Hybrid Manufacturing / Remanufacturing Inventory Model with Two Markets and Price Sensitive Demands with Competition

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Abstract: A hybrid system that produces new and remanufactured products on a common line for two distinct markets is studied in this article. We consider price sensitive demands with competition between the two types of products. The problem is to maximize a profit as a function of economic production quantities and demands or prices decisions. The resulting model is a mixed nonlinear model with linear and nonlinear constraints. A mathematical analysis is proposed to develop an efficient resolution approaches. Numerical study shows that sequential decisions, determining demands first and reorder intervals after, provides solutions closed to optimum. Sensitivity analysis highlights the importance of demand parameters compared to those related to inventory and the importance of considering all costs, not just setup and holding costs, when evaluating order or production quantities.

1 INTRODUCTION

This paper presents models and solutions to determine Economic Production Quantity (EPQ) for hybrid manufacturing / remanufacturing systems. Two types of products, new and remanufactured, are produced on a common production line according to EPQ assumptions. We assume that they are produced in distinct lots due to traceability concerns and different setup requirements. General view of the product flows that we consider is presented in Fig. 1 (detailed presentation of notations is provided after). After being produced, new and remanufactured products are placed in distinct inventories to serve distinct markets. For the two types of products, after a period of use, one part of the products sold is directly disposed of while the other part can be remanufactured and are collected. Not all collected products are remanufactured and one part is disposed of with respect to the decision of the number of product that are remanufactured. Remanufactured products are stored before being processed. The systems thus contains three inventories with their respective holding costs and two production processes with their respective setup and unit cost. We also consider price decisions for new and remanufactured products and the problem is to maximise a profit function including revenues from the two markets, inventory holding and process costs.

As presented in Tang and Teunter (2006), for a real case in automotive industry, remanufactured products can be sold in the same market as new products without distinction in price (as-good-as new assumption). However, in many cases, they are sold at lower prices. Most of the papers related to inventory management for hybrid manufacturing / remanufacturing systems consider as-good-as new assumption. In this paper, we consider price decisions and distinct markets for the two types of products as in Godichaud and Amodeo (2022). Furthermore, some customers may be undecided between the two markets and can change according to selling prices. This means that the products are in competition for a common part of the two markets. This is modelled in price-to-demand relationships. Constraints must be added to the model to respect lower prices for remanufactured products and the maximum part of undecided customer between the markets.

In section 2, we present several works related to our problem. The model with the notations and assumptions are stated in section 3. Mathematical

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analysis and resolution method are developed in section 4. A numerical example is presented in section 5. Conclusion and extensions of this works are summarized in section 6.

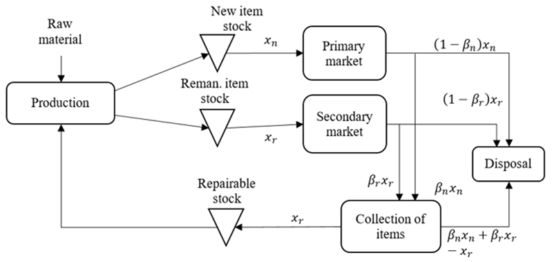

Figure 1: Material flows for hybrid systems with distinct markets and competition.

2 RELATED WORKS

The problem of determining economic lot sizes has long attracted the attention of researchers (Cárdenas-Barrón et al., 2014; Andriolo et al. 2014). Recent works tend to merge different features such as financing practices, deterioration and shortage; see e.g. in Tavakoli and Taleizadeh (2017), Taleizadeh et al. (2020) for review and new models. Three research streams in line with our problem have been identified: inventory models with remanufacturing operations, pricing and inventory-integrated models, pricing and revenue management in reverse logistic with remanufacturing.

In the field of inventory management with returned and remanufactured products, there are many papers and we restrict our attention on models with EOQ/EPQ-like assumptions. The literature review in (Bazan et al., 2016) highlights that there are three clusters of models before the paper date based on the models of (Richter, 1996), (El Saadany and Jaber, 2008) and (Teunter, 2001). The works reviewed in (Bazan et al., 2016) consider a single market for new and remanufactured products. We identified only two works that consider distinct markets (Jaber and El Saadany, 2009; Hasanov et al., 2012). They used an original assumption: the two types of product are stored and sold separately over distinct periods. This leads to stockout situation that cannot be avoided. When one type of product is in stock, demands for the other are backordered.

Several policies are investigated in literature that considerer common production line for processing different product types under EPQ assumptions, The common cycle policies consist in having one EPQ cycle for each product in one global cycle (Tang and Teunter, 2006; Teunter et al., 2008; Teunter et al., 2009; Nobil et al., 2020). They are easier to determine but better results can be achieved by using basic period policies (Zanoni et al., 2012), which consists in determining cycle length for each product as multiple of a basic period. For the case with only two products, the basic period policy can be shown to be optimal (Vemuganti, 1978). Basic approach to determine optimal economic quantity is to derive the related cost function. If this is simple enough without constraint, closed-form equations can be obtained even for the case with remanufacturing option. In the case of multi-product ELS problem, the cost function is more complex with specific constraints for the line occupancy. Heuristics are then proposed in literature (Teunter et al., 2009; Zanoni et al., 2012). We also note that in all papers reviewed in this research stream a cost function is used that contains setup and inventory holding costs. More recently, Soleymanfa et al. (2022) integrate sustainability related parameters (environmental and social aspects) in the cost function but consider same market for new and returned products. (Hasanov et al. 2019) consider a four level supply chain with energy, carbon emission and disposal considerations. The literature review in (Karim and Nakade, 2022) does not mentioned papers assuming competition between new and remanufactured products. In this paper, we consider problems with two products, new and remanufactured, but with price decisions and profit objective function instead of a cost function.

Basis of models integrated pricing and EOQ/EPQ are presented in Kunreuther and Richard (1971) for a single item problem. The model can be extended to consider multi-echelon serial supply chain (Lau & Lau, 2003), wholesale price and discount policies (Viswanathan and Wang, 2003) in the cases of single market for one final product. The linear demand / price function is commonly used because it facilitates property analysis and is easy to adjust to field data. The iso-elastic function presents the same advantages in the cases without market competition (Ray et al., 2005). In our work, we used linear demand function with parameters modelling competition between products, as in Bernstein and Federgruen (2003), and we consider returned products and common production line considerations with EPQ assumptions. For the case where one production line has to process several products, Salvietti and Smith (2008) propose models and methods for ELS problem with pricing decisions. The products are different and sold on distinct market without competition. Our work resumes the common line aspects but integrates and analyses product returns and competition between products. In the context of reverse logistic, Teksan and Geunes (2016) consider the coordination between recovery decisions and selling prices for a single product / market and the works of Pour-Massahian-Tafti et al. (2020), Godichaud and Amodeo (2020) address pricing problems in disassembly systems. Relationship between price and demand can be extended to consider perishable product and promotion (Avinadav et al., 2016), stock and shortage (Mishra et al., 2017). Taleizadeh et al. (2019) consider pricing and EOQ integrated model for substitutable products. In our work, remanufactured and new products can be considered as partially substitutable and additional constraints are necessary. Taleizadeh et al. (2022) propose an EOQ underlying wit partial credit, partial backordering, carbon emissions and demand function of selling price and carbon emissions. A signomial geometric programming approach is proposed to solve the problem. Finally, this paper extends Godichaud and Amodeo (2022) by considering competition between new and remanufactured products, additional decision on inventory and further analysis on decision process in this context.

Competition between products is considered in several papers related to pricing / revenue management in reverse logistic with remanufacturing. Distinct markets with competition for new and remanufacturing products are considered in Majumder et al. (2001), Wu (2012), Wang et al. (2019). Competition between different supply chain agents is investigated in Ranjbar et al. (2020). We note that linear relationship between demand and price are used in all these papers. It facilitates model analysis and derivation of closed-form equations for optimal decision. Except in Guide et al. (2003), all these works consider different agents for the recovery of products and study roles of third party recover partner in closed-loop supply chain. In the case of two-echelon supply chain, wholesale prices are considered as decision variables. Patoghi et al. (2022) developed pricing model with different supply chain actors with demands function of price, quality, collection effort and return policy. The effect of production constraints and inventory costs are however not taken into account. We also note that in these papers only simple unit proportional costs are considered.

Based on the previous reviewed papers, we propose in this paper an inventory model under EPQ assumptions with manufacturing and remanufacturing operations. Common production line and return limitation constraints are considered. Pricing decisions with two markets for newly produced and remanufactured products is analysed.

3 MODEL STATEMENT

3.1 Notations

The material flows of the problem under study are shown in Figure 1. The parameters, presented her after, are related to the two processes, manufacturing and remanufacturing, the three inventories (new, remanufactured and repairable products) and the two markets (primary for new products and secondary for remanufactured products). The data and variables are indexed by n and r for the new and remanufactured items respectively.

Cost data are those of basic EPQ model for each product stream (time unit is written in day but can be changed depending on the context):

- s_n, s_r setup costs, [€/lot],
- h_n , h_r , h_u inventory holding costs(u is for returned products), [€/product.day],
- c_n, c_r unit production costs [€/product].

Process data and variables represent the two processes with returned products and common line utilization:

- m_n, m_r production rates [product/day],
- τ_n, τ_r setup times [day],
- β_n, β_r percentage of available items from primary and secondary markets,
- t_n, t_r cycle lengths [day],
- b basic period [day].

Demands or prices can be used as variables:

- x_n, x_r demand rates [product/day],
- y_n, y_r selling prices [€].

The relationships between demand rates and selling prices are defined by the following relationships:

- $X_n(y_n, y_r) = d_n - a_n y_n + e y_r$ and $X_r(y_n, y_r) = d_r - a_r y_r + e y_n$ are the demand rates for new or remanufactured items, each function of the selling prices of the two types of items,
- $Y_n(x_n, x_r) = u_n - v_n x_n - w x_r$ and $Y_r(x_n, x_r) = u_r - v_r x_r - w x_n$ are the unit selling prices of one new or remanufactured item, function of the demand rates of the two types of items.

For linear demand-to-price relationships, the interpretation of the parameters is the following:

- d_n , d_r maximum demands for new items and remanufactured items (markets size),
- a_n , a_r , e price to demand function parameters (linear relationships),

- u_n, u_r maximum prices,
- v_n, v_r, w demand to price function parameters.

The same competition parameter, e, is used is the two functions, X_n and X_r, based on the following interpretation of the competition. There are some undecided customers between buying a new product or a remanufactured product. Their decision is made based on the price difference $y_n - y_r$. For a given price difference, $e(y_n - y_r)$ represents the number of customers (per unit of time) that change their willing to buy a new product to buy a remanufactured one. Initially, the functions X_n and X_r can be defined by $X_n(y_n, y_r) = d_n - a'_n y_n - e(y_n - y_r)$ and $X_r(y_n, y_r) = d_r - a'_r y_r + e(y_n - y_r)$. By setting $a_n = a'_n + e$ and $a_r = a'_r + e$, we retrieve the previous function. We also note that given this interpretation of the competition, we impose constraint $y_n > y_r$, otherwise undecided customer prefer to buy a new product. The change between demand functions and price functions can be done using the following relationships: $v_n = a_r/(a_n a_r - e^2)$, $u_n = v_n\big(d_n + d_r(e/a_r)\big)$, $w = v_n(e/a_r) = v_r(e/a_n)$, $v_r = a_n/(a_n a_r - e^2)$, $u_r = v_n\big(d_r + d_n(e/a_n)\big)$.

3.2 Assumptions

The problem assumptions follow those of traditional EPQ and ELS-R with remanufactured returns:

- Demands, production and returns are characterized by rates (items per day for instance), and they are constant,
- Inventory are continuously reviewed (continuous time inventory model),
- Cost and profit indicators are defined in average (unit of money per unit of time) on an infinite time horizon,
- There is a fixed cost incurred for each production setup (before starting a production lot) and one unit in inventory per unit of time generates an holding cost,
- Backlogs or lost sales are not allowed in this work.

In addition to the previous presentation of relationship between prices and demands, we add the following assumptions more specific to the problem under study:

- There are two markets, one for new products and one for remanufactured products, but one production line to produce the two types of products,
- Production rate, holding and setup costs are different for the two types of product,
- Demands or prices for the two markets are set once at the beginning of the planning horizon,
- Only the returned products that will be remanufactured are kept in the return inventory, if the returns are superior to the remanufacturing quantity, the surplus is disposed of (i.e. directed to others recovery channels).

These assumptions justify the rate associated to the material flows in Figure 1. The products availability from the production line must be x_n and x_r to satisfy demands in each markets. After the period of use, $\beta_n x_n + \beta_r x_r$ products are available for remanufacturing. However, only x_r are remanufactured and $\beta_n x_n + \beta_r x_r - x_r$ are disposed of. The return inventory is replenished continuously at rate x_r.

Based on previous assumptions, the evolution of the three inventories over time is presented in Figure 2. The curves In, Ir and Iu represent respectively the new, remanufactured and repairable product inventories. Curve shapes are basic saw tooth profiles due to EPQ-related assumptions. There is a repetition of two phases for each inventory: one production-consumption phase and one consumption-only phase. For In and Ir, inventories first increase at rate $m_n - x_n$ and $m_r - x_r$ and then decrease at rate x_n and x_r. The duration of the two phases is t_n (resp. t_r) and the first lasts $(x_n/m_n)t_n$ (resp. $(x_r/m_r)t_r$. For Iu, inventory first increases at rate x_r and then decrease at rate $m_r - x_r$ symmetrically to inventory Ir. Only the returned products that will be remanufactured are kept in the return inventory and there is only a common setup cost to move an item between the two inventories with remanufacturing process.

The low part (under the graph) of Figure 2 shows the production line occupancy. There is one repetitive cycle with two new production lot, one remanufacturing lot and idle time between lots in this example. Setup times take place before each lot.

Basic period policy is used to ensure that the solution defining the cycle length of each product is feasible without overlapping. Basic period policy has the following characteristics:

- The cycle length of each type of product is an integer multiple of the basic period,
- One basic period can contain one production phase and one setup for each type of product.

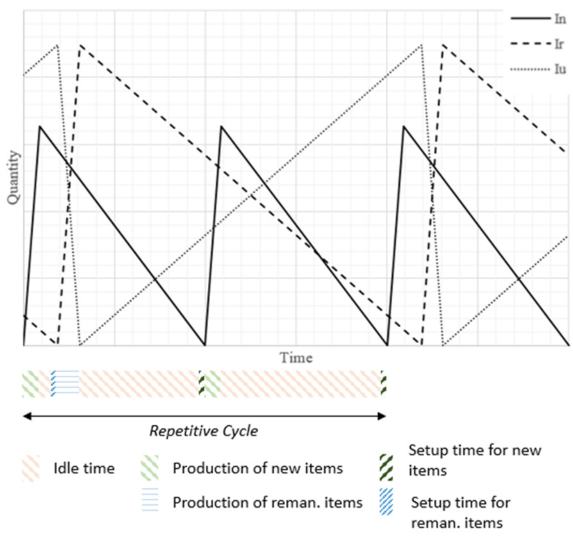

Figure 2: Inventories evolution over time.

Based on the results in (Vemuganti, 1978), basic period policy is optimal for two products if the basic period is a variable. In Figure 2, the repetitive cycle lasts two basic periods with two new product batches and one remanufacturing batch. The first basic period in the cycle contains the productions and setup times for one cycle of each type of products. By denoting with b the length of the basic period and k_n, k_r the integer multiples, the basic period impose that $t_n = k_n b$ and $t_r = k_r b$ and the following constraint:

$$\tau_n + \tau_r + \frac{x_n}{m_n} k_n b + \frac{x_r}{m_r} k_r b \leq b$$

According to the previous assumptions, especially constant demand, production and return rates and continuous time review, one can note that on Figure 2 that inventories are linear functions of time. Average inventory holding cost for the three inventories, denoted by H_n, H_r and H_u ([€/day]) are given by, setting $t_n = k_n b$ and $t_r = k_r b$:
$H_n = \frac{h_n}{2} k_n b x_n \left(1 - \frac{x_n}{m_n}\right)$, $H_r = \frac{h_r}{2} k_r b x_r \left(1 - \frac{x_r}{m_r}\right)$ and $H_u = \frac{h_u}{2} k_r b x_r \left(1 - \frac{x_r}{m_r}\right)$.

As mentioned previously, we consider competition as a part of consumers, positioned between the two markets, which are sensitive to the price difference between the two products. Furthermore, we also consider that if the remanufactured product price is higher than the new product price then no customer will want to buy a remanufactured product. The constraint $Y_n \geq Y_r$ is then added to the model.

The parameter d_r being interpreted as the remanufactured product market size, the constraint $d_r - (a_r - e)Y_r \geq 0$ is necessary. Mathematically, without this constraint, it is possible to have x_r $(= d_r - a_r Y_r + e Y_n) > 0$ and $d_r - (a_r - e)Y_r \leq 0$. Constraint $d_n - (a_n - b)Y_n \geq 0$ is not necessary since we must have $x_n = d_n - (a_n - e)Y_n - e(Y_n - Y_r) \geq 0$ and $Y_n \geq Y_r$.

3.3 Model

The problem is to maximise the system average profit, denoted by $\Pi(x_n, x_r, k_n, k_r, b)$, with the decision variables x_n , x_r , k_n , k_r and b under the previous assumptions. The resulting model is a non-linear mixed integer program with constraints defined by (1)-(5).

$$\Pi(x_n, x_r, k_n, k_r, b) = (Y_n - c_n)x_n + (Y_r - c_r)x_r - \frac{s_n}{k_n b} - \frac{s_r}{k_r b} - \frac{h_n}{2} k_n b x_n \left(1 - \frac{x_n}{m_n}\right) - \frac{(h_r + h_u)}{2} k_r b x_r \left(1 - \frac{x_r}{m_r}\right) \qquad (1)$$

$$-\beta_n x_n - (\beta_r - 1)x_r \leq 0 \qquad (2)$$

$$(-v_r + w)x_r + (v_n - w)x_n \leq u_n - u_r \qquad (3)$$

$$-v_r x_r - w x_n \leq (w/(v_n - w))(u_r - u_n) \qquad (4)$$

$$\tau_n + \tau_r + \frac{x_n}{m_n} k_n b + \frac{x_r}{m_r} k_r b \leq b \qquad (5)$$

The first two terms of the objective function represent profit without inventory related parameters. We denote it by $M(x_n, x_r)$. The following two terms are the average setup costs and the last two are the average holding costs. The objective function is different from (Godichaud & Amodeo, 2022) with consideration of competition (different Y_n and Y_r functions) and b as decision variable. Constraints (2) to (5) arise from previous assumptions. Constraint (2) limits the available returns for remanufacturing. Constraint (3) corresponds to $Y_n \geq Y_r$ with respect to decisions variable x_n and x_r. Constraint (4) forces $d_r - (a_r - e)Y_r \geq 0$. Constraint (5) is the basic period policy constraint.

4 MATHEMATICAL ANALYSIS AND RESOLUTION APPROACHES

The problem models by (1)-(5) is a non-linear programming model (objective function and constraint (5) are non-linear) with integer variables k_n and k_r. We propose to decompose its analysis into sub-problems having good properties for solving and analysis of solutions. The problem without inventory related costs is first studied to obtain an initial solution (section 4.1). This solution is completed sequentially with the determination of inventory related variables with respect to a 2-items ELS problem (section 4.2). The solution is then iteratively improved with non-linear resolution methods.

This decomposition enables to compare three resolution approaches:

- Sequential resolution: this approach use the properties of the pricing presented section 4.1 and ELS resolution method.
- One iteration resolution: it adds one step to adjust the demands based on the cycle lengths find in the sequential resolution. This is a modified pricing problem integrating inventory-holding costs presented in section 4.3.
- Joint optimisation: based on the initial solution found with the one iteration resolution, all the variables are optimised simultaneously based on the procedure presented in section 4.3.

4.1 Problem Without Inventory Related Costs

We consider only the first two terms of the profit function (1). It reduces to the profit function denoted by $M(x_n, x_r)$, with only variables x_n and x_r, which is a sum of common functions in pricing problems. There are two motivations to study this sub-problem apart. In many companies, as mentioned in (Kunreuther and Richard, 1971), a sequential decision process is performed. Prices are decided first, by marketing department, and then, production and inventory decisions are made with the demands associated with the given prices. In our case, k_n, k_r and b would be decided in a second step with x_n, x_r fixed in a first step. Even in case where integrated decisions are made, solving sub-problem without inventory related costs gives initial values.

Replacing Y_n and Y_r as function of x_n and x_r, $M(x_n, x_r)$ is quadratic function:

$$M(x_n, x_r) = (u_n - v_n x_n - w x_r - c_n)x_n + (u_r - v_r x_r - w x_n - c_r)x_r$$

Its stationary points are:

$$\bar{x}_n = \frac{v_r(u_n - c_n) - w(u_r - c_r)}{2(v_n v_r - w^2)} \text{ and } \bar{x}_r = \frac{v_n(u_r - c_r) - w(u_n - c_n)}{2(v_n v_r - w^2)}$$

$M(x_n, x_r)$ is concave for $x_n > 0$ and $x_r > 0$ .The constraints (2)-(4) are linear and the sub-problem of maximising $M(x_n, x_r)$ w.r.t. (2)-(4) is solve rapidly with a solver or feasible direction methods.

4.2 Initial EPQ Problem with Two Products and a Common Line

If x_n and x_r are fixed, the problem reduces to the minimisation of inventory related cost (setup and holding) subject to constraint (5). Furthermore, for given value of k_n and k_r, the optimal value for b is:

$$b_{opt} = \sqrt{\frac{2((S_n/k_n) + (S_r/k_r))}{h_n k_n x_n\left(1 - \frac{x_n}{m_n}\right) + (h_v + h_r) k_r x_r \left(1 - \frac{x_r}{m_r}\right)}} \quad (6)$$

Constraint (5) gives the minimal value for b, denoted by b_{min}, to process the two types of products on the same production line without overlapping of production runs:

$$b_{min} = (\tau_n + \tau_r) / \left(1 - \left(\frac{k_n x_n}{m_n} + \frac{k_r x_r}{m_r}\right)\right) \quad (7)$$

The solution of the problem, for given values of x_n, x_r, k_n and k_r, are then $b = max\{b_{opt}, b_{min}\}$. The value for k_n and k_r are tested iteratively in the overall procedure.

4.3 Joint Optimisation

Previous sub-problem can be extended with inventory-related decision variables (k_n, k_r and b) fixed (setup costs are constant but holding cost vary w.r.t. x_n and x_r). The objective function then remains quadratic and concave (under the following conditions) with one stationary point given directly by the following equations:

$$\hat{x}_n = \frac{v_r^+(u_n - c_n^+) - w(u_r - c_r^+)}{2(v_n^+ v_r^+ - w^2)},$$

$$\hat{x}_r = \frac{v_n^+(u_r - c_r^+) - w(u_n - c_n^+)}{2(v_n^+ v_r^+ - w^2)}$$

with $c_n^+ = c_n + h_n k_n b/2$, $c_r^+ = c_r + (h_r + h_v) k_r b/2$, $v_n^+ = 2v_n - h_n k_n b/m_n$ and $v_r^+ = 2v_r - (h_r + h_v) k_r b/m_r$. The function is concave for $4v_n^+ v_r^+ - 4w^2 > 0$.

With k_n, k_r and b fixed, constraint (5) is linear as constraint (2)-(4). Based on the previous properties, this sub problem can be solved by using any feasible direction methods.

At a second level, k_n, k_r are fixed and a line search is applied with respect to b. Based on initial values of x_n, x_r, the first value of b for the line search is determined. Values of x_n, x_r are then determined for each value of b by solving the problem with k_n, k_r and b fixed. Numerically, based on instances we used, the search of b with built-in optimisation on x_n, x_r gives concave profit function.

At a higher level, values of k_n, k_r are examined with nested iterations. Starting with $k_n = k_r = 1$, the values are increased while the profit function increases. The following procedure can be used to solve the overall problem, we denote by π^* the best profit found during the procedure with k_n^*, k_r^*, x_n^*, x_r^* and b^* the related decisions:

- Step1. Solve the problem without inventory related cost (section 4.1) to have initial value for x_n, x_r. Set $k_n = k_r = k_n^* = k_r^* = 1$, $x_n^* = x_n$, $x_r^* = x_r$ and $b^* = b = max\{b_{opt}, b_{min}\}$ and compute the first best profit π^*.
- Step 2. Set $b = max\{b_{opt}, b_{min}\}$. Adjust value for x_n, x_r with inventory related cost with b fixed.
- Step 3. Apply line search on b with built-in optimisation on x_n, x_r and compute the profit, denoted by π_X, for the obtained value. If $\pi_X > \pi^*$, update the best solution (set $\pi^* = \pi_X$ and k_n^*, k_r^*, x_n^*, x_r^* and b^* to the current value of k_n, k_r, x_n, x_r and b), set $k_r \leftarrow k_r + 1$ and go back to Step 2, else if $k_n = k_n^*$, set $k_n \leftarrow k_n + 1$ and go back to Step 2, else go to Step 4.
- Step 4. Return the best solution found.

5 NUMERICAL ANALYSIS

A first set of instances is generated based on the instance generator proposed in Salvietti and Smith (2008). The following steps are used to generate instances ($U[lower\ upper]$ corresponds to uniform distribution between specified lower and upper value of the parameters). The time unit is one day.

- Market sizes are generated with $d_n = U[4000\ 5000]$ and $d_r = U[0.2\ 0.8] \times d_n$ (we study cases where secondary market is smaller than primary market).
- Maximum price are generated with $Y_{n,max} = U[20\ 50]$ and $Y_{r,max} = U[0.5\ 0.8] \times Y_{n,max}$ (the maximum price of remanufactured products is lower since we assume this constraint in the model).
- To generate values for e (competition parameter), we used a maximum market change parameter, denoted here by X_c (defined for instance generation only). It represents the percentage of customers who can change from new to remanufactured products. We used $X_c = U[0.05\ 0.2] \times d_n$ and $e = X_c / Y_{n,max}$.
- Price-to-demand parameter are deduced from previous parameters with: $a_n = d_n / Y_{n,max} + e$ and $a_r = d_r / Y_{r,max} + e$.
- Unit cost are generated with $c_n = U[0.1\ 0.5] \times u_n$, to avoid infeasible instances with $c_n > u_n$, and $c_r = U[0.2\ 0.9] \times c_n$.
- Based on (Salvietti et al., 2008), set-up and inventory holding costs are generated with $s_n = U[30\ 50]$, $s_r = U[30\ 50]$, $h_n = (U[0.05\ 0.1] \times c_n)/100$, $h_v = (U[0.05\ 0.1] \times c_r)/100$ and $h_r = U[0.1\ 0.5] \times h_v$; production rates with $m_n = U[2000\ 4000]$ and $m_r = U[2000\ 4000]$; set-up times with $t_n = U[0.05\ 0.3]$ and $t_r = U[0.05\ 0.3]$.

These instances can be considered as high-level demands and low-level inventory costs. Table 1 presents data for three instances from this dataset. Instance 0 is obtained with median values of the distribution ranges. We generated more than one thousand instances and the instance 1 and 2 in Table 1 correspond to those that give the best and the worst profit respectively.

The first part of Table 2 presents values of decision variables and profit for instances in Table 1 with the three decision processes presented in section 4.

Table 1: Instances generated based on (Salvietti and Smith, 2008).

Ins.	Data
0	d_n=4500 a_n=144.63 d_r=2250 a_r=114.96 e=16.06 c_n=10.14 c_r=5.58 s_n=40 s_r=40 h_n=7.61E-03 h_{vr}=4.2E-03 m_n=3000 m_r=3000 τ_n=0.175 τ_r=0.175 β_n=0.667 β_r=0.667
1	d_n=4948 a_n=121.65 d_r=3859 a_r=142.25 e=18.56 c_n=4.74 c_r= 3.62 s_n=45.1 s_r=46.6 h_n=4.09E-03 h_{vr}=4.07E-03 m_n=3742 m_r=2747 τ_n=0.21 τ_r=0.29 β_n=0.667 β_r=0.667
2	d_n=4016 a_n=238.95 d_r=1767 a_r=180.65 e=38.15 c_n=9.07 c_r= 7.22 s_n=30.13 s_r=49.04 h_n=4.58E-03 h_{vr}=5.22E-03 m_n=3400 m_r=2409 τ_n=0.13 τ_r=0.16 β_n=0.667 β_r=0.667

In this type of instances, inventory related cost (setup + holding costs) are very small compared to unit proportional costs ($c_n x_n + c_r x_r$). One can note that inventory costs have little effect on the overall profit. The variation of decisions and profit are then small between the three decision processes. In this case, a sequential decision process in two phases is sufficient while having properties of separate models presented in section 4. One limitation is that demands values generated in the first phase can be infeasible for the second phase if $x_n/m_n + x_r/m_r \geq 1$. This is observed in 23% of the instances generated. This limitation can be solved by introducing a constraint $x_n/m_n + x_r/m_r < \alpha$, with $\alpha < 1$, in the problem of the first phase. It also allows having an initial solution for the joint optimization problem. This is the case of instance 1.

In instance 0, basic period constraint is not binding (line occupancy is 0.895 < 1) with the optimal demands. The line occupancy is equal to one in all instances where the first phase of the sequential decisions give infeasible for the second phase. The basic period constraint has a greater effect when the second market demand is small compared to primary market demand when it is not possible to place two or more remanufacturing lot with one manufacturing lot in one basic period.

In instance 2, we note again no significant variation of variables and profit between the three decision processes and the inventory cost represents a small parts of the overall cost. In this instance, two remanufacturing lot can be optimally placed in one basic period for one manufacturing lot. In instance 1, the first phase of the sequential decision gives infeasible solutions. We add the constraint mentioned before to obtain an initial solution for the joint optimisation.

The lines "Stat." in Table 2 presents the means and standard deviations of variables and indicators over 1200 instances generated. We note significant variation of the basic period length (b) due to constraint (5). The constraint is saturated in 34% of instances and, for these ones, the basic period and inventory costs tend to be larger. We also note that only 53% of the returned products are remanufactured. If environmental constraints impose higher remanufacturing rate, the producer have to accept lower profit.

Table 2: Results for instances generated based on (Salvietti and Smith, 2008).

Ins.		Results (decisions and profit)
0	Joint	k_n=1 k_r=1 b=4.39 Profit=26728.76 x_n=1561.35 x_r=885.55 y_n=21.98 y_r=14.94
	Sequ	k_n=1 k_r=1 b=4.39 Profit=26728.76 x_n=1561.27 x_r=885.77 y_n=21.98 y_r=14.94
	1-iter	k_n=1 k_r=1 b=53.07 Profit=68440.76 x_n=1975.60 x_r=885.77 y_n=21.98 y_r=14.94
1	Joint	k_n=1 k_r=1 b=4.39 Profit=26728.76 x_n=1561.35 x_r=1271.53 y_n=27.76 y_r=21.81
	Sequ	Profit=70770.01 x_n=2219.3 x_r=1716.02 y_n=25.23 y_r=18.36 (not feasible w.r.t. (5))
2	Joint	k_n=1 k_r=2 b=3.99 Profit=6580.4 x_n=1061.94 x_r=403.18 y_n=14.04 y_r=10.51
	Sequ	k_n=1 k_r=2 b=3.99 Profit=6580.4 x_n=1062.08 x_r=404.36 y_n=14.04 y_r=10.51
	1-iter	k_n=1 k_r=2 b=3.99 Profit=6580.4 x_n=1061.94 x_r=403.18 y_n=14.04 y_r=10.51
Stat.	Joint	b=11.88(15.58) Profit=26940.97(11003.8) x_n=1497.0 (248.12) x_r=842.78(301.76) y_n=22.45(5.81) y_r=15.32 (4.29)

Joint = joint optimisation of all variables, *Sequ* = demand variables first and reorder interval variables after, *1-iter* = reorder interval variables fixed by *Sequ* and re-optimisation of demand variables, line *Stat* gives mean value and standard deviation over 1200 instances.

0	Joint	InvCost=36.48 LineOcc=0.895 Return=1631.27 Rem./Return=0.543 Market change=113.015
1	Joint	InvCost=176.86 LineOcc=1 Return=2164.77 Rem./Return=0.59 Market change=110.46
2	Joint	InvCost=27.36 LineOcc=0.72 Return=976.75 Rem./Return=0.41 Market change=134.54
Stat	Joint	InvCost=48.22 (25.12) LineOcc=0.90 (0.10) Return=1559.90 (276.5) Rem./Return=0.53 (0.13) Market change=111.82 (46.72)

InvCost = order + holding costs, *LineOcc* = setup + production times on basic period length, *Return* = return rate of products that can be remanufactured, *Rem./Return* = proportion of remanufacturable products which are actually remanufactured, Market change = part of secondary market that comes from primary market.

In the previous dataset, the joint optimisation brings no profit improvement or change in variables values. This is due to the low level of inventory-related costs (setup plus holding costs) compared to that of proportional costs ($c_n x_n + c_r x_r$). In this type of instance the sequential resolution, give solutions closed to optimum and cycle lengths variation does not lead to significant profit variation. We observe this fact, in all dataset form ELS-related literature we have tested (questioning the importance of sophisticated method compared to simple ones like common cycle policy). A second set of examples is generated to analyse others types of practical situations. These examples are adapted from literature. The data are presented in Table 3 and the results are presented in Table 4. These examples have lower market sizes (which must lead to lower demands and production volumes) and higher inventory holding costs (the gap between inventory-related costs and proportional costs increases with respect to the demand). The time unit is one day.

Table 3: Additional instances based on literature on EOQ-pricing and ELS problems.

Ins.	Data
5	d_n=40 a_n=1.608 d_r=32 a_r=2.568 e=0.32 c_n=10 c_r=5 s_n=50 s_r=50 h_n=1 h_{vr}=0.85 m_n=50 m_r=30 τ_n=0.5 τ_r=0.5 β_n=0.67 β_r=0.67
6	d_n=40 a_n=1.608 d_r=32 a_r=2.568 e=0.32 c_n=10 c_r=5 s_n=50 s_r=50 h_n=1 h_{vr}=0.85 m_n=50 m_r=30 τ_n=2 τ_r=2 β_n=0.67 β_r=0.67
7	d_n=100 a_n=0.334 d_r=80 a_r=0.534 e=0.067 c_n=15 c_r=13 s_n=150 s_r=150 h_n=4.5 h_{vr}=4.1 m_n=100 m_r=100 τ_n=0.5 τ_r=0.5 β_n=0.67 β_r=0.67
8	d_n=100 a_n=2.004 d_r=80 a_r=3.34 e=0.4 c_n=15 c_r=13 s_n=150 s_r=150 h_n=4.5 h_{vr}=4.1 m_n=100 m_r=100 τ_n=0.5 τ_r=0.5 β_n=0.67 β_r=0.67
9	d_n=100 a_n=3.34 d_r=80 a_r=3.204 e=0.67 c_n=15 c_r=13 s_n=150 s_r=150 h_n=4.5 h_{vr}=4.1 m_n=100 m_r=100 τ_n=0.5 τ_r=0.5 β_n=0.67 β_r=0.67
10	d_n=20 a_n=0.404 d_r=16 a_r=0.404 e=0.08 c_n=25 c_r=20 s_n=130 s_r=130 h_n=1.2 h_{vr}=0.85 m_n=100 m_r=100 τ_n=0.5 τ_r=0.5 β_n=0.67 β_r=0.67

Instances 5 and 6 are generated as the previous one except that the market size is 40 (products per day) instead of 4000. The difference between the two are the setup times (0.5 and 2 days) to get insights on the effect of constraint (5) and the utilisation of a common production line. For instance 5, inventory-related costs represents 24% of the total costs for the sequential solution (we remind that it is also the initial solution for the joint solution) which is significant. However, the variation in profit and decision variables is not. The line occupancy in less than 1 allowing the basic period length b to be at the optimum and the inventory cost function is flat at this value. On the contrary, in instance 6, the setup times are long, 2 days while the basic period length is 7.52 days, and the constraint (5) is saturated. A gap of 24.5% is observed between the sequential and joint methods while the inventory-related costs represents 29% of the total costs for the sequential solution close to instance 5. In this case, the joint method act simultaneously on the basic period length and the demands to find the optimal values along the constraint. In these two instances, more returned products are used in proportion compared to first dataset, but not all.

Instances 7 to 9 are adapted from (Taleizadeh et al. 2019), who study a real case application of pricing-inventory model for two substitutable products. Instance 7 have high maximum prices compared to unit costs. We also note that the inventory holding costs are very high, assuming the integration of more aspects than the financing part (warehousing, energy, handling resources …). Instances 8 and 9 are generated from instance 7 by reducing the maximum prices in order to have a greater share of inventory cost in the profit and analyse the behaviour of the resolution methods in these cases. For the three instances, inventory-related costs represents around 30% of the total costs for the sequential solution but the variation in the profit is limited: 1.4%, 1.3% and 3.7% for instances 7, 8 and 9 respectively. In instance 7, the variation in inventory-related costs is 30% between sequential and joint solutions showing the simultaneous effect of cycle length and demands. These results also show the quality of the sequential solution as initial solution for the joint resolution. The remanufacturing rate is close to that obtained from the first dataset and remains low (half of the return product are remanufactured).

Instance 10 is adapted from (Zipkin, 2000) and has higher setup and holding costs and low demands. The time basis is the week. The inventory-related costs represents 24% of the total costs but the variation of the profit is again limited to 1.7% between sequential and joint solutions. Constraint (5) is not saturated and the percentage of returned products that are remanufactured is higher than in the first dataset.

Table 4: Results for the second set of instances.

Ins.		Results (decisions and profit)
5	Joint	k_n=1 k_r=1 b=3.65 Profit=121.18 x_n=12.07 x_r=10.77 y_n=19.5 y_r=10.7
	Sequ	k_n=1 k_r=1 b=3.60 Profit=120.8 x_n=12.76 x_r=11.18 y_n=19.01 y_r=10.48
	1-iter	k_n=1 k_r=1 b=3.60 Profit=121.17 x_n=12.08 x_r=10.78 y_n=19.49 y_r=10.69
6	Joint	k_n=1 k_r=1 b=7.25 Profit=104.6 x_n=10.71 x_r=7.02 y_n=20.66 y_r=12.3
	Sequ	k_n=1 k_r=1 b=10.75 Profit=83.99 x_n=12.76 x_r=11.18 y_n=19.03 y_r=10.48
	1-iter	k_n=1 k_r=1 b=10.75 Profit=87.89 x_n=10.52 x_r=9.54 y_n=20.58 y_r=11.31
7	Joint	k_n=1 k_r=1 b=4.49 Profit=10457.68 x_n=45.58 x_r=32.15 y_n=185.44 y_r=112.76
	Sequ	k_n=1 k_r=1 b=6.65 Profit=10311.15 x_n=47.93 x_r=36.03 y_n=176.36 y_r=102.49
	1-iter	k_n=1 k_r=1 b=6.65 Profit=10312.90 x_n=47.95 x_r=37.03 y_n=176.67 y_r=104.39
8	Joint	k_n=1 k_r=1 b=2.13 Profit=662.51 x_n=36.29 x_r=16.84 y_n=36.64 y_r=24.29
	Sequ	k_n=1 k_r=1 b=2.48 Profit=646.43 x_n=37.57 x_r=22.17 y_n=35.64 y_r=22.50
	1-iter	k_n=1 k_r=1 b=2.48 Profit=654.18 x_n=36.76 x_r=17.1 y_n=36.38 y_r=24.17
9	Joint	k_n=1 k_r=1 b=1.98 Profit=211.27 x_n=26.62 x_r=19.91 y_n=26.62 y_r=23.31
	Sequ	k_n=1 k_r=1 b=2.11 Profit=203.68 x_n=29.28 x_r=23.29 y_n=25.58 y_r=22.08
	1-iter	k_n=1 k_r=1 b=2.11 Profit=210.69 x_n=26.42 x_r=19.65 y_n=26.7 y_r=23.4
10	Joint	k_n=3 k_r=4 b=2.18 Profit=106.04 x_n=5.18 x_r=4.42 y_n=44.1 y_r=37.4
	Sequ	k_n=3 k_r=4 b=2.07 Profit=104.24 x_n=5.75 x_r=4.96 y_n=42.34 y_r=35.71
	1-iter	k_n=3 k_r=4 b=2.07 Profit=105.95 x_n=5.21 x_r=4.45 y_n=44.01 y_r=37.31
5	Joint	InvCost=54.82 LineOcc=0.87 Return=15.23 Rem./Return=0.71 Market change =2.82
6	Joint	InvCost=60.86 LineOcc=1 Return=11.82 Rem./Return=0.59 Market change =2.68
7	Joint	InvCost=518.19 LineOcc=1 Return=51.82 Rem./Return=0.62 Market change =4.85
8	Joint	InvCost=312.85 LineOcc=1 Return=35.42 Rem./Return=0.48 Market change =4.94
9	Joint	InvCost=303.25 LineOcc=0.97 Return=31.02 Rem./Return=0.64 Market change =2.21
10	Joint	InvCost=69.7 LineOcc=0.79 Return=6.4 Rem./Return=0.69 Market change =54

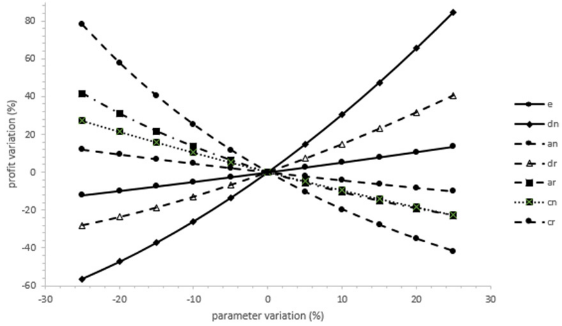

Figure 3: Sensitivity analysis for instance 5.

Figure 3 presents the result of a sensitivity analysis on instance 5. We focus on demand function parameters and unit process costs (c_n and c_r) as we had seen that inventory related had small effect on the profit over all instances. Each parameter is varied one at a time keeping all other parameters constant, from -25% to 25% by step of 5%. The profit is determined with the joint optimisation for each value. We first note that the profit is more sensitive to parameters d_n and a_n (primary market) with opposite direction. The profit increased with respect to e but with smaller amplitude as it concerns less customers. As expected, if the unit process costs increased, the profit decreased. We perform the same analysis on parameter β_n and β_r but no variation in the profit and decision variables is observed. As constraint (2) is not saturated, x_r is constant and the proportion of returned product that are remanufactured simply decreased as β_n or β_r increased. We performed the same analysis in the other instances and the same trends are observed but with different amplitudes.

6 CONCLUSION

In many real situations, remanufacturing offers the opportunity to sell product at lower price when some customers want to pay less for a remanufactured product while the others prefer to buy new one at higher price. We have developed a new model for hybrid system that produces new and remanufactured products for two distinct markets. We have modelled the competition between the two products, with a part of customers that are undecided, in the relationship between prices and demands. We have also considered that the two products are produced on the same production line with EPQ assumptions. The resulting model is a mixed nonlinear problem with linear and nonlinear constraints. The mathematical analysis presented justify an efficient resolution method based on identification of sub problems with good properties. It also make possible to compare decisions processes with different decisions makers (e.g. marketing and inventory) and highlight their importance in the profits of the company. The numerical analysis shows that all instances generated are solved rapidly and infeasibility or negative profit situations are detected. Higher profit can be obtained with joint optimization when inventory holding cost are very high (larger that common assumption of 20% of unit cost over one year) and demand are low. Sensitivity analysis shows the importance of demand function parameters compared to the others. Future research may extend the model to consider additional real case assumptions. The first one would be to consider more segments in the market with respect to different quality levels of returned products. Shortage situations, inventory capacity limitation, multi products and stages are worth developing. Financing parameters are also to be developed for remanufacturing and recovery activities.

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