Serial or Simultaneous? Possible Attack Strategies with an Arsenal of Attack Tools

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Abstract: In various fields such as medicine, management, cyber operations, and military strategy, the choice between sequential and parallel strategies is pivotal in achieving objectives, be it maximizing the likelihood of success or minimizing the time to victory. This study considers a hacker who attempts to destroy a rival system using multiple attacking tools. It is assumed that the success probability of each attack tool to destroy the system is equal and independent of the other tools. Execution of an attack is time consuming and it is assumed that this attack time increases exponentially with the number of tools used simultaneously. We consider different attaching schemes that vary in their design and balance between parallel and sequential steps. Our findings indicate that when the attack time for a multi-tool attack is extremely short, the optimal solution will be a purely simultaneous attack. Conversely, if the attack time approaches the total time required for a sequential attack, then the optimal solution will be a purely sequential approach. In between these extremes, we discover that a mixed strategy is optimal. Interestingly, our numerical analysis reveals that in these mixed cases, it is consistently more advantageous to initiate a simultaneous attack and then complement it with a sequential one. Moreover, we demonstrate that as the probability of success increases, the optimum tends towards a sequential attack.

1 INTRODUCTION

In the realms of many disciplines such as medicine, management, cyber and military operational art, the choice between sequential and parallel strategies plays a crucial role in achieving objectives, whether its maximizing the probability of success or minimizing the time until victory. A sequential strategy involves using the available tools one at a time, progressing to the next only if the previous one fails. In contrast, a parallel strategy entails the employment of many or all available tools simultaneously in a concerted effort.

Practical examples to this choice are abundant. In the field of medicine, for example, an oncologist may find that her patient's tumor responds to only two types of chemotherapies. Should she treat with both simultaneously to maximize the chance of full tumor lysis before the tumor develops defensive mutations? Or perhaps, she should begin with monotherapy so that in the event the tumor develops immunity to it, she has another viable treatment for her patient. In certain fields of medicine, a mix of simultaneous and sequential strategies have been developed. For example, physicians treating HIV carriers have recognized as early as 1995 that a cocktail of drugs has better results than sequential monotherapy, and when a specific cocktail is less effective then another cocktail is used (Lu et al., 2018). Thus, both a simultaneous and sequential approach are used.

This question has intrigued many thinkers in the field of combat and military. For example, Soucy (2018) studies General MacArthur's sequential approach against the Japanese in the South West Pacific during World War II, and compares it with General Schwarzkopf's parallel war against Iraq during Desert Storm. His conclusion is that due to the Unites State's considerable military power, a parallel war is advantageous and allows it to "quickly shatter an enemy's strategic and operational ability to resist (Soucy, 2018, p. 3)".

In this paper, we consider a hacker trying to attack and destroy a system. The hacker has at its disposal a pool of attack tools, whereas the attacked system can (with a known probability) withstand an attack by any one of the tools and develop defense mechanisms

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that allow it to be immune to that specific tool. This setting gives rise to multiple attack strategies ranging from fully sequential attacks (i.e., in each period a single tool is employed until the system is destroyed), to purely simultaneous attack in which all the attack tools are used simultaneously. In between these two extremes there are strategies that mix the sequential and simultaneous approaches in varying degrees.

In our model, we assume that the probability of success of each tool is equal and independent of the other tools and show that if the attack time is independent of the number of tools employed in the attack, then the purely simultaneous strategy is optimal to the hacker. In contrast, if attack time is the sum of the individual attack times of the tools that it employs, then the fully sequential strategy is optimal.

When attack time is increasing concavely with the number of tools then strategies that mix between simultaneous and sequential attacks can be optimal, depending on the total number of tools available to the hacker and the probability of each tool to succeed.

While our model is quite simplistic in its nature, it provides insights that may be useful whenever there are multiple resources available. Staggering resources in contrast to pooling them in a concerted effort is of interest in a large array of applications, including military, cyber, health and managerial.

2 LITERATURE REVIEW

Sequential versus simultaneous strategies have been considered in a wide range of scientific fields. In computer science, Andradóttir et al. (2017) studies a model in which queues or servers may be pooled. It is well known that pooling queues and servers is advantageous when servers are not subject to failure. However, when servers could fail, then there is a tradeoff between efficiency (queue length) and risk (i.e., the probability that a system will be overcrowded). See also Sunar et al. (2021) who considers a similar problem with different constraints. Similarly, Cui et al. (2018) compares between simultaneous attacks and sequential attacks on cyber-physical systems.

Sequential strategies are often likened to a cautious approach and are widely used in the medical field, where the progression of treatment options depends on patient response. A notable example of sequential strategy in medicine is the management of cancer. Oncologists often begin with less aggressive treatments, such as radiation therapy or chemotherapy, and only switch to more invasive procedures like surgery if the initial methods do not yield the desired results. This approach minimizes the immediate risks and side effects while keeping more potent interventions in reserve but in some cases may be more susceptible to mutants. For example, with metastatic breast cancer, international guidelines recommend using sequential monotherapy unless there is rapid disease progression (Cardoso et al., 2014). A systematic review comparing between combination (i.e., simultaneous) and sequential therapy found that there was no difference in overall survival between the two groups but found that when drugs were given one at a time there may be more time before the tumors grew back again thereby achieving longer progression-free survival (Dear et al., 2013). In contrast, for the initial treatment of hypertension MacDonald et al. (2017) found that combination therapy is superior to sequential monotherapy. Another interesting example is the medical management of patients that are HIV positive. It is a long standing consensus that a combination therapy is superior to sequential monotherapy in stopping these patients from acquiring AIDS (Lu et al., 2018). Despite this established clinical approach, it has been suggested that certain HIV subpopulations that are resistant to multi-drug treatment may benefit from a sequential monotreatment approach (Phillips et al., 2003).

Sequential and parallel strategies also have a place in the field of management, where the objective is often to maximize business performance or minimize operational challenges (e.g, Thompson and Kwortnik Jr, 2008). The sequential approach is frequently employed when tackling complex problems or implementing organizational changes (Read et al., 2001). Managers may choose to take one step at a time, evaluating the effectiveness of each action before proceeding to the next. For instance, when faced with declining profits, a company may first focus on cost-cutting measures, followed by rebranding, and subsequently, market expansion. This sequential strategy allows for a more measured evaluation of each phase of the plan. Parallel strategies in management are often associated with rapid and comprehensive changes. For instance, during a business turnaround, a company on the brink of failure may implement a combination of cost-cutting measures, diversification, rebranding, and fundraising simultaneously to expedite a recovery. This approach aims to address multiple critical issues concurrently, potentially leading to a quicker turnaround, but it also involves higher risk and resource allocation Xiong et al. (2019).

Strategic planning is a framework used in various sectors, including business, government, and the military, with the primary goal of achieving long-term objectives. For instance, during a military campaign, a parallel approach might involve deploying all available resources, such as ground forces, air support, and electronic warfare, in a coordinated effort to achieve a swift and overwhelming victory. This approach seeks to minimize the time until winning by overwhelming the adversary. This approach is often evident in large-scale offensives, where all available resources are deployed simultaneously to overwhelm the enemy. The "shock and awe" strategy, for example, aims to incapacitate the adversary through a concentrated and simultaneous show of force (McNaughton, 2019). This parallel approach can be highly effective in achieving a swift victory but comes with higher risks and resource commitment. Conversely, a sequential military strategy is often employed when precision and limited collateral damage are essential. Surgical strikes conducted by special forces units exemplify this approach (Sasikumar, 2019).

The choice between sequential and parallel strategies in medicine, management, and strategic planning depends on the specific objectives, risks, and resources available. Each approach has its merits and drawbacks, and a balanced combination of both may be the most effective solution in many cases. Context-specific analysis is essential to determine the optimal strategy that will either maximize the winning probability or minimize the time until winning, depending on the situation at hand.

3 THE BASIC MODEL

The environment that we present comprises two rival entities. The attacking entity ("the hacker") desires to eliminate a system ("the system") that the hacked entity ("the hacked") has acquired. To do so, the hacker has at its disposal N distinct attack tools, each of which can potentially eliminate the system. The system has a lifespan of T periods after which it is obsolete. Therefore, if all the attacks on it fail, it is expected to survive exactly T periods.

The probability of each attack tool to successfully eliminate the system is q and independently of the other attack tools. In this case, we assume that the system is eliminated at the end of the period. If the attack tool fails to eliminate the system, it is assumed that the attacked system was either prepared or has developed defences against this tool and therefore this attack tool is rendered useless.

In the basic model we assume that the attack time is exactly one period regardless of the number of tools employed. That is, an attack using one tool and attack using $k > 1$ tools will each require one period to set up. In the next section we relax this assumption. Consider N strategies. Strategy Q_k, $k = 0, \ldots, N-1$ follows the subsequent steps:

1. Initialization: Success:=False, Pool:= N attack tools.
2. Serial Phase: For $j := 1, \ldots k$
   (a) In each period, attack with a single attack tool from the pool of tools.
   (b) Remove this tool from the pool.
   (c) If the attack is successful then Success:=True; Go to step 4.
3. Simultaneous Phase: In period $k+1$ attack simultaneously with $N-k$ (remaining) attack tools. If successful Success:=True.
4. Output: Success.

Notice that strategy Q_k, will last at most $k+1$ periods and that when $k = 0$ or $k = N-1$ the strategies are either purely simultaneous ($k = 0$) or only sequential ($k = N-1$).

In Figure 1 we depict the strategy Q_2 when $N = 5$. The hacker attacks in the first period with a single attack tool. If this attack has succeeded then the system's survival was one period. Otherwise, the hacker attacks again with another single tool, and if this attack has succeeded then the system's survival was two periods. Otherwise, the hacker goes all in with the three remaining tools at its disposal. If this simultaneous attack has succeeded then the system's survival was 3 periods and if not then the system's survival was T periods.

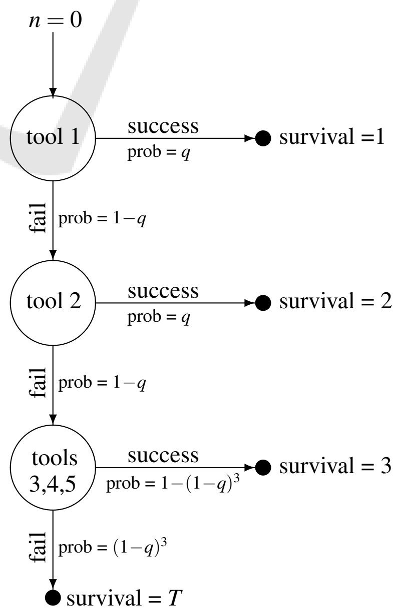

Figure 1: The Q_2 strategy in the basic model when $N = 5$ attacking tools are available.

Denote X as the random variable representing the survival (i.e., length of life) of the attacked system. The expected value of X comprises three elements. First, it must survive the sequential phase. The system is eliminated after j periods, $1 \leq j \leq k$, if and only if it survived the first $j-1$ periods, but was successfully attacked in the jth period. The probability for this to happen is $(1-q)^{j-1}q$ and when this happens the system survived exactly j periods. Thus, the sequential phase contributes to the system's expected survival $\sum_{j=1}^{k} j(1-q)^{j-1}q$. The firm survives exactly $k+1$ units of time if the sequential phase failed, but the simultaneous phase (that comprises $N-k$ attack tools) was successful. The probability of this is

$$(1-q)^k(1-(1-q)^{N-k}) = (1-q)^k - (1-q)^N$$

and therefore the contribution of the simultaneous phase to the expected survival is $(k+1)\big((1-q)^k - (1-q)^N\big)$. Finally, the system survives all the attacks with probability $(1-q)^N$ in which case it survives T periods and therefore this contributes $T(1-q)^N$ to the system's expected survival. Thus, the system's expected survival when strategy Q_k is employed is given by:

$$E_k^Q = q\sum_{j=1}^{k} j(1-q)^{j-1} + (k+1)\big((1-q)^k - (1-q)^N\big) + T(1-q)^N \quad (1)$$

Lemma 1. *The expected survival is increasing with k.*

Proof. To show that a higher k results with longer survival we must show that for each $k<N-1$, it holds that $E_{k+1}^Q - E_k^Q > 0$. By (1),

$$E_{k+1}^Q - E_k^Q = q(k+1)(1-q)^k + (k+2)(1-q)^{k+1}(1-(1-q)^{N-k-1}) - (k+1)(1-q)^k(1-(1-q)^{N-k}). \quad (2)$$

Expanding the latter expressions gives

$$E_{k+1}^Q - E_k^Q = q(k+1)(1-q)^k + (k+2)(1-q)^{k+1} - (k+2)(1-q)^N - (k+1)(1-q)^k - (k+1)(1-q)^N, \quad (3)$$

which can be simplified to

$$E_{k+1}^Q - E_k^Q = q(k+1)(1-q)^k - (1-q)^N + (k+2)(1-q)^{k+1} - (k+1)(1-q)^k. \quad (4)$$

The first and fourth terms above can be combined to $-(k+1)(1-q)^{k+1}$ and when added to the third term results with $(1-q)^{k+1}$. Therefore, (4) simplifies to

$$E_{k+1}^Q - E_k^Q = (1-q)^{k+1} - (1-q)^N, \quad (5)$$

which is positive since $k+1 < N$. □

Lemma 1 implies that the more the strategy is simultaneous (i.e, shorter sequential phase) the better for the hacker. In fact, under the assumptions of the basic model the best strategy is the pure simultaneous strategy, Q_0. This result follows from the fact that there is no probabilistic loss from attacking simultaneously (since the probabilities for success of each attack tool are independent) whereas time-wise a simultaneous approach is advantageous since more attack tools are employed in a shorter time. In the next section, we revisit this feature of the model concerning the time that the hacker needs to execute a simultaneous attack. In addition, we extend the menu of strategies that we consider to strategies that begin with a simultaneous attack employing a subset of the tools followed by sequential attacks using the remaining tools.

4 GENERALIZED MODEL

We consider two extensions to the model described in the previous section.

4.1 Exponential Attack Time

The attack time for simultaneous cyber attacks may not increase at a constant rate with the number of attacking tools due to a concept known as "decreasing marginal cost". This idea comes from economics and refers to the phenomenon where the cost of producing each additional unit of a good or service decreases as the overall quantity increases. In the context of cyber attacks, think of each attacking tool as a unit. As you add more tools, there may be synergies or efficiencies gained in the overall attack execution process. For example, some tools may share common requirements or dependencies, and once those are set up, adding more tools becomes quicker and less resource-intensive.

Additionally, attackers may develop scripts or automated processes that can be reused across different tools, further reducing the attack time for each additional tool. This is similar to economies of scale in manufacturing, where producing more units leads to a lower cost per unit. In line with these ideas we assume that the attack time for a simultaneous attack with k tools is k^α, where α satisfies; $0 \leq \alpha \leq 1$. Note that when $\alpha = 0$, the attack time is constant in the number of attacking tools, which is consistent with the basic model (see Section 3), whereas $\alpha = 1$ assumes that the attack time is linear in the number of attackers (i.e., the attack time increases at a constant rate with the number of attacking tools). In Figure 2 we depict the strategy Q_2 when $N = 5$. The hacker attacks in the first period with a single attack tool. If this attack has succeeded then the system's survival was one period. Otherwise, the hacker attacks again with another single tool, and if this attack has succeeded then the system's survival was two periods. Otherwise, the hacker goes all in with the three remaining tools at its disposal. If this simultaneous attack has succeeded then the system's survival was $2+3^{\alpha}$ periods and if not then the system's survival was T periods.

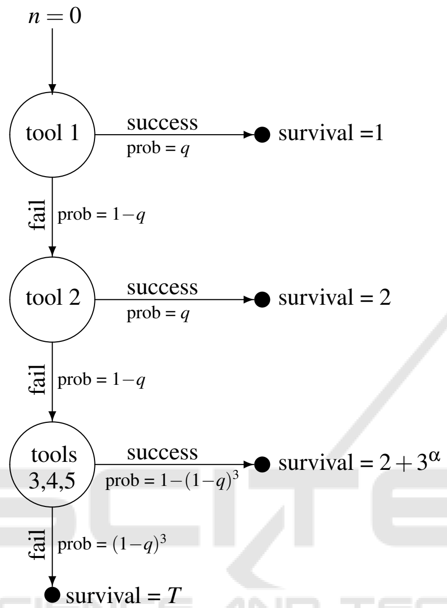

Figure 2: The Q_2 strategy in the Exponential model when $N = 5$ attacking tools are available.

4.2 Simultaneous-Sequential Strategies

In Section 3 and 4.1 we considered a class of strategies $\{Q_k\}$, in which the hacker begins with a sequential approach and then delivers a final simultaneous attack. We now extend our consideration to include a class of strategies $\{M_k\}$, with this order reversed.

Strategy M_k, $k = 0,...,N-1$ follows the subsequent steps:

1. Initialization: Success:=False, Pool:= N attack tools.
2. Simultaneous Phase:
   (a) For $(N-k)^{\alpha}$ periods attack simultaneously with $N-k$ attack tools.
   (b) Remove these tools from the pool.
   (c) If the attack is successful then Success:=True; Go to step 4.
3. Serial Phase: For $j := 1,...k$
   (a) In each period, attack with a single attack tool from the pool of tools.
   (b) Remove this tool from the pool.
   (c) If the attack is successful then Success:=True; Go to step 4.
4. Output: Success.

In Figure 3 we depict the strategy M_2 when $N = 5$. The first round of attack lasts 3^{α} periods and the hacker attacks with three attack tools. If this attack has succeeded then the system's survival was 3^{α} periods. Otherwise, the hacker attacks again with a single tool (attack duration of one period), and if this attack has succeeded then the system's survival was $3^{\alpha}+1$ periods. Otherwise, the hacker attacks with the last available tool. If this attack has succeeded then the system's survival was $3^{\alpha}+2$ periods and if not then the system's survival was T periods.

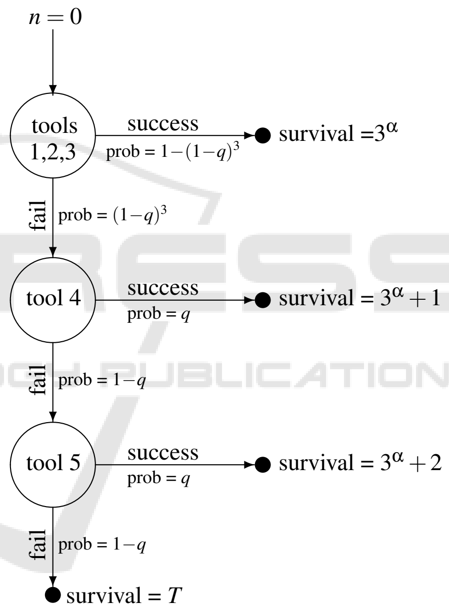

Figure 3: The M_2 strategy in the Exponential model when $N = 5$ attacking tools are available.

Accordingly, the system's expected survival when strategy Q_k is employed is given by:

$$E_k^Q = q\sum_{j=1}^{k} j(1-q)^{j-1} +$$

$$(k+(N-k)^{\alpha})\big((1-q)^k-(1-q)^N\big)+T(1-q)^N. \quad (6)$$

The system's expected survival when strategy M_k is employed is given by:

$$E_k^M = (N-k)^{\alpha}\big(1-(1-q)^{N-k}\big)$$

$$+q\sum_{j=1}^{k}\big((N-k)^{\alpha}+j\big)(1-q)^{N-k+j-1} \; +T(1-q)^N \quad (7)$$

The first term is the contribution of the simultaneous attack, the multiplication of the probability of $N-k$ tools to succeed $(1-(1-q)^{N-k})$ with the survival if the event happens $((N-k)^{\alpha})$. The second term is the contribution of the sequential phase, where notice that the probability for surviving exactly $(N-k)^{\alpha}+j$ periods, $j=1,\dots,k$, is q multiplied by the probability to fail in the earlier rounds $(1-q)^{N-k+j-1}$. The third term is the contribution to the survival if no attack succeeds.

Proposition 1. *The expected survival for each group of strategies* (E_k^Q, E_k^M) *is*

- *increasing with* k *when* $\alpha = 0$
- *decreasing with* k *when* $\alpha = 1$.

Proof. Consider first the Q_k strategies. By (6), the difference $E_{k+1}^Q - E_k^Q$ where $k < N-1$ is

$$E_{k+1}^Q - E_k^Q = q(k+1)(1-q)^k + (1-q)^N\Big((N-k)^{\alpha} - 1 - (N-k-1)^{\alpha}\Big) - (1-q)^k(k+(N-k)^{\alpha}) + (1-q)^{k+1}(k+1+(N-k-1)^{\alpha}), \quad (8)$$

This can be simplified to

$$E_{k+1}^Q - E_k^Q = (N-k-1)^{\alpha}\big((1-q)^{k+1} - (1-q)^N\big) + (1-(N-k)^{\alpha})\big((1-q)^k - (1-q)^N\big). \quad (9)$$

When $\alpha = 1$ this can be further reduced to

$$-q(N-k-1)(1-q)^k,$$

which is negative since $k < N-1$. When $\alpha = 0$ this is positive by Lemma 1.

We now consider the M_k strategies. Using well-known summation formula and algebraic manipulation, (7) can be rewritten as

$$E_k^M = (N-k)^{\alpha} + + (1-q)^N\left(\frac{1}{q}\big(\frac{1}{(1-q)^k} - 1 - kq\big) - (N-k)^{\alpha}\right). \quad (10)$$

Therefore, the difference $E_{k+1}^M - E_k^M$, $k < N-1$ is

$$E_{k+1}^M - E_k^M = (N-k-1)^{\alpha} - (N-k)^{\alpha} + (1-q)^N\left(\frac{1}{q}\big(\frac{1}{(1-q)^{k+1}} - 1 - (k+1)q\big) - (N-k-1)^{\alpha}\right) - (1-q)^N\left(\frac{1}{q}\big(\frac{1}{(1-q)^k} - 1 - kq\big) - (N-k)^{\alpha}\right) \quad (11)$$

This can be simplified to

$$E_{k+1}^M - E_k^M = X(\alpha)\big(1-(1-q)^N\big) + (1-q)^{N-k-1} - (1-q)^N \quad (12)$$

where $X(\alpha) := (N-k-1)^{\alpha} - (N-k)^{\alpha}$. When $\alpha = 0$ then $X(\alpha) = 0$ and therefore $E_{k+1}^M - E_k^M = (1-q)^{N-k-1} - (1-q)^N > 0$. Conversely, when $\alpha = 0$ then $X(\alpha) = -1$ and $E_{k+1}^M - E_k^M = (1-q)^{N-k-1} - 1 < 0$. □

Proposition 1 implies that for extreme values of α the optimal strategy is also at the extreme. If $\alpha = 0$ a purely simultaneous strategy is optimal since it minimizes the system's survival, whereas when $\alpha = 1$ the purely sequential strategy is optimal. Recall the discussion following Lemma 1, when $\alpha = 0$ the simultaneous approach has the advantage of gaining a high probability of success in a short period of time. In contrast, when $\alpha = 1$, the attack time is additive and therefore a sequential approach is better since it permits early elimination of the system if an early attack is successful.

In the next section we numerically analyze the attack strategies when attack time is concave with the number of tools employed in the attack, i.e, when $\alpha \in (0,1)$.

5 NUMERICAL ILLUSTRATION

We now consider numerically the case when the attack time parameter α is intermediate. In (6) and (7), the last term of the expected survival, $T(1-q)^N$, is independent of the strategy and therefore in what follows we consider $\hat{E}_k^Q := E_k^Q - T(1-q)^N$ and $\hat{E}_k^M := E_k^M - T(1-q)^N$ in lieu of E_k^Q and E_k^M, respectively. That is, $\hat{E}_k^Q$ and $\hat{E}_k^M$ represent the *variable components* of the expected survival. Throughout this section we let $N = 10$ and $\alpha = 0.5$. We plot $\hat{E}_k^Q$ and $\hat{E}_k^M$ and determine the optimal strategy when for different values of q.

Figures 4, 5 and 6 describe the variable components of the expected survival ($\hat{E}_k^Q$, $\hat{E}_k^M$) of the different strategies when $q = 0.1, 0.4$ and 0.7, respectively. These examples illustrates two phenomena that holds more generally. First, when q is low, any M_k strategy is less or equal to its companion Q_k strategy. However, as q increases the M_k strategies increase compared to the Q_k strategies and when q is sufficiently high the M_k strategies are equal or above their companion Q_k strategies.

Second, as q increases the optimal strategy shifts to the higher (i.e., larger k) strategies. The intuition to this is that when q is high then a sequential approach has an advantage since it is more likely to eliminate the system within a single or few period (compared to the simultaneous that increases the probability of success but at the cost of more time). In contrast, when

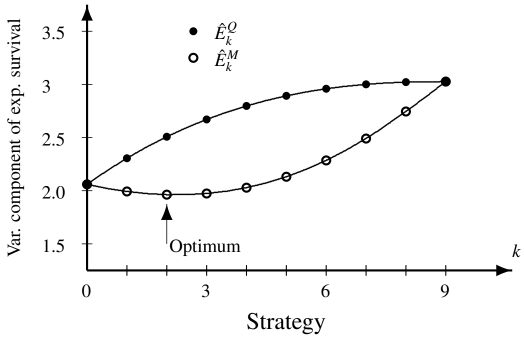

Figure 4: The variable component of the expected survival, $\hat{E}_k^Q$ and $\hat{E}_k^M$, when $N = 10$, $\alpha = 0.5$ and $q = 0.1$.

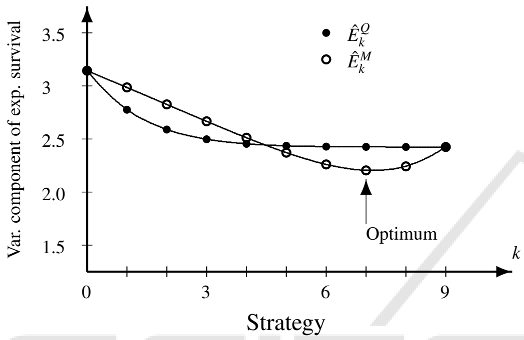

Figure 5: The variable component of the expected survival, $\hat{E}_k^Q$ and $\hat{E}_k^M$, when $N = 10$, $\alpha = 0.5$ and $q = 0.4$.

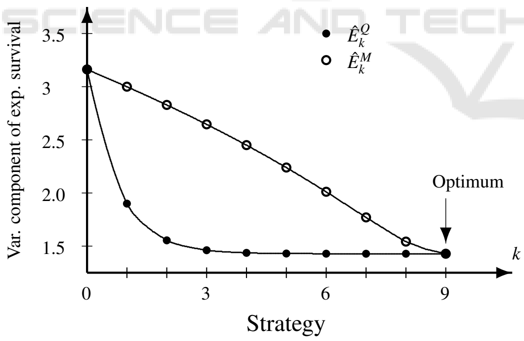

Figure 6: The variable component of the expected survival, $\hat{E}_k^Q$ and $\hat{E}_k^M$, when $N = 10$, $\alpha = 0.5$ and $q = 0.7$.

q is low the advantage of saving time (since $\alpha < 1$) is relatively higher since it is more likely that multiple tools will have to be used anyways due to the low probability of each one to eliminate the system.

While the Q_k strategies are lower than the M_k strategies when q is sufficiently high, it appears that this does not happen when it counts, i.e., the optimal M_k strategy is always less or equal to its companion Q_k strategy. This can be gleaned from Table 1 that describes the optimal strategy for different values of

Table 1: Optimal strategy for different values of α and q.

q	α =0.1	α =0.3	α =0.5	α =0.7	α =0.9
0.1	Sim	M_1	M_2	M_5	Seq
0.2	Sim	M_1	M_4	M_7	Seq
0.3	M_1	M_3	M_6	M_8	Seq
0.4	M_2	M_5	M_7	Seq	Seq
0.5	M_4	M_7	M_8	Seq	Seq
0.6	M_6	M_7	Seq	Seq	Seq
0.7	M_7	M_8	Seq	Seq	Seq
0.8	M_8	Seq	Seq	Seq	Seq
0.9	M_8	Seq	Seq	Seq	Seq

Notes: Sim and Seq denote the purely simultaneous and purely sequential strategies, respectively.

α and q (here, too, $N = 10$).

6 CONCLUSIONS

In this paper, we consider the question of whether "putting all your eggs in one basket" is advisable or not. This dilemma of pooling resources versus saving assets for later needs is applicable to many fields of operation. In our modelling, pooling resources does not create an advantage or disadvantage in the probability of success, since we assume that the success probability of each attack tool is identical and independent. Instead, by pooling resources the hacker trades off between waiting for the attack time to complete before the success can be realized and the fact that this attack time is shorter than a sequence of single-tool attacks of the same number of tools.

Therefore, when the attack time of a multiple-tool attack is very short, there is no advantage to the sequential approach and the optimal solution will be a purely simultaneous attack. In contrast, if the attack time is nearing the total time of a sequential attack, then the optimal solution will be a purely sequential attack. In between, we find that the optimal solution is mixed, where we show numerically that it is always preferable to begin with a simultaneous attack and then complement it with a sequential attack. Additionally, we demonstrate that as the probability of success increases the optimum leans more towards a sequential attack.

While our observations from the numerical example are not formally proved, it appears the formulas that we derive for the M_k and Q_k strategies can be mathematically analyzed to qualify these observations. It is also of interest to examine how changing the modeling assumption that the probability of success is independent of the other probabilities will affect the results. This is left to future study.

REFERENCES

Andradóttir, S., Ayhan, H., and Down, D. G. (2017). Resource pooling in the presence of failures: Efficiency versus risk. *European Journal of Operational Research*, 256(1):230–241.

Cardoso, F., Costa, A., Norton, L., Senkus, E., Aapro, M., Andre, F., Barrios, C., Bergh, J., Biganzoli, L., Blackwell, K. L., et al. (2014). Eso-esmo 2nd international consensus guidelines for advanced breast cancer (abc2). *The Breast*, 23(5):489–502.

Cui, P., Zhu, P., Xun, P., and Shao, C. (2018). Robustness of cyber-physical systems against simultaneous, sequential and composite attack. *Electronics*, 7(9):196.

Dear, R. F., McGeechan, K., Jenkins, M. C., Barratt, A., Tattersall, M. H., and Wilcken, N. (2013). Combination versus sequential single agent chemotherapy for metastatic breast cancer. *Cochrane Database of Systematic Reviews*, (12).

Lu, D.-Y., Wu, H.-Y., Yarla, N. S., Xu, B., Ding, J., and Lu, T.-R. (2018). Haart in hiv/aids treatments: future trends. *Infectious Disorders-Drug Targets (Formerly Current Drug Targets-Infectious Disorders)*, 18(1):15–22.

MacDonald, T. M., Williams, B., Webb, D. J., Morant, S., Caulfield, M., Cruickshank, J. K., Ford, I., Sever, P., Mackenzie, I. S., Padmanabhan, S., et al. (2017). Combination therapy is superior to sequential monotherapy for the initial treatment of hypertension: a double-blind randomized controlled trial. *Journal of the American Heart Association*, 6(11):e006986.

McNaughton, A. (2019). "parallel warfare" in conflicts with limited political aims.

Phillips, A. N., Youle, M. S., Lampe, F., Johnson, M., Sabin, C. A., Lepri, A. C., and Loveday, C. (2003). Theoretical rationale for the use of sequential single-drug antiretroviral therapy for treatment of hiv infection. *AIDS*, 17(7):1009–1016.

Read, D., Antonides, G., Van den Ouden, L., and Trienekens, H. (2001). Which is better: Simultaneous or sequential choice? *Organizational Behavior and Human Decision Processes*, 84(1):54–70.

Sasikumar, K. (2019). India's surgical strikes: Response to strategic imperatives. *The Round Table*, 108(2):159–174.

Soucy, R. (2018). Serial vs. parallel war: An airman's view of operational art. Technical Report ADA274440, School of Advanced Military Studies, US Army Command and General Staff College, Leavenworth, KS.

Sunar, N., Tu, Y., and Ziya, S. (2021). Pooled vs. dedicated queues when customers are delay-sensitive. *Management Science*, 67(6):3785–3802.

Thompson, G. M. and Kwortnik Jr, R. J. (2008). Pooling restaurant reservations to increase service efficiency. *Journal of Service Research*, 10(4):335–346.

Xiong, G., Dong, X., Lu, H., and Shen, D. (2019). Research progress of parallel control and management. *IEEE/CAA Journal of Automatica Sinica*, 7(2):355–367.