the customized model significantly improved the quality of resources, enabling the creation of diverse, representation-rich, and conceptually balanced tasks. Teachers reported that the personalized ChatGPT facilitated transitions between algebraic, graphical, and tabular representations, supported exploratory problem-solving, and provided opportunities for rigorous justification. The findings contribute to a deeper understanding of how AI-driven tools, when aligned with structured pedagogical frameworks, can support mathematics instruction and teacher development.

1 INTRODUCTION AND RATIONALE

The integration of digital technologies in mathematics education presents both challenges and opportunities for teachers' professional development (Drijvers and Sinclair, 2023). Artificial Intelligence (AI), particularly tools like ChatGPT, offers the potential to enhance teaching by enabling customizable and interactive learning resources, though concerns about reliability and accuracy remain (Bernardi et al., 2024a). The effectiveness of these tools depends on teachers' ability to use them strategically, which requires targeted training.

This study applies the Knowledge for Teaching Mathematics with Technology (KTMT) framework (Rocha, 2020) to analyze how AI can be leveraged to support mathematics instruction, specifically focusing on the concept of continuity for real-valued functions. The KTMT framework identifies three key dimensions:


Task Characteristics: Attention to the cognitive demand, the level of structuring, and the role of technology in the proposed tasks.


Balance Among Mathematical Representations: Effective articulation and integration of diverse representations, such as algebraic, graphical, and numerical formats.


Experimentation and Justification: A balance between leveraging technology for experimentation and fostering mathematical justification through argumentation and proof.

In this research, these dimensions serve as a basis to investigate the impact of using ChatGPT in mathematics education. Specifically, the study focuses on teaching the concept of continuity for real-valued functions of a real variable, a central topic for fostering students' analytical thinking.

In this study group of in-service teachers initially used ChatGPT in its default state to generate instructional materials and then experimented with a customized version designed to produce

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pedagogically refined content (Bernardi et al., 2024b).

The research seeks to validate whether the potential identified by researchers aligns with teachers' experiences. To this end, the study is guided by two primary research questions:

Does using a customized ChatGPT model personalized to the topic of continuity provide benefits to teachers compared to using the default, non-customized version?

How do the three key dimensions identified by the KTMT framework—task characteristics, balance among representations, and the interplay between experimentation and justification—manifest and evolve when utilizing the customized ChatGPT?

The analysis is based on qualitative data collected through an interpretive observational study, employing tools such as in-depth interviews to explore the perceptions, experiences, and strategies of participating teachers.

The scientific contribution of this research lies in demonstrating how the KTMT framework can be employed to analyze the potential of AI in mathematics education, providing a structured approach to assessing its pedagogical impact. Unlike previous works that examine ChatGPT's role through general pedagogical models, such as the didactical tetrahedron (Dasari et al., 2024), dialogic learning (Pavlova, 2024), and the Zone of Proximal Development (Govender, 2023), this study specifically situates AI's impact within a framework designed to assess teachers' technological knowledge in mathematics instruction. The findings aim to offer practical insights for teacher training programs, supporting a more effective and reflective integration of AI into mathematics education.

2 THEORETICAL FRAMEWORK

As a theoretical framework, we use The Knowledge for Teaching Mathematics with Technology (KTMT). It is a framework conceptualizing teachers' professional knowledge to integrate research on teacher knowledge and the incorporation of digital technology (DT) into teaching practice (Rocha, 2013). It builds on foundational work by Shulman (1986) and Mishra and Koehler (2006) and identifies three categories of knowledge: base knowledge domains, inter-domain knowledge, and integrated knowledge. The base knowledge domains include Mathematics, Teaching and Learning, Technology, and Curriculum and Context. These domains align with those found in established teacher knowledge models, but KTMT

uniquely positions the Curriculum and Context domain as transversal, influencing all other domains. A distinguishing feature of KTMT is its two inter-domain knowledge sets:

Mathematics and Technology Knowledge (MTK): Focuses on how DT shapes mathematical concepts, either enhancing or constraining particular aspects.

Teaching and Learning and Technology Knowledge (TLTK): Examines how DT influences teaching and learning strategies, similarly, acting as either an enabler or a limitation.

Both MTK and TLTK draw on research into the integration of DT, identifying its impacts on task design, teaching approaches, and mathematical exploration. For example, DT facilitates exploratory tasks where students analyze cases, formulate conjectures, and identify patterns, thereby enriching the teaching and learning process (Goos and Bennison, 2008). DT also offers access to diverse representations, requiring teachers to guide students in interpreting and articulating the information across these formats (Rocha, 2013). Finally, KTMT introduces **Integrated Knowledge (IK)**—a comprehensive synthesis of the base domains and inter-domain knowledge. IK reflects the teachers' ability to interweave all these elements to effectively integrate DT into their practice, ensuring it supports both mathematical understanding and pedagogical goals.

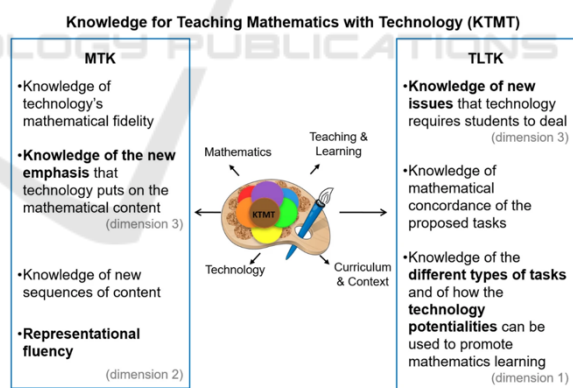


Figure 1: KTMT model and the three dimensions (Rocha, 2020, p.1383).

In Fig. 1, the painter's metaphor is used to represent the KTMT model. The palette represents the knowledge of the curriculum and of the context in which the teacher works. The three primary colors (red, blue, and yellow) represent the base knowledge domains (Mathematics, Teaching and Learning, and Technology). The three secondary colors (orange, green, and purple) represent MTK, TLTK, and PCK (Pedagogical Content Knowledge). Each of these is a

new color (new knowledge) created by the merging of two primary colors and assuming different tonalities depending on the amount of each of the primary colors mixed (different teacher knowledge). This metaphor highlights differences from the idea of intersection present in TPACK, where the development depends directly on the knowledge in the base domains (Mishra and Koehler 2006). In KTMT, an increase in the amount of ink of one primary color may or may not result in a tonality change of the secondary color. The result depends on the quantity of the primary color that is actually mixed with some other primary color (an increase in the knowledge of one of the base knowledge areas need not result directly in an increase of related inter-domain knowledge).

This study adopts a theoretical framework based on three dimensions critical to teaching mathematics with digital technology (DT) in our study ChatGPT, derived from the Knowledge for Teaching Mathematics with Technology (KTMT) model:

Type of Task and Use of Technology: DT enables diverse tasks (e.g., problems, investigations, modeling) that vary in structure and cognitive demand. Teachers must design tasks that leverage DT affordances, balancing procedural and conceptual approaches while focusing on the intended learning outcomes (Ponte, 2005).

Representations and Their Use: DT facilitates dynamic linking of representations (algebraic, graphical, numeric, and tabular), enhancing conceptual understanding (Duval, 2006). Teachers must balance and articulate transitions between representations to maximize learning and avoid privileging specific forms over others (Rocha, 2013).

Experimentation and Justification: DT fosters experimentation but can lead to over-reliance on empirical evidence. Teachers must guide students to move beyond conjectures to deductive reasoning, balancing exploration with rigorous justification to highlight the role of proof in mathematics (Lesseig, 2016).

These dimensions provide a lens to examine how teachers integrate DT effectively in mathematics instruction, focusing on task design, representation use, and the interplay between exploration and justification.

3 METHODS

3.1 Experimental Design

The experiment was conducted with a group of 15 in-service mathematics teachers who had limited prior

experience with ChatGPT but demonstrated a strong interest in exploring innovative teaching practices for their classrooms. The study was designed as an exploratory investigation, organized into distinct phases to evaluate the potential of ChatGPT as a teaching tool. The primary objective was to compare the efficacy of the default ChatGPT with a version specifically personalized for teaching the concept of continuity in real-valued functions.

The experiment comprised the following phases:

Phase 1 (P1): exploration of ChatGPT without instructions;

Phase 2 (P2): exploration of ChatGPT with instructions;

Phase 3 (P3): reflections and final interview.

In the first phase, participants interacted with ChatGPT in its default configuration, without any prior customization or specific instructions. The focus was on the topic of continuity for real-valued functions of a real variable. Teachers explored the system independently, asking questions and generating resources as needed. The primary aim of this phase was to observe how the teachers engaged with the tool in its generic state and to analyze the types and quality of the outputs generated in response to their queries.

In the second phase, participants were introduced to a customized version of ChatGPT, specifically instructed on the topic of continuity for real-valued functions. This personalized version was accessible via a dedicated link and was designed to provide explanations, examples, and tasks aligned with pedagogical goals. The purpose of this phase was to assess how the customization influenced the relevance, accuracy, and educational value of the outputs compared to the first phase. Teachers were encouraged to explore the model's capabilities for generating personalized resources and to reflect on its potential as a tool for supporting mathematical reasoning and instruction.

The final phase involved a structured reflection and feedback session, providing insights into the perceived strengths and limitations of both versions of ChatGPT. They discussed possible applications of the tool in their teaching practices and proposed recommendations for its integration into mathematics education.

3.2 Setup and Configuration of the Continuity-Specific ChatGPT

As part of this research, a specialized ChatGPT model was developed to enhance the exploration and teaching of the concept of continuity for real-valued

functions. This tool was designed not only to support learners but also to inspire educators in designing innovative teaching strategies. The creation of this customized version of ChatGPT required a carefully planned process to ensure that it could address the specific needs of mathematics education in this domain.

The first step in configuring the model was to define its objectives clearly. The idea was to create a conversational assistant capable of engaging users with clear and adaptable explanations, solving mathematical problems related to continuity, and providing pedagogical insights for teaching this fundamental concept. To achieve this, a dataset was prepared containing theoretical content, solved examples, practical exercises, and real-world applications. This dataset covered topics such as the definition of continuity, types of discontinuities (e.g., removable, jump, and infinite), the relationship between continuity and limits, and graphical interpretations of these concepts. To make the tool versatile, examples were included with varying levels of difficulty, and the responses were designed to be dynamic, encouraging users to explore deeper questions and scenarios.

The model was fine-tuned using this dataset to ensure that its responses were accurate, contextually appropriate, and reflective of the pedagogical goals of the project. Special attention was given to the way the model interacts with users: it was designed to adapt to their level of knowledge, providing simpler explanations for beginners and diving into more complex details for advanced users. For instance, when asked about the continuity of a function, the model might start with a basic definition and then guide the user through a step-by-step analysis of a practical example, incorporating graphical visualizations when relevant.

To make the interaction engaging and inspiring, the model includes elements that stimulate curiosity and real-world connections. For example, it might ask users reflective questions like, “Have you ever wondered how the concept of continuity is used to ensure smooth animations in digital graphics?” or propose links to everyday applications of continuity in physics or engineering. Additionally, it was designed to provide motivational feedback, such as highlighting the user’s strengths in problem-solving while suggesting areas for improvement. The tone of the interaction was carefully calibrated to be conversational, encouraging, and inspiring, leveraging emojis and thoughtful phrasing to maintain engagement.

The specialized ChatGPT is equipped to perform a range of functions adapted to the needs of both learners and educators. These include responding to theoretical questions about continuity, solving problems step by step, generating exercises with progressive difficulty, and providing personalized feedback on user-generated solutions. For educators, the tool offers suggestions for teaching strategies and classroom activities, such as designing exercises that link continuity to real-world phenomena or using graphical representations to explain abstract concepts.

This tool is available at the following link: <https://chatgpt.com/g/g6759b2b8242c8191b8243f3fba7099d6-continuita-funzioni-reali-gpt>.

Educators and students alike are encouraged to explore the potential of this specialized ChatGPT. Whether it’s generating interactive lesson plans, testing one’s understanding with custom exercises, or delving into the philosophical beauty of mathematical continuity, this tool is designed to be a versatile and inspiring companion. By combining adaptability, interactivity, and pedagogical value, it represents a step forward in integrating artificial intelligence into mathematics education, creating new possibilities for learning and teaching alike.

3.3 Data Collection Instruments

To ensure a comprehensive analysis of the teachers’ experiences and perceptions about ChatGPT’s usefulness and potential in mathematics education qualitative observations and in-depth interviews were employed. The interactions of teachers with ChatGPT during the sessions were observed, focusing on their problem-solving strategies, patterns of tool usage, and engagement with the AI.

3.4 Data Analysis

The collected data were analyzed using a qualitative approach, focusing on identifying themes and patterns related to the use of ChatGPT in mathematics education, taking into account the three dimensions critical to teaching mathematics with digital technology (DT), derived from the Knowledge for Teaching Mathematics with Technology (KTMT) model.

4 RESULTS AND DISCUSSION

The findings of our study highlight that the use of the customized ChatGPT had a significant impact on the

quality of the instructional materials produced by teachers. This customization contributed to the development of more personalized and innovative lessons, particularly in teaching the concept of continuity for real-valued functions. However, participants' comments during the initial phase reflected concerns about the reliability and appropriate use of such a tool. For instance, T1 noted, "I am very doubtful about the correctness of ChatGPT's responses; I fear it might generate errors or inaccurate content." Similarly, T2 remarked, "I think that the use of AI will, over time, further diminish the teacher's central role in mediating the learning process." T3 expressed uncertainty about their ability to fully utilize ChatGPT, stating, "I don't know if I am capable of using ChatGPT to its full potential; without proper training, I might cause more harm than good, and I don't think it is particularly intuitive to use."

These concerns were partially mitigated as the experiment progressed and participants explored the potential of the customized tool. Below are excerpts from an in-depth interview with three teachers who voluntarily consented to continue the discussion beyond the main phases of the study.

Do you think ChatGPT can help you design innovative lessons on the continuity of functions?

T1: ChatGPT allows me to design innovative lessons on continuity that balance experimentation and justification, stimulate transitions between different representations, and offer customizable tasks with strong educational value.

In what ways?

T1: ChatGPT enables me to create diverse tasks with varying levels of structure and cognitive complexity. I can use it to design exploratory problems where students verify the continuity of a function through real-world scenarios or practical applications. For instance, I discovered that ChatGPT can generate examples of functions representing natural or economic phenomena, allowing students to model and analyze data. This approach supports a balance between procedural tasks, like calculating limits, and conceptual understanding, such as distinguishing between pointwise and global continuity. Additionally, the chatbot helps me customize activities based on students' specific needs, generating tasks that encourage different levels of mathematical reasoning.

T2: ChatGPT is particularly useful for creating activities that integrate multiple representations of functions. I asked the chatbot to provide a textual description of a function accompanied by graphical representations, value tables, and algebraic

formulations. This capability allows me to encourage dynamic transitions between different forms of representation, which is critical for fostering a deep understanding of continuity. Moreover, the chatbot suggested innovative ways to link these representations, ensuring that students don't focus exclusively on one form—such as graphs—while neglecting symbolic reasoning.

T3: ChatGPT facilitates the exploration of hypotheses about continuity by enabling dynamic manipulation of parameters in a function and connecting with tools like GeoGebra for interactive simulations. When teaching discontinuities, I can use ChatGPT to generate examples of functions with removable, jump, or essential discontinuities, describing their algebraic characteristics. For example, the chatbot can define and illustrate a function with a jump discontinuity, such as $f(x) = \frac{x^2-1}{x-1}$, generate the associated graph, and visually highlight where the discontinuity occurs. This helps students connect the symbolic form of the equation with the visual behavior of the function.

Have you noticed differences between responses from the instructed chatbot versus the default version?

T3: Yes, especially when discussing the three types of discontinuities. I documented these differences with screenshots.

T2: One critical advantage of the instructed ChatGPT is its ability to create immediate connections between different representations. For instance, after presenting a function algebraically, I can ask the chatbot to provide textual descriptions explaining the meaning of the discontinuity in language accessible to students. It can then integrate graphs and tables to demonstrate how the discontinuity emerges across various representational contexts, reinforcing the mathematical phenomenon.

T1: I made this request to ChatGPT: "After presenting a function algebraically, could you create textual descriptions that explain the meaning of the discontinuity using language accessible to students?" This was the response from the "non-instructed" chatbot regarding jump discontinuities:

Table 1: “Non-instructed” ChatGPT response and translation in English.

<p>1. Discontinuità di salto Immagina di camminare lungo il grafico di una funzione. Una discontinuità di salto è come un gradino improvviso: camminando lungo la curva, arrivi a un punto in cui devi "saltare" per continuare. Questo succede quando i due pezzi del grafico, uno a sinistra e uno a destra del punto, non si connettono. Ad esempio, se osservi il punto $x = 1$ nella funzione $f(x)$, potresti notare che la curva "salta" da un valore a un altro senza passare per il mezzo. Questo significa che il limite sinistro ($\lim_{x \rightarrow 1^-} f(x)$) è diverso dal limite destro ($\lim_{x \rightarrow 1^+} f(x)$).</p>
<p>1. Jump Discontinuity Imagine walking along the graph of a function. A jump discontinuity is like an unexpected step: as you walk along the curve, you arrive at a point where you must "jump" to continue. This happens when two pieces of the graph, one to the left and one to the right of the point, do not connect. For example, if you observe the point $x=1$ in the function $f(x)$, you might notice that the curve "jumps" from one value to another without passing through the middle. This means that the left-hand limit ($\lim_{x \rightarrow 1^-} f(x)$) is different from the right-hand limit ($\lim_{x \rightarrow 1^+} f(x)$).</p>

T1: This is the response from the instructed chatbot:

Table 2: “Instructed” ChatGPT response and translation in English.

<p>Dopo aver presentato una funzione algebricamente, mi crei descrizioni testuali che spieghino il significato della discontinuità, utilizzando un linguaggio accessibile agli studenti</p> <p>Assolutamente sì! 🌟 Ti presento un esempio per ogni tipo di discontinuità con una funzione algebrica, seguito da una descrizione testuale accessibile agli studenti. Vediamo insieme! 📖</p>
<p>User: After presenting an algebraic function, could you create textual descriptions that explain the meaning of discontinuity, using language accessible to students?</p>
<p>ChatGPT: Absolutely! I will provide an example for each type of discontinuity using an algebraic function, followed by a textual description accessible to students. Let's get started!</p>
<p>1. Discontinuità di prima specie (a salto)</p> <p>Funzione:</p> $f(x) = \begin{cases} 2x + 1 & \text{se } x < 1, \\ 3x - 4 & \text{se } x \geq 1. \end{cases}$ <p>Spiegazione:</p> <p>Questa funzione ha una discontinuità di prima specie in $x = 1$, perché:</p> <ol style="list-style-type: none"> $\lim_{x \rightarrow 1^-} f(x) = 2(1) + 1 = 3$, $\lim_{x \rightarrow 1^+} f(x) = 3(1) - 4 = -1$. <p>I limiti sinistro e destro esistono, ma sono diversi tra loro. Questo crea un "salto" nel grafico in corrispondenza di $x = 1$: se immagini di tracciare il grafico, c'è un'interruzione, come un "gradino".</p>

<p>1. First Kind Discontinuity (Jump Discontinuity)</p> <p>Function:</p> $f(x) = \begin{cases} 2x + 1 & \text{if } x < 1, \\ 3x - 4 & \text{if } x \geq 1. \end{cases}$ <p>Explanation: This function has a first kind discontinuity at $x=1$, because:</p> <ol style="list-style-type: none"> $\lim_{x \rightarrow 1^-} f(x) = 2(1) + 1 = 3$ $\lim_{x \rightarrow 1^+} f(x) = 3(1) - 4 = -1$. <p>The left-hand limit and right-hand limit exist but are different from each other. This creates a "jump" in the graph at $x = 1$. Imagine drawing the graph—there's an interruption, like a "step."</p>
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T1: It also provided a metaphor for the students:
 “🌟 Metaphor for Students: Imagine you're walking along a straight road, but suddenly you find a step change: you have to jump to keep going. This jump represents the discontinuity! 📖”

Does the use of ChatGPT diminish the teacher's role in the classroom?

T1: My role remains crucial for guiding students toward a critical use of technology and consolidating the mathematical rigor required for learning.

T3: One of the main risks, in my view, is over-reliance on empirical verification. For instance, students might conclude that a function is continuous based solely on observing its graph without formally proving it. To avoid this, I would integrate ChatGPT-generated resources into activities that require students to formulate conjectures through exploration but subsequently justify them with rigorous deductions using definitions and theorems. This way, I can balance the exploratory aspect provided by technology with the rigor of mathematical reasoning.

The results of this study demonstrate that the instructed ChatGPT significantly improved the quality of instructional resources produced by teachers, addressing the three key dimensions identified by the KTMT framework: task characteristics, balance among mathematical representations, and the interplay between experimentation and justification. These findings underscore the potential of personalized AI tools to align with and enhance pedagogical goals in mathematics education.

From the perspective of task characteristics, the instructed ChatGPT enabled teachers to design tasks that were not only diverse but also aptable to the

cognitive levels of their students. This aligns with the KTMT framework's emphasis on leveraging digital tools to create tasks with varying degrees of structure and cognitive demand. For instance, teachers used the instructed ChatGPT to generate exploratory problems that required students to verify the continuity of a function in real-world scenarios, such as modeling natural phenomena or economic trends. These tasks encouraged critical thinking by connecting abstract mathematical concepts to practical applications, fostering a deeper understanding of continuity beyond procedural fluency. The personalized model also allowed for incremental scaffolding, enabling teachers to adjust task complexity dynamically. This reflects the importance of technology in supporting differentiated instruction, as highlighted by the KTMT model.

In terms of balance among mathematical representations, the instructed ChatGPT demonstrated its ability to seamlessly integrate algebraic, graphical, and tabular formats. This capability resonates with the KTMT framework's call for dynamic linking of representations to enhance conceptual understanding. Teachers reported that the tool facilitated transitions between these formats, helping students explore the interconnectedness of different representations of continuity. For example, one teacher highlighted how the chatbot could generate a textual explanation of a function's discontinuity alongside its graph and tabular representation, ensuring that students did not overly rely on one form, such as visual graphs, while neglecting symbolic reasoning. This balance is critical for promoting a holistic comprehension of mathematical concepts, a challenge often observed in traditional teaching practices.

The interplay between experimentation and justification was another area where the instructed ChatGPT made a meaningful impact. While digital tools like ChatGPT naturally encourage exploratory learning through simulations and dynamic parameter adjustments, the KTMT framework stresses the need for rigorous justification to prevent over-reliance on empirical observations. Teachers noted that the instructed ChatGPT provided tasks that encouraged students to formulate hypotheses about continuity but also required them to validate their conjectures using formal definitions and proofs. For example, teachers used the tool to guide students through analyzing removable and jump discontinuities, transitioning from initial observations to deductive reasoning. This dual focus on experimentation and justification not only deepened students' understanding of continuity

but also reinforced the importance of mathematical rigor.

Reflecting on the two research questions posed at the outset, the study provides clear answers. First, the use of a customized ChatGPT personalized to the topic of continuity indeed offered significant benefits compared to the default, non-customized version. Teachers highlighted the improved relevance, accuracy, and pedagogical alignment of the resources generated by the instructed model. The personalized instructions enabled the chatbot to address specific learning objectives more effectively, supporting the creation of tasks and explanations that were directly aligned with teaching goals.

Second, the instructed ChatGPT positively influenced all three dimensions of the KTMT framework:

Task Characteristics: The tool facilitated the design of tasks that varied in structure and cognitive complexity, enabling teachers to challenge students at multiple levels of understanding.

Balance Among Representations: It promoted transitions between algebraic, graphical, and tabular formats, fostering an integrated understanding of continuity.

Experimentation and Justification: It supported activities that balanced hands-on exploration with rigorous proof-based reasoning, aligning with the framework's emphasis on connecting empirical exploration with deductive rigor.

These findings highlight the critical role of teacher preparation in maximizing the potential of AI tools like ChatGPT. The initial skepticism expressed by participants—regarding the reliability, appropriateness, and usability of the technology—underscores the need for targeted training to build teachers' confidence and competence in integrating AI into their instructional practices. Without this preparation, there is a risk of reinforcing negative attitudes toward technological innovation and limiting the effectiveness of such tools in educational settings.

In conclusion, the study demonstrates that the customization of ChatGPT not only enhances its functionality but also aligns with the pedagogical priorities outlined in the KTMT framework. By addressing key dimensions such as task diversity, representational balance, and the integration of experimentation with justification, the instructed ChatGPT provides a powerful example of how AI tools can transform mathematics education.

5 CONCLUSION AND FUTURE WORKS

The study faced challenges related to technology adoption and skepticism from participants, particularly regarding the reliability of ChatGPT and their ability to use it effectively. These perceptions influenced how they engaged with the tool, despite ongoing support. The study also highlighted the need for more comprehensive training programs to build teachers' confidence and competence in using AI tools. Another challenge was the limited scope of customization applied to ChatGPT. While improvements were observed compared to the default version, further fine-tuning could yield even better results, emphasizing the importance of iterative development.

Additionally, while the study provided valuable qualitative insights, it would benefit from quantitative analysis to support the findings. A mixed-methods approach, combining both qualitative and quantitative data, would strengthen the evaluation of ChatGPT's impact on teaching practices, such as through pre- and post-assessments of teachers' resource design or students' learning outcomes.

In conclusion, the study explored the potential of a customized ChatGPT model to support teaching continuity for real-valued functions. Using the KTMT framework, the research showed how personalized AI tools can align with pedagogical goals, improving task design, representation balance, and the interplay between experimentation and justification. Significant improvements in instructional resource quality were observed with the customized ChatGPT. Future work will refine the model, expand the participant sample, and integrate a mixed-methods approach to further investigate AI's role in education, contributing to the integration of advanced technologies in mathematics teaching.

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