Enhancing Portfolio Performance: A Random Forest Approach to Volatility Prediction and Optimization

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Abstract: Machine learning has diverse applications in various domains, including disease diagnosis in healthcare, user behavior analysis, and algorithmic trading. However, machine learning’s use in portfolio volatility predictions and optimization has only been recently explored and requires further investigation to prove valuable in real-world settings. We thus propose an effective method that accomplishes both these tasks and is targeted at people who are new to the realm of finance. This paper explores (a) a novel approach of using supervised machine learning with the Random Forest algorithm to predict portfolio volatility value and categorization and (b) a flexible method taking into account users’ restrictions on stock allocations to build an optimized and customized portfolio. Our framework also allows a diversified number of assets to be included in the portfolio. We train our model using historical asset prices collected over 8 years for six mutual funds and one cryptocurrency. We validate our results by comparing the volatility predictions against recent asset prices obtained from Yahoo Finance. The research underlines the importance of harnessing the power of machine learning to improve portfolio performance.

1 INTRODUCTION

Portfolio management refers to the science of selecting investment types that meet the financial objectives of a client. Typically, these objectives involve a combination of maximizing performance and minimizing risk. Portfolio management is critical as institutions need to meet financial obligations daily, to satisfy their own goals and the goals of individuals who are in some way connected to such institutions.

Despite a wide variety of portfolio tools being freely available for use, the vast majority of investors fail to earn portfolio returns that exceed the market rate. Many studies attribute this phenomenon to a lack of diversification, herd behavior, and the efficient market hypothesis, leading to sub-optimal portfolio performance.

Diversification of investment types is one of the most widely known investment strategies. It refers to spreading one’s investments across different asset classes to protect one’s portfolio against adverse market movements. However, many falsely believe that having investments in many asset classes will augment volatility levels (Reinholtz et al., 2021).

Herd behavior, where individuals mimic the actions of a larger group rather than making independent decisions, can lead to market bubbles as investors collectively rush to buy assets, driving up prices without individual asset analysis (Choiil et al., 2022).

Lastly, the efficient market hypothesis states that all available market information is reflected in asset prices, thus suggesting that, in theory, investors shouldn’t be able to achieve above-average returns consistently (Mancuso, 2022).

This paper proposes a model to help investors overcome these challenges and enhance portfolio performance. Our main contributions are two-fold. First, we use the Random Forest algorithm to predict the volatility of a random portfolio (with a variety of investment types), helping to quantify the risk for a certain portfolio and imply suggestions about diversification for an investor. Second, we perform portfolio optimization while allowing users to include their investment allocation restrictions to augment the overall flexibility.
2 RELATED WORKS

Machine learning is a widely used technique for portfolio optimization (Bartram et al., 2021) and volatility prediction or forecasting. Our work derives from the portfolio-related ML contributions that other researchers have conducted.

Kobets et al. (Kobets and Savchenko, 2022) explores the use of long short-term memory neural networks and linear regression models to create an optimal portfolio based on price predictions, finding that taking into account one-month asset prices improved their performance results. They used the Markowitz portfolio model (modern portfolio theory) to optimize the portfolio, which was trained using historical data, similar to what we did.

Ma et al. (Ma et al., 2020) similarly analyze the effectiveness of three different types of deep neural networks in portfolio optimization. They chose semi-standard deviation as the risk indicator, which involves calculating the absolute differences between data points and the central measure, whereas variation involves the squares of these differences. Our study predicted volatility, which is the square root of variation.

Another LSTM-involved portfolio recommendation system was by Leung et al. (Leung et al., 2023), who used a web application to take into account user preferences in their optimization algorithm. We also included a feature with user involvement in our optimization method.

A recent study involving reinforcement learning (Gao et al., 2021) used Deep Q-Network for portfolio management to take into account transaction fees. They measured the cumulative rate of returns for the portfolio. Their model was flexible to accommodate any number of assets, similar to ours.

Furthermore, a literature review (Ertencice and Kalayci, 2018) finds that variance tends to be the most commonly used risk indicator and Markowitz’s mean-variance portfolio theory to be the most commonly used formulation for portfolio optimization. In line with these prevailing practices, we also used these two methods in our study, thus fortifying the robustness of this research.

While most research discusses new portfolio optimization techniques, machine learning is also used for volatility forecasting. Christensen et al. (Christensen et al., 2021) find that machine learning techniques outperform the HAR (heterogeneous autoregressive) model. However, with this in mind, most volatility predictive models involve a temporal aspect, such as intraday volatility forecasting. Zhang et al. (Zhang et al., 2022) employs neural networks to forecast volatility over very short-term periods such as 10 or 30 minutes. Our research instead predicts annualized volatility given random asset allocations.

The majority of the literature covered only uses ML techniques to optimize their portfolio; however, our research uses ML to predict volatility and uses modern portfolio theory with ML to optimize a portfolio. Furthermore, to the best of our knowledge, the dataset we tested our model on was the historical asset prices of mutual funds (and one cryptocurrency), which we believe to be a novelty.

Finally, predicting market volatility using the Random Forest model has been vastly unexplored. Despite this fact, Kumar et al. (Kumar et al., 2018) showed that Random Forest tends to perform the best at predicting stock market activity for large datasets out of five supervised machine learning models studied, justifying our use of this model. Cervelló-Royo et al. (Cervelló-Royo and Guijarro, 2020) similarly concluded Random Forest’s superior capabilities in predicting stock market movement compared to the other ML methods in the study.

3 ASSUMPTIONS

Before we introduce and explain our model, we make the following assumptions:

- Measuring the volatility of a portfolio is a sufficient and suitable metric to gauge the risk level of such a portfolio.
- The bounds for the generated random allocations of the assets (e.g. metal, cryptocurrency, etc.) in our data set represent common investment practices and recommendations (Liu and Tsyvinski, 2021).
- Sharpe Ratio assumes a constant risk-free rate (Sharpe, 1998), which may not reflect real market conditions.
- The Random Forest model is a suitable choice for this study based on its interpretability, robust performance, and well-researched ability to handle various data types. While much other research uses neural networks, we find Random Forest most appropriate for our scenario.
- The features used in the RF model are representative of the factors affecting portfolio volatility.
Table 1: Summary of Volatility Prediction Approaches.

<table>
<thead>
<tr>
<th>Author(s)</th>
<th>Model Used</th>
<th>Dataset</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Vidal and Kristjanpoller, 2020)</td>
<td>Convolutional Neural Networks with Long Short-Term Memory (CNN-LSTM)</td>
<td>Gold Market</td>
</tr>
<tr>
<td>(Idrees et al., 2019)</td>
<td>Autoregressive Integrated Moving Average (ARIMA)</td>
<td>Indian Stock Market</td>
</tr>
<tr>
<td>(Hu et al., 2020)</td>
<td>Generalized Autoregressive Conditional Heteroskedasticity (GARCH),</td>
<td>Copper Market</td>
</tr>
<tr>
<td></td>
<td>Long Short-Term Memory with Artificial Neural Networks (LSTM-ANN)</td>
<td></td>
</tr>
<tr>
<td>(Kim and Won, 2018)</td>
<td>Long Short-Term Memory, Generalized Autoregressive Conditional Heteroskedasticity (GARCH)</td>
<td>KOSPI (200 Korean stocks)</td>
</tr>
<tr>
<td>(Walther et al., 2019)</td>
<td>Generalized Autoregressive Conditional Heteroskedasticity variant of Mixed Data Sampling (GARCH-MIDAS)</td>
<td>CrIX (Cryptocurrency index) and five high-revenue cryptocurrencies</td>
</tr>
<tr>
<td>(Hwang and Hong, 2021)</td>
<td>Multivariate Heterogeneous Autoregressive Realized Volatility model with Generalized Autoregressive Conditional Heteroskedasticity (HAR-RV-GARCH)</td>
<td>S&amp;P 500 Index, KOSPI, Russell 2000, and EURO STOXX 50</td>
</tr>
<tr>
<td>(Wen et al., 2016)</td>
<td>16 HAR (Heterogeneous Autoregressive)-type models</td>
<td>WTI (West Texas Intermediate) Crude Oil Futures</td>
</tr>
</tbody>
</table>

4 PROBLEM METHODOLOGY

4.1 Overview

We will now provide a brief overview of our research methodology. We began by identifying the problem, noting that much of the research on volatility prediction and portfolio optimization is relatively recent; hence, new methods for performing these tasks should be explored. Next, we collect the data with a Python library, allowing access to historical prices for assets. After collection, we process the data to filter it such that only the information needed is kept, and we prepare it to be read by our model. Once the data is fully cleaned, we feed it into our model, the majority of which is used for training and the rest for testing. Next, we use our model for volatility prediction and portfolio optimization. Finally, we evaluate our model performance and make any tweaks as necessary.

Next, we look at other research approaches that accomplished a similar task as our research for volatility prediction. The majority of the studies in Table 1 show the following two commonalities:

- The use of neural networks or time-series related models. CNN and ANN are neural networks, while ARIMA and GARCH are employed for time-series data.
- The dataset tends to be an entire market in either one geographic area or encompassing one asset type.

Our research is unique in that we use Random Forest for volatility prediction (which is much less explored). Furthermore, our dataset spans a variety of asset types and isn’t restricted to one geographic area. Table 2 compares our approach for portfolio optimization to other methods. We note that while using mean-variance optimization was a commonality among most approaches, few used the Random Forest model as well and included a cryptocurrency in the data set used for training.

The Random Forest model is an ensemble learning method, where multiple instances of a base learning algorithm, specifically decision trees, are employed to enhance the model’s predictive capabilities. This comes with numerous advantages over other models, including a smaller likelihood of overfitting (when an ML model generates accurate results for the training data set but not for testing) and improved accuracy. These two factors and other advantages of RF also improve its interpretability.

We thus argue that our model has high inter-
Table 2: Comparison of Machine Learning-based Portfolio Optimization Methods.

<table>
<thead>
<tr>
<th>Author(s)</th>
<th>Model</th>
<th>Cryptocurrency</th>
<th>Mean-Variance Optimization</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Ma et al., 2021)</td>
<td>RF, SVR,</td>
<td>×</td>
<td>✓</td>
</tr>
<tr>
<td></td>
<td>LSTM, CNN &amp; MLP</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Chen et al., 2021)</td>
<td>XGBoost &amp; IFA</td>
<td>×</td>
<td>✓</td>
</tr>
<tr>
<td>(Aboussalah and Lee, 2020)</td>
<td>SDDRRL</td>
<td>?</td>
<td>×</td>
</tr>
<tr>
<td><strong>Our model</strong></td>
<td><strong>RF</strong></td>
<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>

pretability, referring to the extent to which humans can predict a model’s outcome and understand the method by which the outcome was produced (Erasmus et al., 2021). For example, given a higher allocation of cryptocurrency, our model will predictably output a higher volatility value compared to a smaller allocation. In contrast to black-box models, our Random Forest model predicts portfolio risk given certain asset allocations, making its decision-making process very understandable. The features and target variables are carefully selected. We can also dissect the decision trees that the ML model used to get a breakdown of how the model reached a certain risk assessment. Each node in the decision tree represents a decision point, and one’s ability to easily trace this tree-based structure improves our model’s interpretability.

While a single decision tree is inherently more interpretable than multiple decision trees, this may come at the cost of capturing more nuanced patterns and trends in the data provided. Therefore, we choose to balance transparency and complexity to provide a more robust prediction mechanism while still being highly interpretable.

4.2 Data Set

For our Random Forest model, we used the following seven tickers: VTSAX, VTIAX, VBTLX, VDE, VGSLX, OPGSX, and BTC-USD. Note that the first six are mutual funds, and the last ticker is a cryptocurrency. These seven tickers cover a wide range of investment types, i.e., real estate, stocks, bonds, etc. We collected historical ticker prices over an 8-year range (from 2015 to 2023) using the yfinance Python API (which uses Yahoo Finance market data). Using the asset prices, we found the daily returns (percentage change in price for consecutive days) as follows:

\[
\text{Daily Returns} = \frac{P_k - P_{k-1}}{P_{k-1}}
\]

where \(P_k\) denotes the price of an asset at day \(k\). From Figure 1, we can visualize the daily returns distributions (encompassing all eight years of data) for each of the seven tickers.

4.3 Data Architecture

To transform the data into a form in which our model could predict volatility, we created 5000 random instances of possible ticker allocations. Each instance has the following criteria:

\[
w_1 + w_2 + w_3 + w_4 + w_5 + w_6 + w_7 = 1
\]

\[
w_i \geq 0, \quad i \in \{1, 2, 3, 4, 5, 6, 7\}
\]

\[
w_5 \leq 0.20, w_6 \leq 0.20, w_7 \leq 0.10
\]

where \(w_i\) denotes the ticker weight (allocation amount) for ticker \(i\), \(w_1, w_2, w_3, w_4, w_5, w_6, \text{and } w_7\) refer to the weights of tickers VTSAX, VTIAX, VBTLX, VDE, VGSLX, OPGSX, and BTC-USD, respectively. Our model is still compatible without the restrictions on \(w_5, w_6, \text{and } w_7\), but we choose to include them to better represent common real-life investment practices.

For each one of these 5000 random allocations, we found the dot product of the daily returns with the allocations, calculated the standard deviation (the statistical measure of market volatility) of these dot products, and annualized this standard deviation:

\[
d_1 = r_1^1 \cdot w_1 + r_2^1 \cdot w_2 + \cdots + r_7^1 \cdot w_7
\]

\[
d_2 = r_1^2 \cdot w_1 + r_2^2 \cdot w_2 + \cdots + r_7^2 \cdot w_7
\]

\[
\vdots
\]

\[
d_a = r_1^a \cdot w_1 + r_2^a \cdot w_2 + \cdots + r_7^a \cdot w_7
\]

Here \(d_n\) denotes the dot product of the returns \(r_i^a\) and weight \(w_i\) for a certain day \(n\) and asset \(i\). Next, to
calculate the annualized volatility, we can do the following:

\[
\mu = \frac{1}{n} \sum_{i=1}^{n} d_i
\]  

\[
\sigma_d = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (d_i - \mu)^2}
\]  

\[
\sigma_a = \sigma_d \times \sqrt{252}
\]

We multiply the daily volatility, \(\sigma_d\), by the square root of 252 to obtain the annual volatility, \(\sigma_a\), because there are approximately 252 trading days in a year. We repeat these calculations for 5000 random allocations to obtain the annualized volatility for 5000 instances.

Next, we sort the annualized volatility values from increasing to decreasing order. Using the sorted values, we categorize the volatility values based on quartiles, in which the largest 25% are classified as “High”, the second-largest as “Moderate,” the second-smallest as “Medium”, and the smallest 25% as “Low”.

We obtained Table 3 for the lower and upper bounds of our risk categorization. Note that while our data produced a minimum volatility value of 3.856% and a maximum value of 24.540%, the actual volatility values could theoretically range from 0% to an extremely high volatility value, so we thus adjust our table accordingly.

### Table 3: Volatility Bounds by Risk Category.

<table>
<thead>
<tr>
<th>Risk Category</th>
<th>Lower Bound</th>
<th>Upper Bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low</td>
<td>0%</td>
<td>6.389%</td>
</tr>
<tr>
<td>Medium</td>
<td>6.390%</td>
<td>9.323%</td>
</tr>
<tr>
<td>Moderate</td>
<td>9.324%</td>
<td>12.656%</td>
</tr>
<tr>
<td>High</td>
<td>12.657%</td>
<td>(\infty)</td>
</tr>
</tbody>
</table>

To improve the utility of our model’s predictive abilities and make it more user-friendly, we created a feature allowing users to enter any number of portfolios with their random allocations for the given assets. We then rank the portfolios from highest volatility to lowest volatility.

### 4.4 Random Forest Model

We employed a Random Forest classifier and regressor to predict the volatility value category and amount, respectively. Both algorithms use an 80-20 split in which 80% of the data is used for training the model and 20% for testing, as empirical studies show that allocating 20% to 30% of data for testing results in optimal model performance (Dunford et al., 2014). In other words, 4000 random allocation instances were used for the training data set, and the remaining 1000 for the testing set. For the classifier, the input features were the instances of random asset allocations, and the target variable was the risk rating. The regressor had the same input features, but its target variable was the annualized volatility value.

We also performed hyperparameter tuning using an exhaustive grid search but found the changes in model performance to be negligible. Our current model has 100 estimators, no max depth, and 1 max feature.

Figure 2 identifies the regressor’s performance on the testing data set. Our model consistently outputs a predicted volatility which is very close in value to the ideal prediction (actual volatility).

### 4.5 Additional Features

To optimize the portfolio, we use the Efficient Frontier model (Elton and Gruber, 1997), which is the set of portfolios that either (a) maximize the returns of a portfolio given a certain level of risk or (b) minimize the risk of a portfolio given a certain level of returns. To find these sets of portfolios, we aim to maximize the Sharpe Ratio given the set of constraints. The Sharpe Ratio can be calculated as follows:

\[
R_p = (w_1 \cdot r_1) + (w_2 \cdot r_2) + \cdots + (w_j \cdot r_j)
\]

\[
\sigma_p = \sqrt{\sum_i \sum_j w_i \cdot w_j \cdot \sigma(R_i(t)) \cdot \sigma(R_j(t)) \cdot \text{cov}(R_i(t), R_j(t))}
\]

\[
S = \frac{\mathbb{E}[R_p - R_f]}{\sigma_p}
\]

Here, \(R_p\) represents the portfolio expected returns, \(w_i\) is the weight of asset \(i\), \(r_i\) is the expected returns for asset \(i\), \(\sigma_p\) is the portfolio volatility, \(R_i(t)\) is the time series of returns for asset \(i\), \(\text{cov}\) is the covariance of
two assets, $R_f$ is the risk-free rate, and $S$ is the Sharpe ratio (Pav, 2021).

The covariance of two assets can be calculated as follows:
\[
\text{cov}(R_i(t), R_j(t)) = \frac{1}{N-1} \sum_{t=1}^{N} (R_i(t) - \bar{R}_i)(R_j(t) - \bar{R}_j)
\]
where $\bar{R}_i$ represents the mean of $R_i(t)$.

The expected returns for asset $i$, which we call $r_i$, represents the compound annual growth rate (CAGR). CAGR is calculated as follows:
\[
r_i = (\frac{P_f}{P_i})^{\frac{1}{t}} - 1
\]
where $P_i$ and $P_f$ represent the initial and final prices for asset $i$ and $t$ is the number of years.

We let the user enter either a target maximum volatility value or target minimum return value and then provide them with the optimum portfolio using this method. The user can also add their allocation restrictions; for example, a user can include a condition such that some ticker $i$ has an allocation of at least 20%, and the algorithm will consider this when reoptimizing.

5 RESULTS

We first present some performance metrics to measure the accuracy of our volatility-predicting Random Forest model.

Table 4: Random Forest Performance Metrics.

<table>
<thead>
<tr>
<th>RF Model</th>
<th>Metric Type</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Classifier</td>
<td>Accuracy Score</td>
<td>0.946</td>
</tr>
<tr>
<td>Regressor</td>
<td>R-squared value</td>
<td>0.998</td>
</tr>
<tr>
<td>Regressor</td>
<td>Mean Squared Error</td>
<td>$3.80 \times 10^{-6}$</td>
</tr>
</tbody>
</table>

Based on Table 4, our model shows promising strength as both the classifier and regressor were overall accurate and precise.

Upon comparison of our RF model to a simple ANN, we find that the ANN performs slightly better for volatility level classification.

We also found the feature importances for the regressor. The three most influential tickers were VBTLX with 79.22% importance, VDE with 18.58%, and BTC-USD with 1.36%. The other four tickers had a combined importance of less than 1%.

Next, we compare our model to real-life data. We access the most recent half-year’s worth of historical asset prices to do so. We then repeat the following steps for 1000 iterations:

1. Generate one sample of random weights

Figure 3 shows how our model is stronger at predicting portfolios of lower “actual” volatility than a higher “actual” volatility, likely because portfolios with higher volatility tend to be more unpredictable.

Next, looking at Figure 4, we see that the curved line represents the optimal portfolios given a certain level of maximum risk or minimum return. Here, we define “efficient risk” as the portfolio giving the minimum amount of risk for a certain level of expected returns and “efficient return” as the portfolio with the maximum returns for a certain level of risk.

Note that we choose to show an example where the user adds a certain asset weight restriction given that their target risk level was 8% (orange circle); hence, the Sharpe ratio for this portfolio, as calculated in Equation 9, was lower than without any restrictions (blue circle). The green triangle represents another sample case where the user entered a target volatility level classification.
Table 5: Actual vs. Predicted Volatility for Three Random Portfolio Allocations (in Percentages).

<table>
<thead>
<tr>
<th>W1</th>
<th>W2</th>
<th>W3</th>
<th>W4</th>
<th>W5</th>
<th>W6</th>
<th>W7</th>
<th>Actual</th>
<th>Predicted</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.56</td>
<td>17.67</td>
<td>51.04</td>
<td>12.63</td>
<td>4.78</td>
<td>9.16</td>
<td>2.16</td>
<td>8.07</td>
<td>10.35</td>
</tr>
<tr>
<td>6.54</td>
<td>24.15</td>
<td>32.26</td>
<td>8.77</td>
<td>10.84</td>
<td>17.24</td>
<td>0.20</td>
<td>10.02</td>
<td>12.82</td>
</tr>
<tr>
<td>0.55</td>
<td>1.84</td>
<td>79.93</td>
<td>7.57</td>
<td>3.47</td>
<td>0.69</td>
<td>5.95</td>
<td>6.82</td>
<td>6.98</td>
</tr>
</tbody>
</table>

Figure 4: Efficient Frontier Portfolios Visualization.

Figure 5: Optimize Portfolio vs. Equal-Weighted Portfolio Cumulative Returns.

From Figure 5, we see that our portfolio optimizer performs nearly triply as strong as a generic, equal-weighted portfolio in terms of cumulative return. The optimized portfolio (blue line) represents the portfolio with the seven tickers whose allocations maximize the Sharpe Ratio. For the equal-weighted portfolio, each ticker had an allocation of approximately 14.286% (100/7), as we trained our model with seven tickers.

6 CONCLUSIONS AND FUTURE WORK

In this paper, we present (a) a new method of predicting volatility for a portfolio and create (b) a portfolio optimizer that allows user input on the portfolio asset allocations. Our data set consists of 7 tickers – 6 mutual funds and one cryptocurrency. We use Yahoo Finance historical prices for data and find that the mean difference between our Random Forest volatility predictor and the actual volatility value is 3.570%. Furthermore, our portfolio optimizer performs strongly against a generic portfolio. We use modern portfolio theory to do so, calculating the Sharpe Ratio while considering user input.

First, the Efficient Frontier assumes that investments generate returns that follow a primarily normal distribution. However, this isn’t always true of the market as the distribution often shows fat tails (Eom et al., 2019), in which the likelihood of extreme events occurring is higher than expected and predicted in a normal distribution. Hence, this could impact our model’s allocations after performing the portfolio optimization.

Second, our volatility-predicting model only uses closing prices of assets from day to day, as this was what was available through the Yahoo Finance API. However, volatility can also be influenced by intraday price fluctuations. Hence, our results may not fully encompass the price changes relevant to different time frames. This also limits the scope to which our results can be generalized.

For future work, first, we plan to incorporate aspects of neural networks (Sharkawy, 2020) with Random Forest to strengthen the model. Using LSTM will allow us to do a time-series analysis to capture potential temporal factors involved in stock prices, which we couldn’t accomplish with just Random Forest. Neural networks also prove valuable with time series forecasting.

Second, we also plan to build upon more modern portfolio optimization techniques including hierarchical risk parity, the Black-Litterman model, and Monte Carlo simulations. Mean-variance optimization often leads to portfolios overly concentrated in certain assets and may not adequately account for tail risk. However, by implementing more modern models, we hope to consider a broader range of factors that could affect stock market activity, including probabilistic scenarios.
REFERENCES


