

Simultaneous Optimization of Edge Bundling and Node Layout Using Genetic Algorithm

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Abstract: This paper describes an evolutionary visualization method that performs edge bundling during the execution of the genetic algorithm. There are several node layout algorithms and edge bundling, however, there are no methods considering both algorithms simultaneously. This paper proposes an algorithm to optimize the fitness function of GABEB, which is genetic algorithm-based edge bundling, and Zhang's node layout simultaneously. The experiments for the sample graphs show the improved result from the viewpoints of several evaluation criteria.

1 INTRODUCTION

The graph, consisting of edges representing connections between nodes, is used to visually represent various relationships among data represented as nodes, such as in aviation charts or relation diagrams (Saga 2012). However, as the scale and complexity of the graph increase, it becomes challenging for users to comprehend the relationships between nodes, leading to visual clutter problem. To address such issues, various visualization techniques have been proposed over time, including force-directed graph drawing (Kamada, 1989) and compound graph layout (Sugiyama 1991)(Dogrusoz 2009).

In recent years, edge bundling has been proposed as a method to solve visual clutter problem. Edge bundling involves spatially grouping edges according to certain rules, reducing the visual complexity of the graph and making it easier to visually grasp the connection relationships between nodes.


Various edge bundling techniques have been proposed, and their effectiveness has been demonstrated (Lhuillier 2017). On the other hand, in recent years, stochastic optimization approaches such as Genetic Algorithms (GA) have also been implemented to address these problems. The advantages of using such GA approaches include the potential for obtaining visualization results not

anticipated by humans and the ability to apply stochastic optimization methods regardless of the continuity of the visualization problem space.

The studies like Ferreira et al. (Ferreira 2018) and Saga et al. (Saga 2020) have proposed edge bundling using GA, receiving certain evaluations. While various edge bundling techniques have been proposed, those that include the process of adjusting node positions in the edge bundling process have been rarely suggested. When the process of adjusting node positions is not included in edge bundling techniques, even if the method effectively bundles edges before application, the visualization effects of edge bundling may not be fully realized due to the inappropriate positions of nodes in the graph before the application of the method, especially in cases with a high number of edge crossings.

Considering the aspects, Meikari et al. (Meikari 2022) proposed an initial approach to simultaneously optimize node layout and edge bundling using a genetic algorithm. However, the solution to the algorithm is the position of the nodes and the combination of edges for bundles, and actual edge bundling takes place after the completion of the genetic algorithm. Therefore, the process of bundling edges and node layout can be considered separate.

In this paper, our goal is to propose an evolutionary visualization method where node layout and edge bundling are optimized simultaneously.

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Additionally, we aim to validate the visualization effects of this method by running the implemented program and comparing the obtained results with those of the methods by Meikari's method.

2 RELATED WORKS

2.1 Edge Bundling

Edge bundling is a graph visualization technique that involves spatially grouping edges within a graph according to certain rules. This bundling aims to reduce the visual clutter of the graph, making it easier to visually comprehend the connection relationships between nodes connected by edges. Figure 1 illustrates an example of a graph with edge bundling.

Originating in 1989 with the simplification of graph edges through spatial concentration (Newbery 1989), edge bundling has evolved into various methods such as Hierarchical Edge Bundling (Holten 2006) and others, demonstrating its effectiveness (Telea, 2010) (Ersoy, 2011) (Hurter, 2012) (Cui, 2008) (Lhuillier, 2017). Another well-known method is Force-Directed Edge Bundling (FDEB), proposed by Holten et al. (Holten, 2009). FDEB employs a mechanical approach, modeling edges as springs that attract each other. It evenly allocates splitting points along the edges, calculates the magnitude and direction of forces acting on each splitting point based on the distance between points and the geometric similarity between edges, and forms smooth curves through iterative movements of splitting points and information updates.

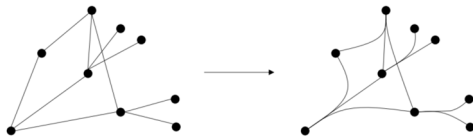


Figure 1: Edge Bundling.

2.2 Edge Bundling Using Genetic Algorithm

In recent years, probabilistic optimization approaches such as Genetic Algorithms (GA) have been implemented. The benefits of using such approaches include the potential to obtain visualization results not anticipated by humans and the ability to apply stochastic optimization methods regardless of whether the visualization problem space is continuous. Additionally, there is an implementation advantage in terms of ease.

Evolutionary Edge Bundling (EEB), proposed by Ferreira et al. (Ferreira 2018), is an edge bundling method that utilizes genetic algorithms to determine bundles (sets of edges) that should be grouped together. Therefore, the process of bundling edges is not included in the genetic algorithm's process, and the actual bundling process uses FDEB after the genetic algorithm concludes. The graph is evaluated based on the similarity in angle and length of edges within bundles that should be maximized and the number of bundles that should be minimized.

Evolutionary Node Layout and Edge Bundling (ENLEB), proposed by Meikari et al. (Meikari 2022), integrates EEB and node layout methods into a single genetic algorithm. This was the first approach aiming to simultaneously optimize edge bundling and node layout in evolutionary visualization. Therefore, it allows obtaining visualization effects considering both edge bundling and node layout. The solution obtained by ENLEB includes the positions of nodes and bundles of edges, and similar to EEB, the process of bundling edges using FDEB occurs after the genetic algorithm concludes. Hence, **the processes of node layout and edge bundling are executed separately.**

Genetic Algorithm Based Edge Bundling (GABEB), proposed by Saga et al. (Saga 2020), is an edge bundling method that uses genetic algorithms by placing control points on edges and optimizing their positions. Control point movements are treated as genes, and genetic operations such as crossover and mutation are applied. The evaluation of individuals includes four aesthetic criteria related to edge bundling: Mean Edge Length Difference (MELD), Mean Occupation Area (MOA), Edge Density Distribution (EDD) proposed by Saga et al. (Saga 2016), and Path Quality proposed by Cui et al. (Cui 2008).

2.2.1 Mean Edge Length Difference

Mean Edge Length Difference (MELD) is a criterion to express the difference from the original edges after edge bundling. A smaller change of edge lengths indicates superior edge bundling because of over-bundling, whereas a large change often leads to a loss of the meaning of the original network. MELD is calculated as

$$MELD = \frac{1}{n} \sum_{e \in E} |L'(e) - L(e)| \quad (1)$$

where n is the number of edges, E is the edge set, and $L(e)$ and $L'(e)$ are the lengths of edge e before and after edge bundling, respectively. In our approach, we aim to minimize the MELD.

2.2.2 Mean of Occupation Area

Mean of Occupation Area (MOA) indicates the degree among the compressed areas before and after edge bundling. Based on the idea that better bundling can compress the area occupied by the edges, MOA is calculated as

$$MOA = \frac{1}{N} \left| \bigcup_{e \in E} O(e) \right| \quad (2)$$

where N is the number of total areas, $O(e)$ is the set of areas occupied by edge e based on an occupation degree (we use 5% of unit area), and $||$ indicates the number of elements contained by a set. Minimising the MOA is one of our optimization goals.

2.2.3 Edge Density Distribution

Edge Density Distribution (EDD) is rooted in the idea that a better edge bundling method can gather edges within a unit area and that the density per unit is high. EDD is calculated as

$$EDD = \frac{1}{|P|} \sum_{p \in P} (H(p) - H)^2 \quad (3)$$

where P is a set of pixels, $H(p)$ is the number of edges pathing pixel p , and H is the average of $H(p)$. We aim to maximise the EDD.

2.2.4 Path Quality

Path Quality (PQ) expresses the degree of zigzag. The lower the PQ, the better the edge bundling. PQ is calculated by the summation of angle differences between neighbours as

$$PQ = \sum_{e \in E} \left(- \sum_{i=3}^m \gamma_i |\Delta_i| \right) \quad (4)$$

with

$$\Delta_i = \begin{cases} A_i - A_{i-1} & \text{if } -\pi < |A_i - A_{i-1}| < \pi \\ |A_i - A_{i-1}| - 2\pi & \text{if } |A_i - A_{i-1}| > \pi \\ 2\pi + |A_i - A_{i-1}| & \text{if } |A_i - A_{i-1}| < -\pi \end{cases} \quad (5)$$

and

$$\gamma_i = \begin{cases} 0 & \text{if } \text{sign}(\Delta_i) = \text{sign}(\Delta_{i-1}) \\ 1 & \text{if } \text{sign}(\Delta_i) \neq \text{sign}(\Delta_{i-1}) \end{cases} \quad (6)$$

, where m is the number of segments divided by control points+1, and A_i is the angle between the original edge and the segment edge. In our GA, we try to maximize PQ. We use the above four criteria separately and perform multi-objective optimization.

3 PROPOSED METHODS

In this paper, we address the issue present in EEB, where the processes of edge bundling and node layout were executed separately. We propose an evolutionary visualization method that

simultaneously performs edge bundling and node layout while optimizing both using a genetic algorithm. To achieve this, we integrate the genetic algorithm-based edge bundling method GABEB and Zhang et al.'s node layout method (Zhang 2005) into a single genetic algorithm.

The reason for using GABEB is that it operates as a genetic algorithm solely for manipulating control points on edges, without considering the impact on node positions. This makes it less likely to encounter problems when simultaneously performing node layout and edge bundling. In the context of our method aiming to perform edge bundling and node layout concurrently, GABEB is well-suited for conducting edge bundling within the genetic algorithm.

Additionally, we choose Zhang et al.'s node layout method for two main reasons. First, it is a genetic algorithm-based node layout method. Second, by adjusting the weights of the evaluation function in Zhang et al.'s method, our proposed method can more easily reflect elements such as the number of edge crossings that need improvement in the graph evaluation, especially in the node layout part.

3.1 The Process of Genetic Algorithm

Figure 2 illustrates the flow of the genetic algorithm for this method. The genetic algorithm of this method repeats the process of crossover, mutation, joining, individual evaluation after generating the initial individuals, and generation update until the termination condition is satisfied. When a control point is subject to crossover or mutation, if the control point is bound to one of the other control points, it is unbound to prevent an anomaly in the information on the movement of the control point.

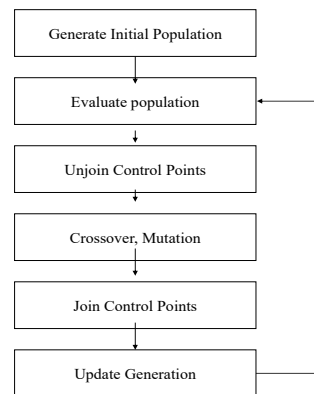


Figure 2: Process of Generic Algorithm.

3.2 Chromosomes

Figure 3 shows an image of the individual representation of the method. Based on GABEB, the individual representation incorporates the concept of control points in FDEB. In FDEB, edges are equally divided by the number of control points, and edge bundling is performed by moving control points based on calculations based on mechanical rules such as the distance between control points. In contrast, GABEB treats the amount of movement of each control point as a gene, which represents each individual. As in FDEB, the original positions of the control points are assumed to be equally divided by the number of control points. In addition to the information on the amount of control point movement for edge bundling, this method also treats the node position of each edge as a gene for the purpose of node layout.

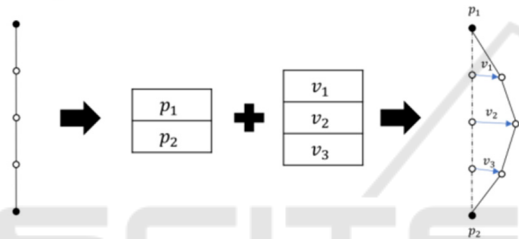


Figure 3: Genetic Representation.

3.3 Crossover

The crossover is performed with respect to the operation on the position of a node and the amount of movement of the control point of an edge. For the former, a simple crossover is performed, in which the position of one node is exchanged between two parent individuals, and an inversion, in which the position of a node of one parent is manipulated. For the latter operation on the mobility of the control point, we perform the blended crossover (BLX- α) ($\alpha=0.5$) (Eshelman 1993).

The inversion of this method is an operation to change the position of a node by numbering the collected positions of the nodes at both ends of an individual edge, and then inverting the numbers within a randomly set interval of numbers. Let S be the list of node positions before applying inversion, S' be the list of node positions after applying inversion, i to j be the interval of node positions to be manipulated, and Pos_i be the collected node position information, the operation by inversion is as in equations (7) and (8). After the crossover is applied to the position of a node, the change in the position of the node is reflected in the entire gene. Each

crossover is assigned an independent crossover probability.

$$S=(Pos_1, \dots, Pos_{i-1}, Pos_i, \dots, Pos_j, Pos_{j+1}, \dots, Pos_n) \quad (7)$$

$$S' = (Pos_1, \dots, Pos_{i-1}, Pos_j, Pos_{j-1}, \dots, Pos_{i+1}, Pos_i, Pos_{j+1}, \dots, Pos_n) \quad (8)$$

3.4 Mutation

Mutation involves two genetic manipulations to change the position of the node as indicated by the node layout method of Zhang et al. and one genetic manipulation to change the amount of movement of the control point.

The mutations shown by Zhang et al. include *non-uniform mutation*, in which the shift of node position becomes smaller with each generation, and *single-vertex-neighborhood mutation*, in which the position shifts within a circular range from the original node position.

Let Pos_{nx} be the X-coordinate of the n th node of an individual and Pos_{nx}' be the X-coordinate of the node after the move, the shift of a node's position due to non-uniform mutation is calculated by the equations (9) and (10).

$$Pos_{nx}' = \begin{cases} Pos_{nx} + \Delta(t, b - Pos_{nx}) & k = 0 \\ Pos_{nx} + \Delta(t, Pos_{nx} - a) & k = 1 \end{cases} \quad (9)$$

$$\Delta(t, y) = y(1 - r^{1-t/T}) \quad (10)$$

In this case, b is the maximum value of the X-axis of the set graph plotting range, and a is its minimum value. k is a value of 0 or 1 randomly determined each time, T is the maximum number of generations, and t is the current number of generations. For the sake of illustration, we have only described the operation on the X coordinate, but the operation on the Y coordinate is similar, and they are performed simultaneously.

The shift of a node's position by single-vertex-neighborhood mutation is calculated by the equations (11) and (12).

$$\begin{cases} Pos_{nx}' = Pos_{nx} + r \cos \theta \\ Pos_{ny}' = Pos_{ny} + r \sin \theta \end{cases} \quad (11)$$

$$r = d_{ideal} * (1 - t/T) \quad (12)$$

In this case, θ is randomly determined between 0 and 2π . And d_{ideal} is calculated by $\sqrt{(s/n)}$ where s is the area of the drawing area. As in crossover, after the mutation on the node position is applied, the change in node position is reflected in the whole gene.

Mutation of a control point is performed by assigning a new random displacement to the target control point that is less than the set maximum displacement. The mutation probability for the position of the node and the mutation probability for the displacement of the control point are assigned

independent mutation probabilities. When it is decided that a mutation on the node position is to be made, one mutation is made at random among the two mutations.

3.5 Control Points Joining

To alleviate the problem of insufficient edge bundling because control points that existed in GABEB rarely exist at the same coordinates, the *join* operation, which places control points that are close enough together at exactly the same coordinates (Saga 2023). The join process of the control points is performed by the following process (Figure 4).

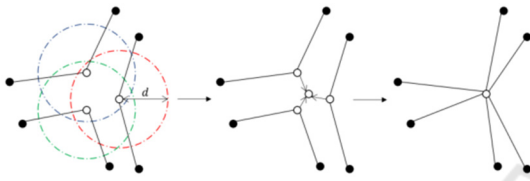


Figure 4: Join process of control points.

1. Find all control points in each edge for which the distance between control points is less than or equal to the set distance (d) that admits a join, and for which no other control points of the edge containing the joined control points are joined to any of the control points.
2. The pair of control points with the shortest distance between them is first joined. If a pair of control points that has been joined before has an edge in common, the pair is not joined.
3. Calculate the average coordinate of each set of control points for which the coupling has been determined and assign the amount of movement from the reference point of the control point to the average coordinate as the amount of movement of each control point. In such a case, the amount of movement before the coupling is stored for use when the coupling is released.

If a control point that is subject to crossover and mutation in edge bundling or a control point whose edge contains a node that is subject to crossover and mutation in node layout is bound to another control point, the binding is unbundled. The reason for not performing the crossover and mutation operation while the control points are still joined is to prevent any of the joined control points from moving beyond an acceptable distance from the edge containing the control point or node position due to the crossover or mutation, which would cause visual confusion.

The unjoin of the control points between two points is done by assigning to each control point the amount of movement before the coupling was

performed. To unjoin two or more control points, only the control points that have been subject to crossover or mutation are assigned the amount of movement before the join, and a new join is processed for the remaining control points, starting from the position of the control points before the join.

3.6 Fitness Function

The fitness function f of this method (equation (13)) is the sum of f_G , which measures the quality of edge bundling with reference to the individual evaluation index used in GABEB, and the evaluation function f_Z shown in the node layout method by Zhang et al. where w_G and w_Z are weights for each evaluation function.

$$f = w_G f_G + w_Z f_Z \quad (13)$$

The fitness function for edge bundling f_G is the sum of the values for edge bundling used in GABEB: EDD and PQ should be maximized, but MELD and MOA should be minimized, so the inverse is taken for these two values.

$$f_G = \frac{1}{MELD} + \frac{1}{MOA} + EDD + PQ \quad (14)$$

The fitness function (equation (15)) presented in the node layout method of Zhang et al. provides an aesthetic evaluation of node locations and the edges affected by them. Here, d_{ij} is the distance between nodes p_i and p_j , E is the entire set of edges, and m is the number of edges. Also, $\angle(p_j p_i p_k)$ is the angle between the edge with p_j and the edge with p_k that share node p_i , $degree(p_i)$ is the degree of node p_i . And d_{ic} is the distance between the location of the center of the drawing area and node p_i . Finally, w_i is a weight that the user can set to emphasize any aesthetic feature.

$$f_Z = \frac{w_1}{\sum_{i=1}^{n-1} \sum_{j=i+1}^n \frac{1}{d_{ij}^2}} + \frac{w_2}{\sum_{(v_i, v_j) \in E} d_{ij}^2} + \frac{w_3}{\sum_{(v_i, v_j) \in E} \frac{(d_{ij} - d_{ideal})^2}{m}} + \frac{w_4}{\sum_{\substack{(v_i, v_j) \in E \\ (v_i, v_k) \in E}} \frac{1}{(\angle p_j p_i p_k)^2}} + \frac{w_5}{\sum_{\substack{(v_i, v_j) \in E \\ (v_i, v_k) \in E}} \frac{(\angle p_j p_i p_k - \frac{2\pi}{degree(p_i)})^2}{degree(p_i)}} + \frac{w_6}{\sum_{\substack{(v_i, v_j) \in E \\ (v_i, v_k) \in E}} \frac{Cross(\overline{p_i p_j}, \overline{p_k p_i}) + 1}{1}} + \frac{w_7}{\sum_{i=1}^n \frac{1}{d_{ic}^2}} + \frac{w_8}{\sum_{i=1}^n \frac{(d_{ic} - \frac{1}{n} \sum_{i=1}^n d_{ic})^2}{n}} \quad (15)$$

Each term with these symbols of the equation (15) has the following meaning. The first term increases as the distance between nodes increases, and the second term increases as the distance between edges decreases. This prevents the distance between edges from becoming too large while still evaluating low node densities. The third term aims to unify the edge lengths by bringing them closer to d_{ideal} . The fourth and fifth terms are evaluations of the angles between

edges and are intended to increase the angles between edges and unify the angles between edges. The sixth term, $Cross(\overline{p_i p_j}, \overline{p_k p_l})$, is the value calculated by the equation (16) and is intended to reduce the number of edge crossings. Finally, the seventh and eighth terms evaluate the symmetry of the graph.

$$Cross(\overline{p_i p_j}, \overline{p_k p_l}) = \begin{cases} 0 & \text{If straight-line segments } \overline{p_i p_j} \text{ and } \overline{p_k p_l} \text{ don't intersect.} \\ 1 & \text{If straight-line segments } \overline{p_i p_j} \text{ and } \overline{p_k p_l} \text{ intersect.} \\ \infty & \text{If straight-line } \overline{p_i p_j} \text{ and } \overline{p_k p_l} \text{ overlap.} \end{cases} \quad (16)$$

3.7 Update and Termination

From the population that has undergone crossover and mutation and the population of the current generation, the top individuals are selected in a number equal to the number of individuals in the current generation to form the population of the next generation. The termination condition is the completion of the generation update for the set maximum number of generations, or when the highest evaluation value is not updated for 100 consecutive generations.

3.8 Example

The parameters of the genetic algorithm used to generate the examples are shown in Table 1. The parameters of the weights in f_Z are set to (0.02, 10, 1, 20, 200, 100000, 0.001, 1). The parameters are very different from each other in order to equalize the magnitude of each item's evaluation value. In particular, the weight of w_G is large, but this is because the evaluation of crossing is treated low as the graph scale increases, and because this method places particular importance on the problem of edge crossing, which cannot be solved by edge bundling. The weights w_G for the evaluation function of GABEB and w_Z for the evaluation function of the node layout method of Zhang et al. are set to 1, respectively.

Table 1: Parameters.

Parameters	Value
Populations	100
Generations	100000
Crossover probability of Edge Bundling	0.9
Mutation Probability of Edge Bundling	0.05
Crossover probability of node layout	0.75
Inversion probability of node layout	0.25
Mutation probability of node layout	0.25
Distance to allow coupling (d)	10
Number of Control point	3
Length of one side of the MOA unit area	5

The graphs of Figure 5 to Figure 7 (the left are examples of graphs before applied, and the graphs on

the right are examples after applied) show that the effect of the parameter on the number of edge crossings, which was set particularly high, is applied, and the number of edge crossings is greatly reduced. This is because the parameters for the evaluation value that increases the distance between nodes and the parameter for the evaluation value that unifies the distance between nodes are set so large, depending on the scale of the graph, that the other evaluation values are not evaluated properly. This is because the parameter setting prevents other evaluation values from being evaluated appropriately because they become very large depending on the scale of the graph. In addition, the effect of edge bundling is not so apparent in Figure 5 because the number of edges is small and the effect of node layout is sufficient, but in Figure 7, dense edges are bundled together in the lower left corner and the effect of edge bundling is fully shown.

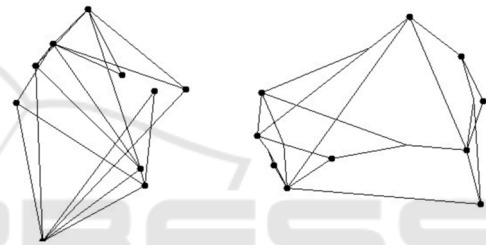


Figure 5: An example applied to a graph with 10 nodes and 20 edges.



Figure 6: An example applied to a graph with 20 nodes and 40 edges.

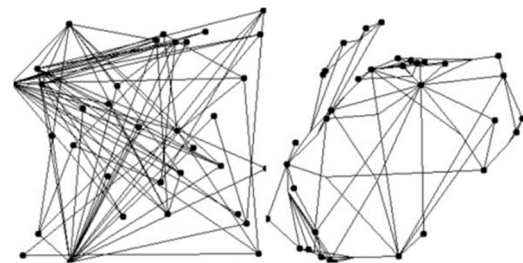


Figure 7: An example applied to a graph with 34 nodes and 78 edges.

4 EXPERIMENTS

4.1 Environment

In order to compare the visualization effects with ENLEB, the target graphs for the experiments were the same as those used in the ENLEB experiments. That is, randomly generated graphs G1 (10 nodes, 20 edges) and G2 (20 nodes, 40 edges), and three graphs representing real world graphs G3 well-known as *karate club* (34 nodes, 78 edges) (Zachary 1977), G4 as *le miserable* (62 nodes, 160 edges) (Girvan 2002), G5 (77 nodes, 254 edges) (Knuth 1993) is the target of this experiment. The criteria used for the measurements were the same five aesthetic evaluation criteria as in ENLEB. The evaluation criteria are shown in Table 2, and the right column of the table shows the desired results.

Table 2: Evaluation criteria.

	Criteria	Desired outcome
C1	Number of edge crossings	Decrease
C2	Number of bundles	Decrease
C3	Average of similarity of edge lengths within a bundle	Increase
C4	Average of the similarity of the angles of the edges in the bundle	Increase
C5	Graph symmetry	Increase

In the experiments, the algorithm of the proposed method was implemented and run on a PC with an Intel Core i7-7500 3.4GHz CPU and 8 GB RAM. The parameters of the genetic algorithm and fitness function in the experiments are the same as those shown in the application example of the proposed method.

4.2 Result and Discussion

The results of the experiment are shown in Table 3. The first row in each graph is for ENLEB, and the second row is for this method. The results in Table 3 show that the symmetry of the graphs is consistently improved compared to ENLEB.

For G1 to G3, where the graph scale is smaller, there is an improvement in the number of edge crossings (C1) and the similarity of the angles of the edges in the bundle (C4). It can be attributed to the positive effect of the inversion operation, which was not implemented in ENLEB. On the other hand, the deterioration in C2 and improvement in C4 can also be attributed to the fact that the appropriate node placement results in more opportunities for the control points in the edges with high similarity to be

spatially close to each other, and less opportunities for the control points in the edges that are not so close to each other.

On the other hand, as the graph scale increased like G4 and G5, the number of edge crossings (C1) and the similarity of the angles (C3) and lengths of the edges (C4) in the bundles deteriorated. The number of bundles (C2) improved. The increase of C1 as the graph scale increases can be attributed to the fact that the evaluation values other than the number of edge crossings increased as the number of edges and nodes increased, and the evaluation value for the number of edge crossings was treated low, resulting in insufficient optimization for C1. The change in C2 through C4 can also be attributed to the fact that as C1 increases, control points in edges have more opportunities to come closer together, resulting in more bundling, but has also increased the chances of bundling edges with low similarity between edges.

Table 3: Experimental results.

-	Node	Edge	C1	C2	C3	C4	C5
G1	10	20	21.0 10.5	10.2 15.8	0.87 0.84	0.83 0.86	336.1 1414.1
G2	20	40	103.1 50.1	19.7 25.7	0.87 0.79	0.83 0.85	189.5 563.3
G3	34	78	375.3 94.6	31.1 40.8	0.85 0.80	0.80 0.90	44.9 253.2
G4	62	160	1711.9 2394.8	65.8 64.0	0.84 0.74	0.83 0.77	27.2 116.9
G5	77	254	4932.0 8298.0	109.2 78.4	0.81 0.74	0.82 0.79	42.5 112.0

Table 4: Calculation Time(s).

-	Node	Edge	Times(s)
G1	10	20	12.6 2797.7
G2	20	40	48.4 3957.1
G3	34	78	270.4 7361.0
G4	62	160	1496.0 22551.6
G5	77	254	4405.1 32437.5

The above results indicate that the number of edge crossings, the similarity of the angles of the edges in the bundles, and the symmetry of the graphs improved for small-scale graphs, but as the scale increased, the number of edge crossings increased, which had a negative impact on the visualization effect. Therefore, it is considered necessary to improve the weighting of evaluation values and the calculation method to deal with the phenomenon that evaluation values other than the number of edge crossings, which is the cause of the increase in the

number of edge crossings, become larger as the number of edges and nodes increases.

The obtained computation time (Table 4) shows that the computation time increased significantly compared to ENLEB. This is thought to be due to the extremely large computation time for coupling and uncoupling, as well as the computation time for the GABEB evaluation values. Therefore, it is necessary to improve the search algorithm using kd-tree, etc. and to reduce the computation time using GPGPU.

5 CONCLUSIONS

In this paper, to solve the problem that the processes of edge bundling and node layout are actually executed separately in ENLEB, we proposed an evolutionary visualization method that performs simultaneous optimization of edge bundling and node layout based on GABEB and Zhang's algorithm. To examine the effectiveness, the experiment results of our method were compared with those of ENLEB.

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