

Grammar Verification for Students: A Grammar Design Recipe with Verification Steps

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Keywords: Context-Free Grammars, Grammar Design, Grammar Correctness, Formal Languages Education.

Abstract: Formal Languages and Automata Theory courses introduce students to grammars as formal systems to generate words in a language. Although grammars appear conceptually simple, students struggle to develop them given that production rules are nondeterministically applied, which leads to uncertainty about grammar correctness. This struggle may be mitigated by offering students a framework for grammar design. This article advocates that such a framework may be provided using a design recipe in a programming-based approach to Formal Languages and Automata Theory. It puts forth a novel initiative to include grammar verification in such courses. The proposed approach is outlined and illustrated using an extended example. Reflections on expectations and on why the initiative can be successful are presented.

1 INTRODUCTION

In most Formal Languages and Automata Theory (FLAT) textbooks and, therefore, courses, students are exposed to regular (rg) and/or context-free grammars (cfg) (e.g., (Vázquez and Sáiz, 2022; Hopcroft et al., 2006; Linz, 2011; Martin, 2003; Morazán, 2024; Rich, 2019; Sipser, 2013; Sudkamp, 2006)). As any FLAT instructor has witnessed, many students struggle developing grammars. Sipser, for example, states that *context-free grammars are even trickier to construct than finite automata* (Sipser, 2013). To help students, Sipser outlines techniques to help students construct grammars that include divide-and-conquer for languages that are the union of simpler languages, a transformation algorithm for languages that are regular (i.e., build a dfa and transform the dfa to a grammar), and the use of recursion (e.g., generate the beginning and ending terminal symbols of a word and recursively generate the middle symbols). Rich (Rich, 2019), in contrast, offers 3 strategies to develop context-free grammars: generate related word regions in tandem (e.g., the beginning and the end), generate unrelated regions using divide-and-conquer, and use recursion. Neither Sipser nor Rich, however, offer a systematic methodology to develop grammars. Most textbooks, in fact, expect students to learn by example with no attention to systematic design principles nor to grammar correctness.

A new trend in FLAT education takes a design-

and programming-based approach (Morazán, 2024). In this approach, students are presented with a design recipe for grammars and a domain-specific language, FSM (Morazán and Antunez, 2014), embedded in Racket (Flatt et al., 2024), to easily implement their designs. In addition to directly constructing rgs and cfgs, students can also employ unit testing to validate grammars and can implement the algorithms they develop as part of their constructive proofs. An undeniable benefit of this approach is that FSM appeals to Computer Science students trained to write programs and provides immediate feedback on grammar implementation, such timely feedback has proven useful in other areas of Computer Science when exposing students for the first time to a body of work (Race, 2001; Venables and Haywood, 2003).

A design recipe is a series of steps, each with a specific outcome, that guides students from a problem description to a well-designed and validated program. This approach was first pioneered by Felleisen et al. for a first introduction to programming course (Felleisen et al., 2018) and later expanded by the author to the first two introductions to programming courses (commonly referred to in the USA as CS1 and CS2) (Morazán, 2022a; Morazán, 2022b).

The initiative presented in this article builds on previous work done to integrate design and programming into FLAT education (Morazán, 2024), which includes design recipes for building state machines and for building grammars. There is, however, a sharp

contrast between these two design recipes. The former includes steps for machine verification whilst the latter does not include verification steps for grammars. This article proposes that grammar verification ought to also be part of FLAT courses just as machine verification. This is appropriate in FLAT courses, because of the emphasis placed on proving construction algorithms correct. In essence, a grammar defines a program to construct words in a language. As such, it ought to be verified much like students verify programs when studying Discrete Mathematics (Akerkar and Akerkar, 2007) or Software Engineering (Blanchard et al., 2024; Huisman and Wijs, 2023). To this end, the design recipe for grammars is extended with verification steps. These new steps guide students to establish grammar correctness.

The article is organized as follows. Section 2 briefly presents grammar design using FSM. Section 3 discusses the proposed new verification steps to help students establish grammar correctness. Section 4 offers reflections on expectations and on why the proposed initiative is likely to be well-received by students. The article ends with concluding remarks in Section 5.

2 GRAMMAR DESIGN USING FSM

2.1 Constructors and Observers

FSM provides constructors to build `rgs` and `cfgs`:

```
make-rg: N T R S → rg
make-cfg: N T R S → cfg
```

N denotes the set of nonterminal symbols (i.e., syntactic categories). T denotes the set of terminal symbols (i.e., the input alphabet). R denotes the set of production rules. S denotes the starting nonterminal. Each production rule has a left and righthand side. The left hand side always consists of a single element in N . For an `rg`, the righthand consists of one or two elements. Only S may generate, ϵ , the empty word. Any nonterminal may generate a terminal symbol or a terminal symbol and a nonterminal symbol. For a `cfg`, the righthand side consists of an arbitrary number of symbols in $\{N \cup T \cup \{\epsilon\}\}^+$. We denote the language generated by a grammar G as $L(G)$.

There is a generic observer to extract each grammar component. For instance, `grammar-rules`, given a grammar, returns the grammar's production rules. In addition, `grammar-derive`, given a grammar and a word, returns the word derivation if the word is in the grammar's language. Otherwise, it returns a string

1. Pick a name for the grammar and specify the alphabet
2. Define each syntactic category and associate each with a nonterminal clearly specifying the starting nonterminal
3. Develop the production rules
4. Write unit tests
5. Implement the grammar
6. Run the tests and redesign if necessary

Figure 1: The Design Recipe for Grammars.

indicating that the word is not in the grammar's language.

2.2 The Design Recipe for Grammars

The design recipe for grammars is displayed in Figure 1 (Morazán, 2024). It has 6 steps and each requires a concrete outcome to advance grammar construction. Step 1 requires specifying the name for the grammar and its input alphabet. Step 2 requires the definition of syntactic categories, the mapping of these syntactic categories to nonterminal symbols, and the clear identification of the starting nonterminal. The definition of syntactic categories requires identifying what needs to be generated by each.

Step 3 requires specifying the production rules. These rules are designed based on what is generated by each syntactic category as defined in Step 2. Students must reason abstractly in terms of the meaning of each syntactic category and not in terms of input alphabet elements. Step 4 requires writing unit tests. For `rgs`, tests must include words that are in and are not in the grammar's language. For `cfgs`, tests must include words in the language. Words that are not in the language must be tested with caution, given that some derivations may be infinite. The unit tests are written using the syntax provided by `RackUnit` (Welsh and Culpepper, 2024). Tests written for $w \notin L(G)$ determine that the string returned by `grammar-derive` corresponds to a string indicating that w is not in $L(G)$. Tests written for $w \in L(G)$ determine that the last element of the derivation returned by `grammar-derive` corresponds to w .

Step 5 requires implementing the grammar based on the results for the previous steps. Finally, Step 6 requires running the tests and redesigning if errors are detected or if tests fail. We note that debugging is performed in the context of design. That is, students are encouraged to revisit the steps of the design recipe to refine their programs.

```

1 ;; Syntactic Category Documentation
2 ;; S: generates words with number of b > number of a, starting nonterminal
3 ;; A: generates words with number of b ≥ number of a
4 (define numb>numa (make-cfg '(S A)
5                       (A → ε)      (A → bA)
6                       (A → AbAaA) (A → AaAbA) )
7
8 (check-equal? (grammar-derive numb>numa '(a b)) "(a b) is not in L(G).")
9 (check-equal? (grammar-derive numb>numa '(a b a)) "(a b a) is not in L(G).")
10 (check-equal? (grammar-derive numb>numa '(a a a a a)) "(a a a a a) is not in L(G).")
11 (check-equal? (last (grammar-derive numb>numa '(b b b))) 'bbb)
12 (check-equal? (last (grammar-derive numb>numa '(b b a b a a b))) 'bbabaab)
13 (check-equal? (last (grammar-derive numb>numa '(a a a b b b b))) 'aaabbbb)

```

Figure 2: A cfg for $L = \{w|w \in (a b)^* \wedge w \text{ has more bs than as}\}$.

2.3 Illustrative Example

To illustrate the result of applying the steps of the design recipe for grammars, consider implementing a grammar over the alphabet $\{a, b\}$ for the language containing all words with more bs than as. A solution is displayed in Figure 2. The results for Step 1, name and alphabet, are part of the implementation as displayed, respectively, on lines 4 and 5.

To document the syntactic categories for Step 2, students must think about what needs to be generated. The starting nonterminal, S, must generate words that have more bs than as. This is an invariant property that any production rule for S must guarantee. This means that S may generate a b with words on either side that have a number of bs that is greater than or equal to the number of as. Students observe that the words on either side of S define a different (sub)language and, therefore, needs to be represented with, A, a new syntactic category. The invariant property, generate words with a number of bs that is greater than or equal to the number of as, must be guaranteed by any production rule for A. Words in $L(A)$ are generated as follows:

- generate ϵ
- generate an a and a b in any order with words in $L(A)$ before and after both of them
- generate an arbitrary number of bs

Students observe that no new (sub)languages need to be generated and, therefore, only two syntactic categories are needed. The results for this step are displayed on lines 1–3 in Figure 2.

The production rules to satisfy Step 3 are developed based on the results from Step 2. S generates words with more bs than as using the rules displayed on line 6 in Figure 2. A generates word with a number of bs greater than or equal to the number of as using the rules displayed on lines 7–8.

A sample of unit tests for words both in and not in L are developed to satisfy step 4 of the design recipe.

- 7 For each syntactic category design and implement an invariant predicate to determine if a given word satisfies the role of the syntactic category
- 8 For words in $L(G)$ prove that the invariant predicates hold for every derivation step.
- 9 Prove that $L = L(G)$

Figure 3: Verification Steps for Grammars

These are displayed on lines 11–16 in Figure 2. The FSM code in Figure 2 illustrates how Step 5 is satisfied. Finally, Step 6 is satisfied by running the code and observing that no errors are thrown and that no tests fail.

3 GRAMMAR VERIFICATION STEPS

3.1 Proposed Expanded Design Recipe

The design recipe for grammars provides students with a framework for grammar implementation and validation. In addition, it provides a *lingua franca* for students and instructors to discuss grammar design. This design- and programming-based approach makes Automata Theory more relevant to Computer Science students and provides instructors with a framework to understand/grade grammars developed by students. The design recipe for grammars does not, however, address guiding students through the process of verification, which is at the heart of FLAT. This means that students are left unsure about the correctness of their implementation. Therefore, design recipe steps for grammars verification are proposed.

The proposed grammar verification steps, 7–9, are displayed in Figure 3. Step 7 asks students to develop an invariant predicate for each syntactic category satisfying the following signature:

word \rightarrow Boolean

```

1 ;; word → Boolean
2 ;; Purpose: Determine if the given word ought to be
   generated by S
3 (define (S-INV w)
4   (let [(as (filter (λ (s) (eq? s 'a)) w))
5         (bs (filter (λ (s) (eq? s 'b)) w))
6         ]
7     (> (length bs) (length as))))
8 (check-equal? (S-INV '()) #f)
9 (check-equal? (S-INV '(a b a)) #f)
10 (check-equal? (S-INV '(a a a a)) #f)
11 (check-equal? (S-INV '(b)) #t)
12 (check-equal? (S-INV '(a a a b b b)) #t)
13 (check-equal? (S-INV '(b b a a b a b)) #t)
14
15 ;; word → Boolean
16 ;; Purpose: Determine if the given word ought to be
   generated by A
17 (define (A-INV w)
18   (let [(as (filter (λ (s) (eq? s 'a)) w))
19         (bs (filter (λ (s) (eq? s 'b)) w))
20         ]
21     (>= (length bs) (length as))))
22 (check-equal? (A-INV '(a a b)) #f)
23 (check-equal? (A-INV '(a a a a)) #f)
24 (check-equal? (A-INV '(b b a a a b a)) #f)
25 (check-equal? (A-INV '()) #t)
26 (check-equal? (A-INV '(b)) #t)
27 (check-equal? (A-INV '(a a b a b b)) #t)

```

Figure 4: Invariant predicates for $\text{numb} > \text{numa}$.

Each predicate determines if the given word satisfies the conditions specified in Step 2 of the design recipe for its syntactic category. Step 8 asks students to prove that the invariant predicates hold for each derivation step. This is done by induction on the number of derivation steps. Step 9 asks students to prove that the language of the grammar is correct. This is achieved by building on the proof developed for Step 8. That is, students assume that the invariant predicates for the nonterminals in the righthand side of a production rule hold and show that the use of the rule means that the invariant predicate for the nonterminal on the left hand side of the rule holds.

3.2 Illustrative Example

To illustrate the development of answers for the new steps of the proposed expanded design recipe, we shall continue with the design started in Section 2.3. To satisfy Step 7, students are expected to write an invariant predicate for, S and A , the grammar's two syntactic categories. For both predicates, the as and the bs of the given word may be extracted. For S , the length of the bs must be greater than the length

of the as . For A , the length of the bs must be greater than or equal to the length of the as . A sample implementation of the results for this step are displayed in Figure 4. Observe that students are expected to *design* the predicates. Among other things, this means that each predicate must have a signature, a purpose statement, and unit tests using words that ought and that ought not be generated by the syntactic category.

For Step 8, the proof that invariant predicates hold for any derivation is done by induction on the height of a derivation tree for an arbitrary word w . A sample proof that students are expected to produce is displayed in Figure 5. The minimum height is 1, which is always the base case and means that any production rule used must use the starting nonterminal. For $\text{numb} > \text{numa}$, this means the only possible derivation tree is generated using $S \rightarrow b$. An argument must be made that the generated word satisfies $S\text{-INV}$. To establish this, the righthand side of each rule is analyzed to establish that the generated word has more bs than as . For the inductive step, students must analyze any rule that may be used after the first production rule and establish that the generated word satisfies the invariant predicate for the nonterminal on the left hand side of the used production rule. Once again, to establish this students analyze the righthand side of the production like done for the base case. The detailed arguments are found in Figure 5. The reader can appreciate that the induction is not beyond the grasp of an advanced undergraduate Computer Science student that has taken an introduction to Discrete Mathematics course.

For Step 9, we denote the language targeted as L , the constructed grammar as G , and the language for the constructed grammar as $L(G)$. The proof for grammar correctness builds on the proof in Step 8. It is divided into two lemmas. The first establishes that, for an arbitrary word, $w \in L$ if and only if $w \in L(G)$. The second establishes that, for an arbitrary word, $w \notin L$ if and only if $w \notin L(G)$. A sample proof students are expected to develop is displayed in Figure 6. We note that proving that predicate invariants always hold simplifies the argument for grammar correctness. Students build an argument based on invariant predicates holding after every derivation step. Once again, the reader can appreciate that the proof is not beyond the grasp of an advanced undergraduate Computer Science student.

4 REFLECTIONS

The primary expectation for the proposed initiative to include grammar verification in FLAT education is

Theorem 1. For $n_{b>a} > n_{a=b}$ invariant predicates hold.

Proof. Base case: $n = 1$.

If a single rule is used, it must be $(S \rightarrow b)$.

This rule generates the word b , which is a word that has more b s than a s. Thus, S-INV holds.

Inductive Step:

Assume: State invariant predicates hold for $n=k$. Show: State invariant predicates hold for $n=k+1$.

$n=k+1$ means that any rule other than $(S \rightarrow b)$ is used. We establish that invariant predicates hold for each:

$(S \rightarrow AbA)$

The generated word is obtained by concatenating a word with $|b| \geq |a|$, the word b , and a word with $|b| \geq |a|$. The generated word, therefore, has at least one more b than a s. Thus, S-INV holds.

$(A \rightarrow \epsilon)$

The generated empty word has an equal number of a s and b s. Thus, A-INV holds.

$(A \rightarrow AbAaA)$

The generated word is obtained by concatenating a word with $|b| \geq |a|$, the word b , a word with $|b| \geq |a|$, the word a , and a word with $|b| \geq |a|$. The generated word, therefore, has $|b| \geq |a|$. Thus, A-INV holds.

$(A \rightarrow AaAbA)$

The generated word is obtained by concatenating a word with $|b| \geq |a|$, the word a , a word with $|b| \geq |a|$, the word b , and a word with $|b| \geq |a|$. The generated word, therefore, has $|b| \geq |a|$. Thus, A-INV holds.

$(A \rightarrow bA)$

The generated word is obtained by concatenating the word b and a word with $|b| \geq |a|$. The generated word, therefore, has $|b| > |a|$. Thus, A-INV holds. \square

Figure 5: Proof for invariant predicates.

to improve student understanding. This better understanding is to be evidenced in at least two ways. The first involves the development of predicate invariants. Such development is evidence that students have reasoned carefully about what a syntactic category represents and understand their formal role enough to formulate a validation predicate. The predicates students developed are not intended to statically reflect their understanding. Students will also be able to provide them as input to a dynamic visualization tool to simulate the construction of the derivation tree for a given word generated by the grammar. The visualization tool shall indicate for each derivation step, using node coloring, if the invariant for the expanded nonterminal holds. In this manner, students will be able to debug their grammars when words not in the target language are generated by a grammar.

The second involves the development of a correctness proof. Such a development reflects that students understand the relationship between syntactic categories. In the vernacular, we can say that they under-

stand how the pieces of the puzzle fit together. Such a development is evidence that students understand the derivation process. An additional benefit is that such a development further hones their proof-development skills. The development of these skills are usually not emphasized enough in undergraduate Computer Science education and grammar development in a FLAT course presents a perfect opportunity to do so.

It is well-known that mental resistance to formal methods (i.e., proofs) exists (Gries, 1987; Zingaro, 2008). A natural question to ask is: *Why do we believe we can be successful with the proposed initiative?* Indeed, this is an important question that cannot be fully answered until the initiative is deployed in the classroom and student perceptions are measured. Nonetheless, we have reason to believe that the initiative is likely to be successful. This cautious optimism is based on data collected from two US-based universities, Seton Hall University (SHU) and Worcester Polytechnic Institute (WPI), using the design-based methodology (Morazán, 2024) to teach their FLAT

Lemma 1. $w \in L \Leftrightarrow w \in L(\text{numb} > \text{numa})$.

Proof.

(\Rightarrow) Assume $w \in L$.

This means that w has $|b| > |a|$. Given that the production rules can generate more b s than a s in any order, there is a sequence of derivation steps whose yield is w . Thus, $w \in L(\text{numb} > \text{numa})$.

(\Leftarrow) Assume $w \in L(\text{numb} > \text{numa})$.

Given that predicate invariants always hold, this means that w has $|b| > |a|$. Thus, $w \in L$. \square

Lemma 2. $w \notin L \Leftrightarrow w \notin L(\text{numb} > \text{numa})$.

Proof. (\Rightarrow) Assume $w \notin L$.

This means that w has $|a| \geq |b|$. Given that predicate invariants always hold, this means that S cannot generate w . Thus, $w \notin L(\text{numb} > \text{numa})$.

(\Leftarrow) Assume $w \notin L(\text{numb} > \text{numa})$

This means that S does not generate w . Given that the grammar can generate word elements in any order and that predicate invariants always hold, w must have $|a| \geq |b|$. Thus, $w \notin L$. \square

Theorem 2. $L = L(\text{numb} > \text{numa})$.

Proof. The proof follows from Figures 6 to 6. \square

Figure 6: Proof for $L = L(\text{numb} > \text{numa})$.

courses. In these courses, students employed a design recipe for state machines that includes verification steps similar to the ones proposed in this article. That is, students wrote proofs for state predicate invariants holding and for machine correctness.

The two courses enrolled a total of 106 students, 11 at SHU and 95 at WPI. A total of 53 students volunteered to participate in the survey, 10 from SHU and 43 from WPI, making the overall response rate among registered students 50% (the response rate among registered students is 91% at SHU and 45% at WPI). The day the survey was administered, a total of 43 students attended class at WPI (31 in-person and 12 remotely) and 10 students attended class at SHU (all in-person). This attendance rate is typical for both institutions. The responses, therefore, reflect the perceptions of students that regularly attend class and not the perceptions of all students registered for these courses. Among the respondents, 14 (26%) self identified as female, 36 (68%) self identified as male, and the remaining 3 (6%) self identified as: 1 nonbinary, 1 agender, and 1 declined to respond. Finally, students at both institutions received neither payment nor benefits for participating in the survey. Using a Likert scale (Likert, 1932) to respond, 1 (Strongly Disagree) to 5 (Strongly Agree), with 3 as a neutral response, these students were presented with the following statements related to proofs:

Q1: I feel empowered by knowing how to reason and write proofs about formal languages.

Q2: Writing programs in FSM helped me develop constructive proofs.

The distribution of responses is displayed in Figure 7. For Q1, we observe that 70% of respondents tend not to disagree with the statement (responses 3–5). For Q2, we observe that 63% of respondents tend not to disagree with the statement (responses 3–5). These results are very encouraging and indicate that the design- and programming-based approach used is making inroads in dismantling the mental resistance to formal methods. The success achieved with machine verification is the basis for our cautious optimism regarding the success of the proposed initiative.

5 CONCLUDING REMARKS

This article presents a novel initiative to include grammar verification in Formal Languages and Automata Theory education. The design and development process is illustrated using a programming-based approach that requires students to document the role of their syntactic categories, to develop invariant predicates based on these roles, and to provide a proof of grammar correctness. As the reader can appreciate, none of these requirements are beyond the abilities of an advanced undergraduate Computer

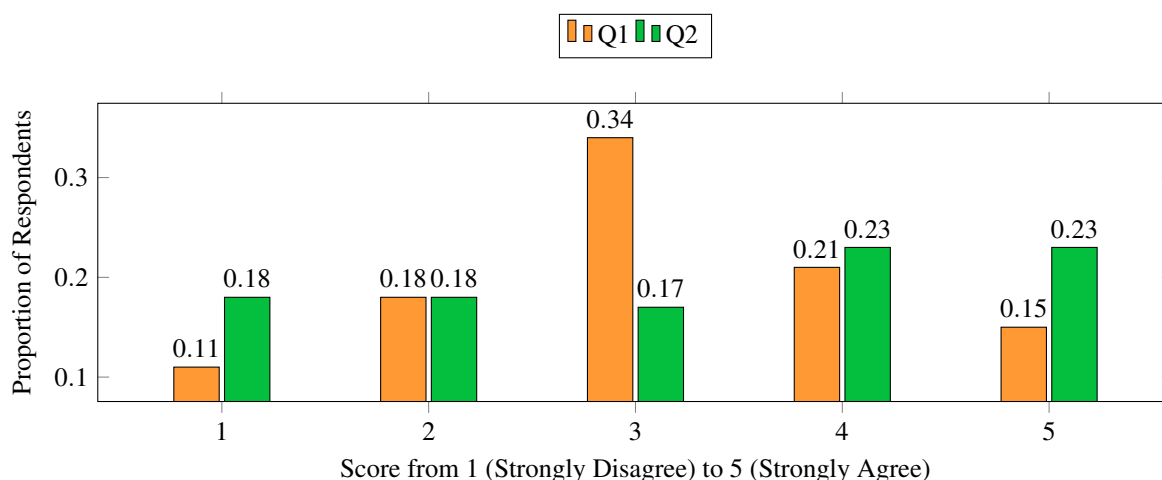


Figure 7: Empirical data on machine proofs.

Science student. Although the proposed initiative is based on a programming-based approach to Formal Languages and Automata Theory, the design steps are transferable to courses still using a pencil-and-paper approach. That is, grammar design is not restricted to programming-based courses.

We are cautiously optimistic that the proposed initiative can be successful for two reasons. The first, as mentioned above, is that none of the verification steps are beyond an advanced undergraduate student that has taken an introduction to Discrete Mathematics course. The second, is that classroom work done with verification of state machines has been successful and well-received by students. Together, these reasons suggests that any Formal Languages and Automata Theory instructor can successfully make grammar verification interesting and palatable to Computer Science students.

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