Evaluation of Approximate Reflectional Symmetry

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Abstract: When an object can be split by a plane into two symmetrical parts, one being the mirrored image of the other, the object has a reflectional symmetry with respect to that plane. The symmetry is often only approximate and not necessarily global. Many algorithms exist for the detection of symmetries and there are various applications utilizing symmetrical properties. Yet there are not so many ways to measure the amount of approximate reflectional symmetry. In this paper, we introduce a method for the evaluation of approximate symmetry for objects represented as a point cloud. The method consists of three parts - a relative symmetry distance for measuring the amount of approximate reflectional symmetry, a plot of relative errors, and visualization of errors. This method offers a way how to compare different objects by the amount of symmetry and improves understanding of the symmetrical properties of objects, both quantitatively and visually.

1 INTRODUCTION

Symmetry is an important property of many real-world objects as well as artificially created objects. An object is symmetrical, if there is a transformation (distance preserving, except identity) that maps the object onto itself, hence the object is invariant under the transformation. A typical example is reflectional symmetry when an object can be split into two parts perfectly mirrored by a plane. However, real-world objects are hardly ever perfectly symmetrical. Even if an object can be perceived as symmetrical, it can have minor variations and imperfections, some parts may be missing, etc. The symmetry can still be recognized but it is only an approximate symmetry.

Many methods for the detection of approximate symmetries exist. In our work, we use robust, fast and flexible symmetry plane detection based on differentiable symmetry measure by (Hruda et al., 2022b). For an object in a point cloud representation, we can hence obtain one or more planes of its reflectional symmetries.

The most important part of Hruda's symmetry detection method is a function for symmetry measurement. It is a function evaluating the given object and a plane of reflectional symmetry. Finding the best plane is then done as locating the maxima of this function in transformation space. The contribution of each reflected point to the overall amount of symmetry is computed by summing up the distances to neighboring points. It is a weighted sum, where more distant points get lower weights than closer points. Wendland's function is used instead of Gaussian for weights because it has limited support and is differentiable. The final value of the symmetry measure depends on the number of points, Wendland's function support radius, and the local density of points. These parameters are constant for a single object but they may be different for others. Therefore, Hruda's symmetry measure is not well suited for comparing multiple objects.

The problem addressed by this paper is how to measure the quality of the detected symmetry. If we detect approximate symmetries in several mutually different objects, we want to distinguish more symmetrical objects from the less symmetrical ones. We propose a new relative symmetry distance for measuring the amount of reflectional symmetry of an object

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and for the comparison of different objects. The distance is based on relative errors. It is invariant to the rotation, translation and scale of the input object. Furthermore, we introduce an error plot and a coloring scheme, which help to understand the symmetrical content of the object both quantitatively and visually. The proposed method is demonstrated in a few examples and compared against Hruda's differentiable symmetry measure.

2 RELATED WORK

Symmetry has been studied extensively from many perspectives, e.g., psychological (Bertamini et al., 2018), mathematical (Bizzarri et al., 2022), computational, possible applications, etc. Many different methods for the detection of reflectional symmetries (and also other types of symmetry) have already been developed, e.g. PRST transform by (Podolak et al., 2006), a few methods by (Mitra et al., 2006) (Mitra et al., 2013), including an interesting application (symmetrization) for transforming otherwise asymmetric shapes to their initial symmetric poses by (Mitra et al., 2007), or the method by (Hruda et al., 2022b) using a differentiable symmetry measure. Recently, (Hruda et al., 2022a) presented an idea for the detection of rotational symmetry, using two reflectional symmetry planes.

Regarding approximate symmetry evaluation, there are a few options. An object A and its transformed image $B = T(A)$ could be compared for similarity using standard statistical measures, e.g., intersection over union

$$J(A,B) = \frac{|A \cap B|}{|A \cup B|} \tag{1}$$

(also known as the Jacard/Tanimoto coefficients). However, this approach would require objects with closed boundaries. Another option is to use Hausdorff distance. Although this distance is very sensitive to outliers, a solution exists (Maiseli, 2021). Nevertheless, missing or extruding object parts are the problem with approximate symmetry, not single-point outliers.

Perhaps more suitable is the Metro tool by (Cignoni et al., 1998). This tool was developed for measuring the error for meshes and their simplifications, e.g., multiple levels of detail. It outputs, among other characteristics, the mean error and the maximal error between two surfaces (one of them is sampled and the sample-to-surface distances are computed for each sample). The mean error is computed as the error distance integral over the whole surface divided by the surface area. These two surfaces would correspond to our object A and its image B. The maximal error is the Hausdorff distance. This tool has been used successfully by (Hruda et al., 2022b) for evaluating the symmetry planes from their symmetry detection framework against a view-based symmetry detection approach by (Li et al., 2016) and against a clustering-based approach by Shi et al. (Shi et al., 2016). Besides absolute errors, Metro also outputs relative errors expressed as the percentage of the bounding box diagonal. However, the bounding box diagonal is a global characteristic sensitive to the overall shape of the object. Therefore, we do not consider it so much suitable for the evaluation of approximate symmetry. Unless, of course, the object fits tightly to its bounding box.

There also is the symmetry distance proposed by (Zabrodsky et al., 1993). The idea is that symmetry should be treated as a continuous measure rather than a binary decision (symmetric/asymmetric). They suggest measuring the minimum effort necessary to turn a shape into a symmetric shape. This effort is measured as the sum of squared distances moved by points of the shape, i.e.,

$$SD = \frac{1}{n}\sum_{i=0}^{n-1} ||P_i - \hat{P}_i||^2, \tag{2}$$

where P_i is a point of the shape P and $\hat{P}$ the symmetry transform. The distance is invariant to rotation and translation. Invariance to scale is provided by scaling the object so that the maximum distance from the object centroid is constant. As we already mentioned, such scaling can be very sensitive to outliers and extruding object parts.

Coloring objects for the visualization of symmetry plane and surface errors is not new. He et al. (He et al., 2020) use a technique similar to ours for highlighting object parts in the corresponding symmetry half-spaces. A heat map is used in the Metro tool (Cignoni et al., 1998) to visualize errors. This is a common technique, nevertheless, we consider our coloring scheme worth including because it clearly illustrates the development of approximate symmetry on a given object.

Sometimes, coloring can be used just as a hint where symmetry occurs locally. The symmetry detection method by (Podgorelec et al., 2023) is specialized for Earth observation data. The acquisition of such data (e.g. LiDAR) cannot provide exact pairs of symmetric elements and, therefore, the method is strictly focused on approximate symmetries, which is accomplished by voxelization. The algorithm detects all partial symmetries for a chosen voxel resolution, identifies local and global symmetries among them, and measures the amount of each individual symmetry within the entire input dataset. The method does not evaluate approximate symmetry explicitly, but it gives the obvious hint that in partial or local symmetry, it is sufficient to consider the points contained in the voxels of symmetric pairs.

3 PROPOSED METHOD

Here we present our method for the evaluation of approximate reflectional symmetry of objects with respect to their symmetry planes. The input to our method is an object represented as a point cloud. If no symmetry plane is given, it will be computed by (Hruda et al., 2022b) method. The output consists of quantitative and visual characteristics, suitable for inter-object comparisons and evaluation. The key aspects are discussed in the following subsections.

3.1 Relative Symmetry Error

Let $\mathbf{p}$ be any point of the input object and R be the given plane of reflective symmetry. The symmetry transforms $\mathbf{p}$ to its image $\mathbf{p}'$. Since the object may be only approximately symmetrical, $\mathbf{p}'$ will not necessarily be part of the input object but it may be close. We take the closest point of the input object (from the same side of R where also $\mathbf{p}'$ lies) as the best symmetrical counterpart of $\mathbf{p}$. Since the input object is a point cloud, the closest point is the nearest neighbor of $\mathbf{p}'$ as measured by the Euclidean distance from $\mathbf{p}'$ as depicted in Figure 1. The error is measured as

$$E_{relative} = \frac{d_2}{d_1}, \tag{3}$$

where d_1 is the Euclidean distance of $\mathbf{p}'$ from $\mathbf{p}$ and d_2 is the Euclidean distance of $nn(\mathbf{p}')$ from $\mathbf{p}'$. The error is relative. If we consider a tolerance threshold, e.g., 5%, small symmetry transformation distances (d_1) would have a small absolute tolerance and larger transformation distances will have larger absolute tolerances.

To quickly find the nearest neighbor of a point, we utilize the R-tree spatial index (Guttman, 1984) from the C++ Boost Geometry Library with the average time complexity $O(log\ n)$ per query. Other indexing structures could be used as well, e.g., a spatial grid, a k-d tree, an octree, etc.

3.2 Symmetry Error Plot for all Points

Inspecting the relative error at each point individually would not provide much information about the symmetry of the whole object. We need to look at these

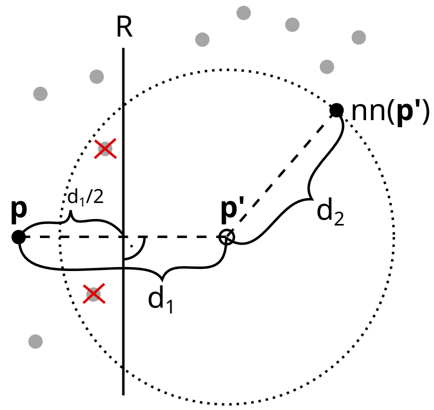

Figure 1: The relative error for the point p is the fraction d_2/d_1. Here, nn(p') is the nearest neighbor of p', the symmetric image of p, found in the half-space containing p'.

pieces of information together. Therefore, we first sort the points by their relative errors in ascending order. Then we consider a 2-dimensional error plot as illustrated in Figure 2, where the x-axis represents the sorted points in the normalized range $< 0, 1 >$ and the y-axis represents their relative errors.

If the whole object was perfectly symmetrical, the vast majority of values in the error plot would be zero. In reality, they will be almost zero, shifted a little bit in the positive direction of the y-axis, because of the point cloud representation of the object and the inherent discrete sampling. Even for an approximately symmetrical object, the vast majority of values in the error plot will be close to some tolerance threshold (a constant on the y-axis) distinguishing symmetry from asymmetry. However, it can be tricky to set a specific (low enough) tolerance threshold beforehand, because the object may only be partially symmetric.

3.3 The Symmetry Threshold Point

The error plot of a strongly symmetrical object will resemble the vertically mirrored letter L as illustrated in Figure 2. We define the symmetry tolerance threshold point $T = (T_x, T_y)$ as the point of the error plot minimizing the Euclidean distance from the lower right corner $(1, 0)$ and measure the distance

$$d = \sqrt{(1 - T_x)^2 + T_y^2} \tag{4}$$

to characterize the overall amount of approximate symmetry of a given object. The more symmetrical the object is, the more its error plot approaches the x-axis, T approaches $(1, 0)$, and d approaches zero. Only the points to the left of T_x in the error plot are symmetrical with relative errors up to T_y. The distance d combines both these quantities into one. It also gives us an initial guess that at least $1-d$ of the whole object is within the error limit d.

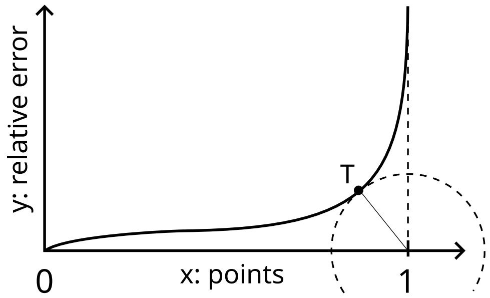

Figure 2: The error plot and the threshold point T.

3.4 Visualization

We use a coloring scheme, where points on the opposite sides of the symmetry plane have two different basic colors, e.g., red and blue. These basic colors are then linearly interpolated to colors highlighting the relative symmetry error, e.g., yellow and white. If we want to emphasize relative errors in a certain range, e.g., up to 20%, we will set the interpolation factor

$$f = min(rel\_error/0.2, 1). \quad (5)$$

However, this range may be too generous for strongly symmetric models or too strict for weakly symmetric models. In such situations, we can set

$$f = min(rel\_error/T_y, 1) \quad (6)$$

and the whole interpolation range will be dedicated to points that are considered symmetric with respect to the automatically detected threshold point T.

4 EXPERIMENTS AND RESULTS

The general applicability and properties of the proposed symmetry evaluation approach are demonstrated on a few models - the Stanford Armadillo and Bunny (Levoy et al., 2005), two historical buildings, namely the Maribor Cathedral and the University of Maribor at GPS location 46.5592, 15.6442 (Žalik, 2023), a statue and a component, both coming from the Thingi10K dataset (Zhou and Jacobson, 2016). Armadillo, Bunny and the statue are two examples of bilaterally symmetrical beings in more or less asymmetrical poses. Approximate symmetry in the two buildings and the component is present due to architectural or constructional intents. Whereas the point clouds of Armadillo, Bunny, the statue, and the component were obtained artificially by sampling the triangular meshes, the point clouds of buildings come from aerial LiDAR scanning and hence they may have some imperfections, e.g., under-sampled or missing vertical regions, irregular sampling, noise, etc.

For all these models, we detected the major symmetry plane using (Hruda et al., 2022b) symmetry detection framework, computed the corresponding error plots, found the symmetry threshold point T and its relative symmetry distance d, and colored the point clouds with respect to the detected plane and errors. The results are depicted in Figures 3, 4, 5, 6 and 7.

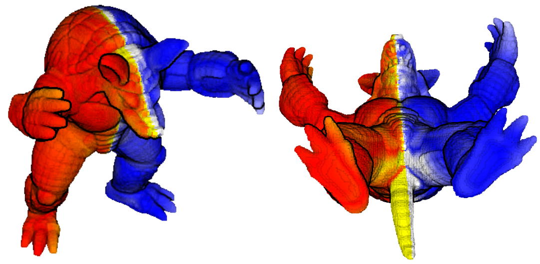

Figure 3: Armadillo - relative symmetry distance d = 0.132.

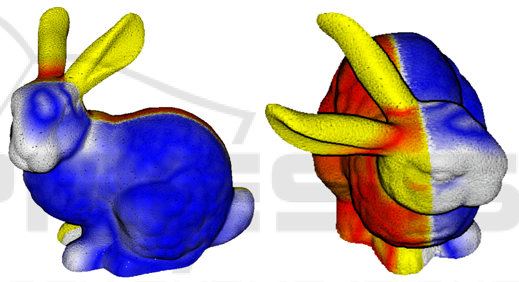

Figure 4: Bunny - relative symmetry distance d = 0.267.

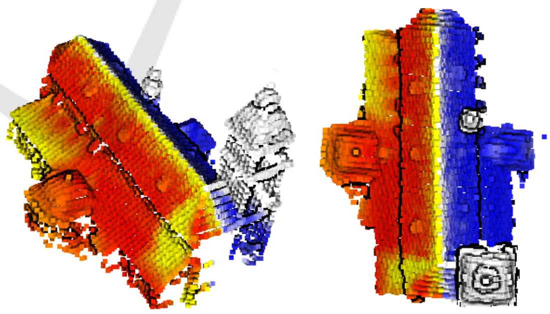

Figure 5: Cathedral - relative symmetry distance d = 0.273.

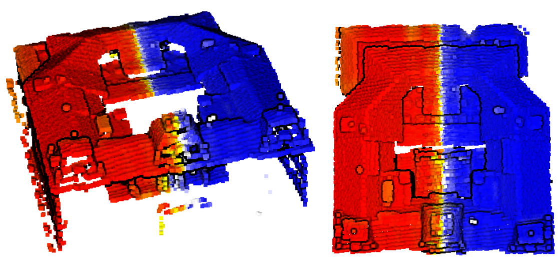

Figure 6: University - relative symmetry distance 0.098.

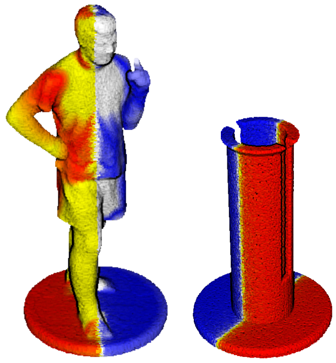

Figure 7: Statue - $d = 0.370$; Component - d $= 0.111$.

When these objects are sorted by the proposed relative symmetry distance, the most symmetric object is the university building (0.098), followed by the component (0.111), Armadillo (0.132), Bunny (0.267), the cathedral (0.273), and the statue (0.370). Red and blue regions indicate strongly symmetric regions, whereas strictly yellow and white regions indicate relative errors of 0.2 (20%) or more. The visualization shows that Armadillo is in a symmetric pose with a slight asymmetry of arms, fingers and tail. Bunny is in a more asymmetric pose with a slightly rotated head and ears and these body parts are relatively larger when compared against Armadillo. The symmetry of the cathedral is violated by its main tower, a smaller tower, and a side roof. The symmetry of the statue is dominated by the rounded pedestal and ruined by the rotated torso and body parts. The symmetry of the component seems to be almost perfect. Its nonzero relative symmetry distance can be attributed to a relatively low sampling density and a slightly higher percentage of points near the symmetry plane. The corresponding error plots of these objects are depicted in Figure 8 together with the threshold points and their attraction line to the ideal point $< 1, 0 >$.

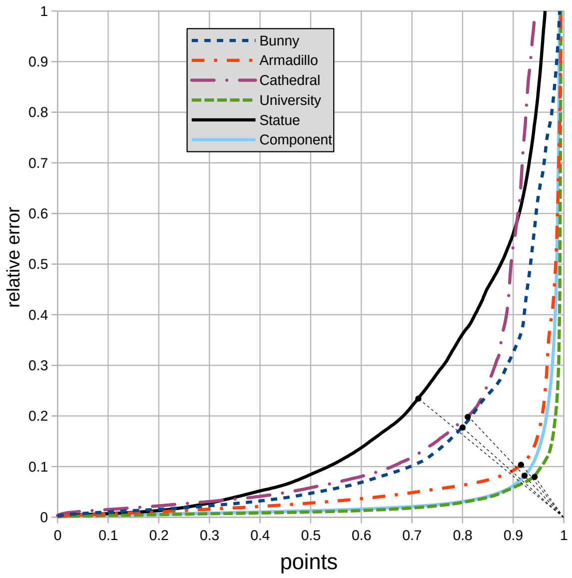

Figure 8: Error plots for the tested models.

Visualization also shows that points near the symmetry plane usually have a large relative error despite their absolute error may be small. Sampling irregularity can play some role here but it is more likely that the detected symmetry plane cannot fully reflect all the small violations of symmetry, e.g., Armadillo's tail and muzzle in Figure 3 are slightly off the detected plane but the plane is globally better for many more other points. The effect of such symmetry imperfections on the relative error gets stronger near the symmetry plane.

Coloring with thresholds can also be used for more visual highlighting of the symmetry error propagation as illustrated in Figure 9. Using a single threshold, even the automatically detected one, would make the symmetry error propagation almost invisible. As the threshold is being moved to lower values, the most symmetric parts of the model will remain in red and blue colors whereas the less symmetric parts of the model will be in yellow and white colors. We can interpret, e.g., that Armadillo's forearms are in a quite symmetric pose, with the right palm shifted up a little bit against the left palm (by one or two fingers). The error in most of the points on the palms is within $2\% - \%5$ of the palm-to-palm distance.

Sampling density naturally has some influence on

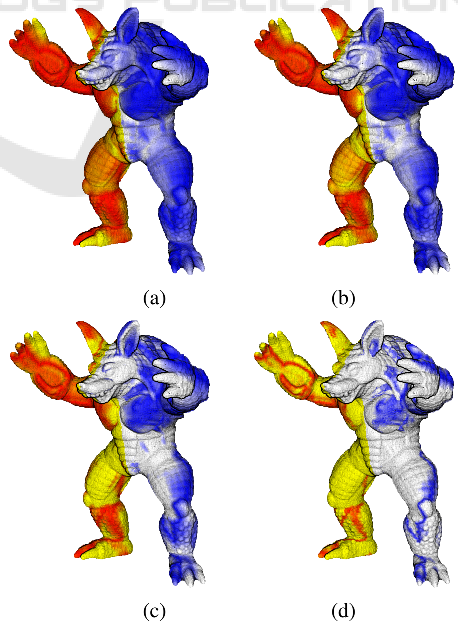

Figure 9: Armadillo colored with different thresholds shows the development of relative symmetry error. Thresholds: (a) 0.10 - found automatically from T; (b) 0.08; (c) 0.05; (d) 0.02.

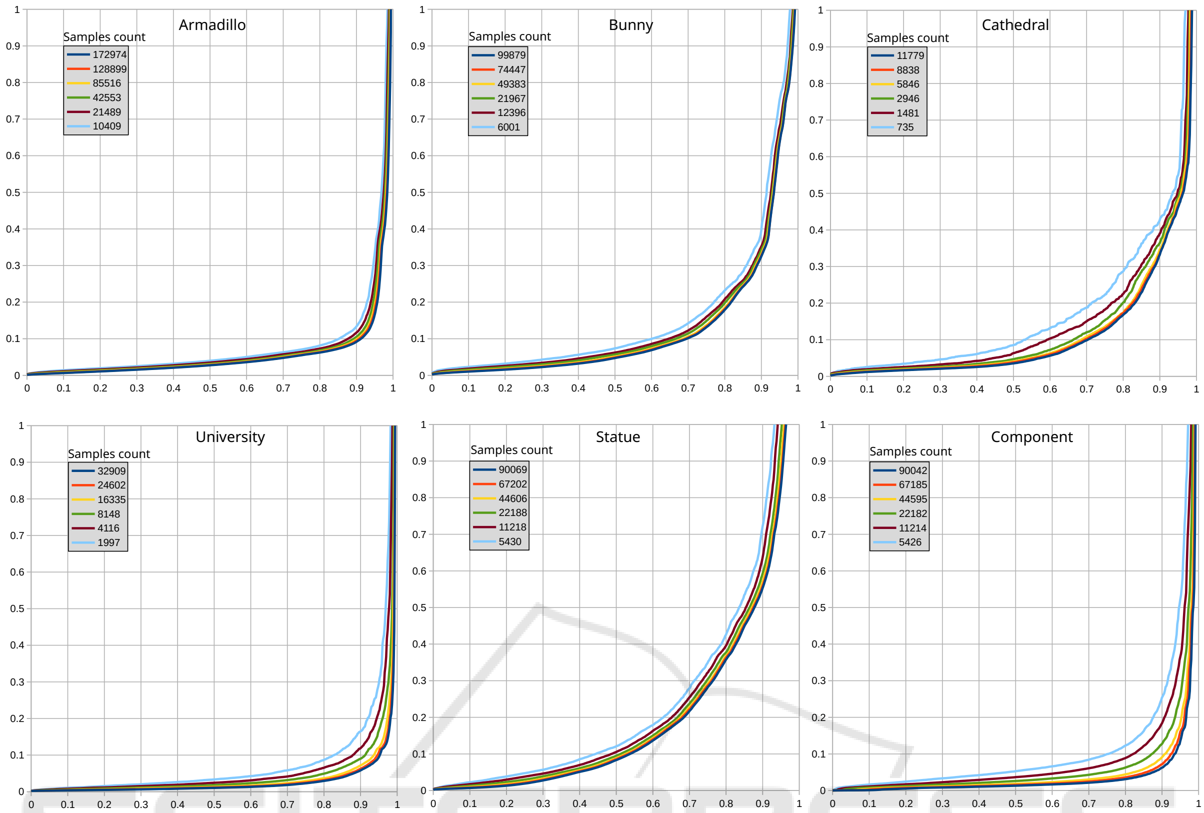

Figure 10: The behavior of relative symmetry error plots when they are computed on 100%, 75%, 50%, 25%, 12.5%, and 6% random samples taken from the same model. Decreasing the sampling density increases the relative symmetry error.

the shape of symmetry error plots and hence also on relative symmetry distances. If the number of samples on the surface of an object is reduced, nearest neighbor distances will generally increase and the error plot will lift up. This is illustrated in Figure 10, where we reduced the sampling of tested models to 75%, 50%, 25%, 12,5%, and 6% of the original amount of points. The symmetry plane was detected only once for each model (using 100% samples). Although the error plots may seem to change only marginally for some models, we can clearly see that the comparison based on the value of d can be influenced by sampling densities. For example, the university with 75% of samples will be evaluated as being more symmetrical than the component with 12.5% of samples:

$$d(Uni, 75\%) = 0.11 < d(Comp, 12.5\%) = 0.19. \quad (7)$$

On the other hand, the university with only 12.5% of samples will be evaluated less symmetrical than the component with 75% of samples:

$$d(Uni, 12.5\%) = 0.15 > d(Comp, 75\%) = 0.12. \quad (8)$$

Therefore, judgements based on the relative symmetry distance of different objects of unknown sampling densities should be made with caution.

In the next experiment, we compared the proposed relative symmetry distance against Hruda's measure of symmetry on a subset of the Thingi10K dataset. We selected over 350 models having 90 000-110 000 samples each. Results are depicted in Figure 11. Each point in the plot represents one model from the Thingi10K dataset. The proposed relative symmetry distance does not correlate much with Hruda's mea-

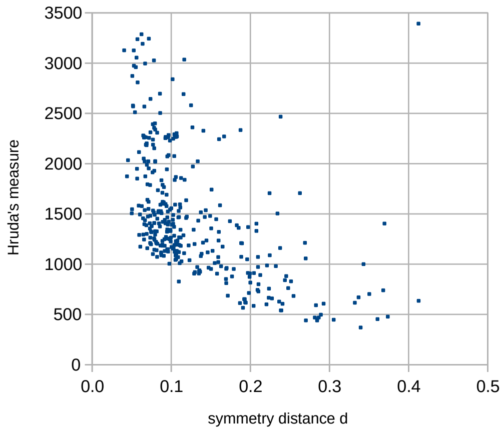

Figure 11: Relative symmetry distance d against Hruda's measure on a subset of the Thingi10K dataset.

sure. The correlation coefficient is -0.3974 and it did not change substantially (-0.3939) even when Hruda's measure was normalized by dividing the value by the number of input points.

Interestingly, Hruda's measure for the statue (with 90 069 points) is 1390.355; and for the component (with 90 042 points) it is 1454.743. The similarity of these two values supports the observation that Hruda's measure is not very suitable for comparing the symmetry of different objects. The relative symmetry distance d, however, may also have its limits, as can be seen in Figure 8 (similar values for the cathedral and Bunny). Therefore, instead of using d as a single value for comparison, it may be worth using some supplementary information from the error plot, e.g., the relative amount of points at several relative error levels, e.g., at 0.01, 0.02, 0.05, 0.10, 0.15 and 0.20. Getting more points at low error levels will then indicate the error plot approaches to zero, i.e., a better symmetry.

5 CONCLUSIONS

The proposed method is usable for comparing objects (point clouds) by the amount of approximate reflectional symmetry if the symmetry plane is provided or computed. The idea of measuring errors relatively makes this method invariant with respect to the global object scale, e.g., the bounding box diagonal. If a global normalization was used instead, the results would be sensitive to the overall shape of the object due to extruding or missing parts. Low values of the proposed relative symmetry distance d, e.g., in the range $< 0,\ 0.15 >$, mean that a high percentage of points have their symmetrical counterpart with low relative errors. Higher values, e.g., $d > 0.2$, mean that the symmetry is more seriously violated. To better understand the cause, it may be worth further analyzing the error plot or seeing the visualization of errors on the object. Values of $d > 0.5$ are very high because the symmetrical counterpart could lie in the same half-space with a non-zero probability.

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