Comparison of Dimension Reduction Methods for Multivariate Time Series Pattern Recognition

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Abstract: Large volumes of time series data are frequently analyzed using unsupervised algorithms to identify patterns. Multivariate time series's time and space complexity poses challenges in this context. Dimensionality reduction, a common technique in data science, provides a viable solution to improve time and space complexity. Nevertheless, a crucial question arises concerning how the time advantage compares to the information loss. This paper compares dimension reduction methods within unsupervised time series pattern recognition, including rule-based, spectral, probabilistic, and unsupervised learning-based approaches. The comparison involves both synthetic and real-world datasets for a comprehensive evaluation. The findings reveal the potential to accelerate pattern recognition algorithms by 90 %, with only 18 % information loss in the sense of the F1 score.

1 INTRODUCTION

Time series analysis and pattern recognition are essential in many fields such as healthcare (Kam et al., 2010), automotive (Petersen and Sax, 2022), and finance (Chan, 2007). A time series is a collection of data points logged in chronological order. It is characterized by trends, seasonality, and cycles (Dodge, 2008). These characteristics can be used to find patterns in the data and predict what will happen in the future. Time series data offers valuable information on system behavior and state, enabling Time Series Pattern Recognition (TSPR) methods to identify recurring load situations. By identifying patterns in time series data, it is possible to better understand how different factors interact and how they influence the overall system. In addition to the quality of the algorithms, handling big data, especially in case of Multivariate Time Series (MTS), is a challenge. TSPR algorithms in MTS require time-consuming numerical computations, such as similarity measures and optimization algorithms (Yeh et al., 2016; Bascol et al., 2016), which can be costly and time-consuming. These algorithms are iterative and require significant computational resources like CPU time and memory. The time complexity for pattern search algorithms ranges from $O(m^2 \log(m))$ to $O(m^2)$, depending on the length m of a Univariate Time Series (UTS) (Yeh et al., 2016; Yan et al., 2017). For MTS, the time complexity increases to $O(n \log(n) m^2)$ with n being the number of dimensions (Alaee et al., 2020). While resources are available through data centers and services, they are limited and expensive. Besides the computational effort for MTS, many algorithms lack the multi-dimensionality ability to detect patterns between different time series. Dimension reduction (DR) methods are potential solutions for this challenge. Clustering techniques have been successfully applied to DR methods. However, to the best of the author's knowledge, DR has yet to be used in TSPR. This contribution investigates how different DR methods influence the impact on the information loss and the time efficiency for TSPR. This paper gives an overview of different DR approaches and their advantages and disadvantages. These methods are tested on a synthetic dataset and on the real-world *Commercial Vehicle Sensor Dataset* (AB,).

Section 2 defines necessary terms and outlines the state of the art of unsupervised TSPR algorithm and DR methods. In Section 3, the concept of the unsupervised TSPR framework is described. The evaluation of the methods described is presented in Section 4. Section 5 concludes the result of this contribution.

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2 STATE OF THE ART

2.1 Unsupervised Pattern Recognition

Unsupervised TSPR of time series is a knowledge discovery problem. For this problem, the following terms need to be defined:

Definition 1 (Time Series $\mathcal{T}$ (Mörchen, 2006))**.** A d-dimensional time series w.r.t. a series of time points $\mathbb{T} = \{t_1, ..., t_n\}$ of length $n \in \mathbb{N}$ is $\mathcal{T} = \{(t_i, y_i) | y_i = (y_{i,1}, ..., y_{i,d})^T \in \mathbb{R}^d, t_i \in \mathbb{T}, i = 1, \cdots, n\}$. If $d = 1$, $\mathcal{T}$ is called *univariate*, for $d > 1$ it is a multivariate time series.

Definition 2 (Subsequence $t_{i,\mathrm{l}}$ (Noering, 2022))**.** A subsequence $t_{i,l} \in \mathbb{R}^{n \times d+1}$ of a time series $\mathcal{T}$ is a subset of values from $\mathcal{T}$ of length l starting from index i. This subsequence $t_{i,l}$ contains consecutive tuples $\begin{bmatrix} t_i & t_{i+1} & \dots & t_{i+l-1} \end{bmatrix}$.

Definition 3 (Pattern $\mathcal{P}$ (Noering, 2022))**.** A time series pattern P is a group of m (with $m \geq 2$) similar subsequences in a time series $\mathcal{T}$ excluding all trivial matches. A subsequence $t_{i,\mathrm{l}}$, that is included in a pattern P, is called a member M. Formally, $P = [M_1, M_2, \dots, M_m]$ with each M_x being a subsequence with a starting index i and a length l. This includes a distance threshold τ with $dist(M_x, M_y) \leq \tau, \forall x, y \in [1, \dots, m]$.

Definition 4 (Full-Dimensional Pattern $\mathcal{P}_{\text{full}}$)**.** A full-dimensional pattern $\mathcal{P}_{\text{full}}$ of dimension d is a pattern, where the distance of two members M_x and M_y in dimension k is $dist(M_{x,\mathrm{k}}, M_{y,\mathrm{k}}) \leq \tau$, with $\{k \in \mathbb{N} | 1 \leq k \leq d\}$ in $\mathcal{T}$.

Definition 5 (Sub-Dimensional Pattern $\mathcal{P}_{\text{sub}}$)**.** A sub-dimensional pattern $\mathcal{P}_{\text{sub}}$ is a pattern of dimension $b < d$, where d is the dimensionality of the time series.

The goal of TSPR is to find patterns of unknown shape, length, frequency of occurrence, and level of detail in time series (Noering, 2022). Existing methods for identifying those patterns include Dynamic Time Warping (Berndt and Clifford, 1994), discretization (Noering, 2022), or matrix profiles (Yeh et al., 2016). Matrix profiles have found great interest in unsupervised TSPR in recent years and are used as a benchmark in this contribution. Nevertheless, the nature of multivariate time series makes TSPR computational intensive. To counteract the curse of dimensionality, DR methods are proposed in the following.

2.2 Dimensionality Reduction of Multivariate Time Series

The goal of DR methods is to find a low-dimensional data representation that retains the most relevant properties for a given problem. Such methods can be divided into four categories: rule-based, spectral, probabilistic, and unsupervised learning-based methods. Rule-based methods apply rules to reduce dimensions of time series, such as averaging the values of each index over all time series dimensions or unifying them under one symbol based on their discretized symbolic representation (Noering, 2022). Spectral methods (Chaudhuri, 2006; Hotelling, 1933) are based on the idea that higher-dimensional data can be represented in a lower-dimensional domain using a linear or nonlinear transformation. Probabilistic methods (McInnes et al., 2018; van der Maaten and Hinton, 2008; Cattell, 1965) view the data as samples from an unknown probability distribution. A low-dimensional latent random variable can be assumed on which the data depends. Unsupervised learning-based methods learn the internal structure of data when supplied with it. AutoEncoder (AE) (Rumelhart et al., 1986) architectures can create so-called bottlenecks through which the data is routed. Thereby, data is compressed to its essentials, which is equivalent to a DR. Additionally, Long Short-Term Memory (LSTM) (Hochreiter and Schmidhuber, 1997) networks can process sequential data. Dictionary Learning (Mairal et al., 2009) allows sparse data encoding by a linear combination of basis vectors, a so-called Dictionary. For this, an optimization problem must be solved. An overview of all methods considered in this paper can be seen in Table 1.

Table 1: Overview of Dimension Reduction Methods.

Rule-Based	Spectral	Probabilistic	Unsupervised Learning
Unification	PCA	t-SNE	Autoencoder
Averaging	FPCA	UMAP	LSTM Autoencoder
		Factor Analysis	Dictionary Learning

Although the strengths and weaknesses of these methods are well known, there is no research on the impact of DR on TSPR in terms of information loss and computational efficiency. For that purpose, the concept of a comparative analysis is described in the following.

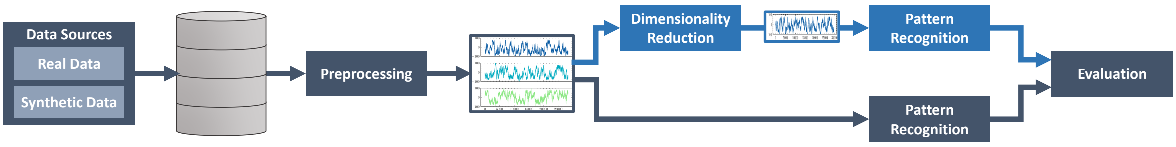


Figure 1: Overview of the concept for evaluating dimension reduction for pattern recognition.

3 CONCEPT

3.1 Process

This paper evaluates ten DR methods for unsupervised TSPR in MTS to investigate the advantages and disadvantages of processing without DR. This is done in an organized manner by predefined concept (see Figure 1). It consists of five steps: data acquisition, preprocessing, optional DR, TSPR, and evaluation.

This paper uses synthetic and real datasets. Further discussions are in Section 3.3 and 3.2. Z-normalization, Piecewise Aggregate Approximation (PAA), and Symbolic Aggregate approXimation (SAX) can be used as optional preprocessing methods. This study aims to accelerate the search for patterns in MTS using DR. However, this can result in a loss of information in data representation (Wang et al., 2016), making it necessary to examine the suitability of reduced data for unsupervised TSPR. Matrix profiles as TSPR algorithm is applied after preprocessing, considering both the dimension reduced (upper path) and original (lower path) data. This paper uses the implementation *stumpy* (Law, 2019), which allows TSPR for UTS and MTS. The application of TSPR to the dimensionally reduced and the original data allows statements to be made about the influence of DR on TSPR.

This study examines the results of dimension-reduced time series compared to non-dimension-reduced ones. Since the synthetically generated data contains much less incorporated pattern sample points compared to the overall number of sample points, is considered to be unbalanced. Thus, F1 score (Sokolova et al., 2006) is used a metric to evaluate the precision and recall of TSPR algorithms. In addition, the gain or loss of $\Delta F1_a$ of the reduced data F1 score $F1_{\mathrm{RPR}}$ in comparison to the original data F1 score $F1_{\mathrm{PR}}$ is calculated by

$$\Delta F1_a = \frac{F1_{\mathrm{RPR}} - F1_{\mathrm{PR}}}{F1_{\mathrm{PR}}}, \tag{1}$$

with $\Delta F1_a \in [-1,1]$, where -1 means 100 % loss and 1 means 100 % gain of the dimension reduced data. The computational efficiency Δt_{a} is analyzed by comparing the runtime of the dimension reduction t_{DR} and the TSPR on the reduced data t_{RPR} with the runtime of the TSPR the original data t_{PR}:

$$\Delta t_{\mathrm{a}} = \frac{t_{\mathrm{PR}} - (t_{\mathrm{DR}} + t_{\mathrm{RPR}})}{t_{\mathrm{PR}}}, \tag{2}$$

with $\Delta t_{\mathrm{a}} \in [-1,1]$, where -1 means 100 % computational efficiency loss and 1 means 100 % computational efficiency gain of the dimension reduced data.

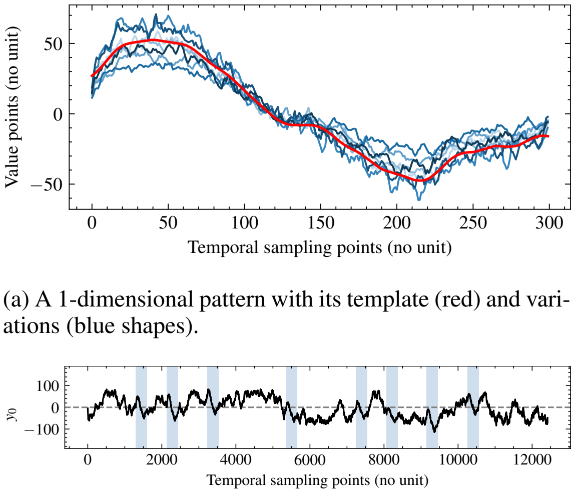
3.2 Discussion on Synthetic and Real-World Data

One of the main benefits of synthetic datasets is that they are entirely labeled by nature. That is why they require less time and effort to prepare for analysis and experimentation. They are helpful in study fields with privacy problems, since they also eliminate privacy and confidentiality difficulties. Although synthetic datasets can be tuned to produce flawless results, they can also accidentally skew data or overfit models. Despite their benefits, synthetic datasets may fail to capture real-world processes accurately. Because of their inability to capture intricacies in real-world events when generalizing conclusions based on synthetic data, undertaking extensive validation in real-world contexts is still necessary.

Real-world datasets provide an excellent opportunity for investigating characteristics that synthetic datasets may miss. Their variety and diversity allow for investigating features, discovering unexpected patterns, and improving the validity and application of study findings. Real-world datasets have advantages but drawbacks, such as the time and money required to gather and manage big, diverse, and representative datasets. Data collection and labeling take time and effort to assure quality and dependability. Concerns about privacy and confidentiality occur when dealing with sensitive material, limiting access, and placing legal and ethical constraints on research operations.

3.3 Synthetic Data Generation

To generate synthetic time series, a random walk (Pearson, 1905) is utilized, as is also recommended for pattern recognition in the literature (Noering, 2022). Therefore, a broader examination for



(a) A 1-dimensional pattern with its template (red) and variations (blue shapes).

(b) A synthetically created 1-dimensional time series with a single pattern incorporated in several occurrences (blue background).

Figure 2: Synthetic data generated via the random walk approach.

use cases other than driving TSPR may be conducted. The test patterns may include all randomly produced forms. Nonetheless, the synthetic data must be verified as a suitable foundation for a valid comparison of DR. As a result, restrictions are put on the synthetic time series data to meet the given objectives, which are defined as follows in the current study: The synthetic MTS can either contain full-dimensional or sub-dimensional patterns without time warping or shifts in pattern variances. Pattern class instances should have different noise levels and amplitude values, as sensor signals are noisy. Time series data should be within a predefined range, but seasonality and wear and tear may be present.

Using the random walk approach, the n-dimensional random walk generates random sequences of arbitrary length. It creates pattern templates and variations to assemble multiple patterns in a time series with random connections. Five distinct trends are created: strong-up, up, stay, down, and strong-down. When the trend time is exceeded, a new trend type is selected randomly, but two super trends cannot follow each other. The resulting curve is smoothed using linear convolution and scaled or squashed according to predefined values.

Variations are made by adding white Gaussian noise and amplitude scaling individually for each dimension, resulting in a variable signal-to-noise ratio (see Figure 2a). The desired order of pattern occurrences is randomly created, and multi-dimensional patterns appear within the time series. A symmetrical random walk with random length and white Gaussian noise connects the patterns. The link and patterns are shifted along the value axis, preventing value jumps within the series (see Figure 2b). The resulting series is smoothed and covered with white Gaussian noise, and the same procedure is applied for all dimensions. The method generates 100 three-dimensional time series with 10k data points with obvious patterns by adjusting scaling or noise parameters. Amplitude differences or noise levels can control the obviousness. To ensure application relevance, the randomly generated data must be correlated to some degree, allowing for multivariate TSPR and DR. Real-world sensor data is correlated to some degree, allowing for multivariate TSPR and DR. For example, the angle of an accelerator pedal and measurement data of an Inertial Measurement Unit (IMU). Synthetic data can embed simple patterns, which search algorithms can easily find due to their conciseness. However, this is unrealistic.

Three datasets with different cross-instance distances CR_{dist} and pattern amplitudes $\texttt{MA}$ were created to test the effectiveness of DR and TSPR in different difficulty levels (see Table 2).

$$\texttt{CR}_{\text{dist}} = \frac{1}{N}\sum_{n=1}^{|P|}\sum_{i,j=1}^{m}|Z(M_i)-Z(M_j)|, \quad i \neq j, \quad (3a)$$

$$\texttt{MA} = \frac{1}{N}\sum_{n=1}^{|P|}\sum_{1}^{m}\max(Z(M_i)), \quad (3b)$$

where Z is the Z-Score normalization, $|P|$ the number of patterns, m the number of members in each pattern, and N the total number of members over all patterns.

Table 2: Overview of the cross-instance distances and pattern amplitudes of three generated benchmark datasets.

Case	$\texttt{CR}_{\textbf{dist}}$	$\texttt{MA}$
I	2.95	0.94
II	3.57	0.86
III	4.32	0.75

4 EVALUATION

The evaluation aims to examine ten used DR methods for unsupervised TSPR in MTS. Therefore, the benefits and drawbacks of DR for TSPR versus non-reduction processing are investigated. A synthetic dataset (see Section 4.1) and a real-dataset (see Section 4.2) are used to compare the DR methods in TSPR.

4.1 Synthetic Data

4.1.1 Selection of Suitable Methods for Dimension Reduction

Three-dimensional time series are synthetically generated as a full-dimensional TSPR problem. As preprocessing, only z-normalization has been applied. These dimensions were reduced to one dimension to evaluate the effectiveness of different DR methods. The hypothesis that larger amplitude and smaller distance make patterns easier to find is confirmed by pattern search results. For cases I and II, $F1$ values are good, while for case III, the values are worse. This correlation is also observed for reduced time series, indicating that pattern difficulty is not a significant factor in evaluating reduction methods (see Figure 3).

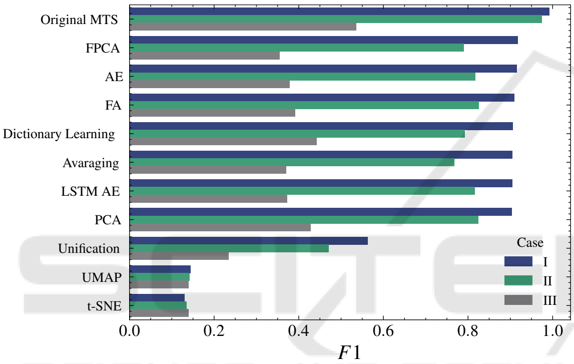


Figure 3: Results of pattern recognition with and without prior dimensionality reduction on different pattern properties (Table 2).

It can be seen from the results that Factor Analysis (FA), Functional PCA (FPCA), and AE are the most promising DR methods due to performing the best throughout the different cases.

4.1.2 Runtime Analysis of Dimension Reduction

DR in TSPR aims to save time by analyzing dimensionally reduced time series compared to original data, speeding up the analytical process. The evaluation of dimension reduction methods involves comparing their runtime with (2). Time series are synthetically generated as a full-dimensional TSPR problem with dimensions from 2 to 14 reduced to 1, 2, and 3 dimensions. Each reduction process is tested ten times to compensate for measurement variations. For this analysis, the top three reduction methods regarding their F1 score, as mentioned in Figure 3, are used.

The loss and gain of computational efficiency Δt_a of dimension reduction by Principal Component Analysis (PCA), AE, and FA is shown in Figure 4.

In this scenario, the training of the AE is done in three epochs with a batch size of six, and there is no need to retrain after each sample.

The inference times for individual methods are insignificant compared to the time required for TSPR. A benefit in speed is nearly always present compared to multivariate TSPR on all initial dimensions, regardless of the combination of initial or reduced dimensions. The lower the number of reduced dimensions, the higher the benefit. The percentage benefit also increases with the number of original dimensions. However, AE training can have a negative impact when the distance between the number of original dimensions and the number of reduced dimensions is less than three.

In conclusion, a reduction generally leads to a speed benefit. The higher the distance between the original MTS dimensions compared to the reduced time series, the higher the temporal benefit. From Figure 4, it can be seen that the decision on how many dimensions the original MTS should be reduced has an impact on the overall performance of the TSPR algorithm. Therefore, it is recommended to investigate the optimal number of dimensions before analysis.

4.1.3 Dealing with Sub-Dimensional Patterns in Reduced Dimension

To find the optimal number of reduced dimension $n_{\mathrm{y,opt}}$, the dimension reduction method FA is tested by reducing a 10-dimensional MTS to all possible dimensions $n_{\mathrm{y}} \in [1, 2, \ldots, 9]$. The study aims to find the optimal number of components n_y, which is the number of dimensions of the reduced time series $\mathcal{T}$, to achieve the best possible results in TSPR. However, an additional problem arises during TSPR: the patterns in the reduced times series $\mathcal{T}$ do not necessarily have to be a full-dimensional TSPR problem. Specifying the minimum number of dimensions in which a sub-dimensional pattern should occur in a multi-dimensional pattern search is necessary. The test is performed in three cases:

1. Sub-dimensional TSPR problem with $|k| \geq 1$.
2. Sub-dimensional TSPR problem with $|k| \geq \left\lfloor \frac{n_y}{2} \right\rfloor$.
3. Full-dimensional TSPR problem with $|k| = n_y$.

The test results are displayed as heatmap matrices, with each heatmap showing one of the three test cases (see Figure 5).

The patterns' dimensions n_m are horizontally displayed, and the number of reduced time series' dimensions n_y are vertically displayed. The $F1$ score

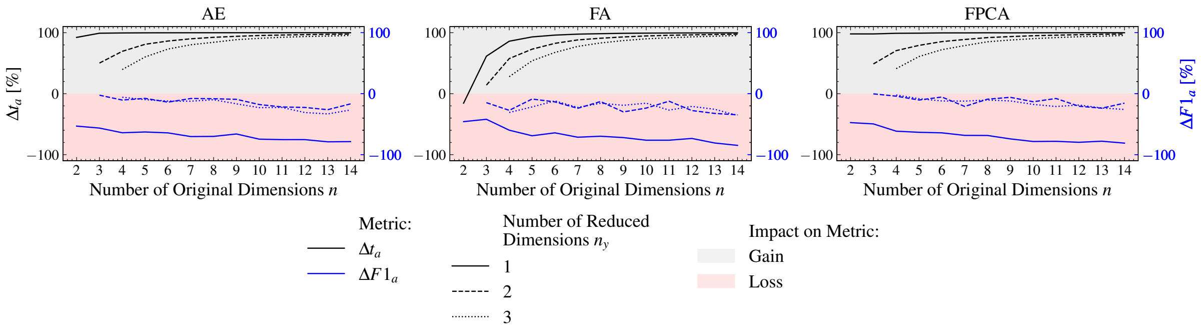


Figure 4: Loss and Gain of $\Delta F1_a$ and computation time Δt_a of different dimension reduction methods in pattern recognition.

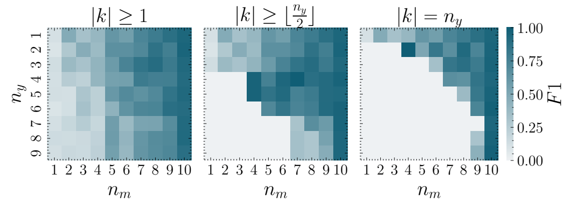
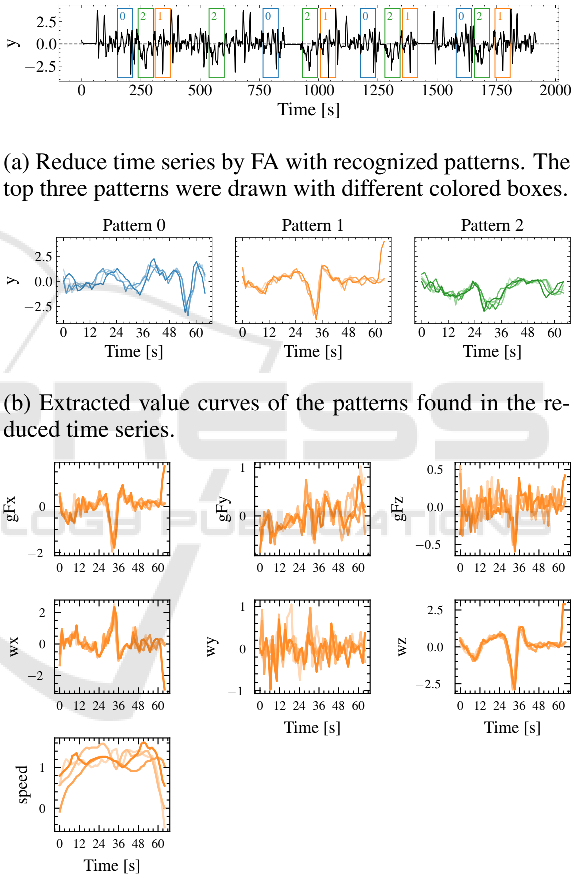


Figure 5: Heatmaps displaying the $F1$ score of pattern recognition after reduction with FA regarding different numbers of dimensions as well as dimensions of patterns.

values are indicated in color, with dark values representing high scores and light values representing low scores. The smaller n_m, the more difficult it is to find the pattern in the reduced time series. This is because the fewer dimensions contribute to a pattern, and the less FA can identify and reproduce it. The relationship between the number of dimensions in a reduced time series and the size of the pattern is influenced by the number of dimensions. The smaller the pattern dimensions in the original MTS and the larger the number of dimensions reduced, the less well the pattern is found. The best possible combination of n_m, n_y, and $\min n_{m,r}$ cannot be guaranteed and must be determined at the beginning of each new analysis.

4.2 Real-World Data

The *Commercial Vehicles Sensor Dataset* (AB,) is used to validate TSPR on a reduced dataset. The dataset classifies the activities of two dumpers during regular use. Dumpers have various states, such as idle, driving, loading, dumping, and engine-off, labeled in their MTS data. These states are less detailed and suitable for classifying operating modes, but recurring events within these modes require unsupervised TSPR. The data is z-normalized and analyzed using a 100 Hz sampling rate for MTS acquisition. Data reduction is performed using a 128-sequence SAX dictionary with 256 symbols converted from letters to floating point numbers, as both matrix profiles and DR methods work with numeric data. Time frames during idle or engine-off mode are removed, as they lack useful patterns and are irrelevant to the analysis. The dataset has seven dimensions, reduced using FA for synthetic data and MTS reduced to one dimension. Pattern classes are represented by the same box color, with value curves of pattern instances represented by different shades (see Figure 6a). Three patterns with the smallest z-normalized Euclidean dis-



(a) Reduce time series by FA with recognized patterns. The top three patterns were drawn with different colored boxes.

(b) Extracted value curves of the patterns found in the reduced time series.

(c) Sensor values at the location of the found pattern 2.

Figure 6: Results of real-world data pattern recognition.

tance are marked with colored boxes in the reduced time series. Value curves are extracted and displayed in separate diagrams for all classes (see Figure 6b).

The question remains whether these patterns in the reduced time series are also present in the original MTS. Therefore, as an example, the original segments of the identified pattern 2 are investigated (see Figure 6c). Positive and negative correlations are evident between sensor values, mainly between gyroscope data in x and z direction and accelerometer data in x and z direction. Patterns are found between variables of gyroscope data in x direction, accelerometer data in x, and z direction, indicating a link between correlation and patterns. However, some sensors have little to no correlation to other sensors, such as speed and accelerometer data in y direction. A correlation between sensor values is necessary for DR, especially for PCA methods, which use correlation as a primary factor. Patterns 0 and 2 show similar results, indicating that patterns found for reduced time series also occur in the MTS. However, higher variances or noise among pattern instances occur in a subset of the dimensions, making them sub-dimensional patterns. Utilizing upstream DR for TSPR results in a time saving of 89 % compared to multivariate cases.

5 CONCLUSION & OUTLOOK

This paper compares various methods for dimension reduction in the context of unsupervised pattern recognition. As a result, Autoencoder, Functional Principle Component Analysis (FPCA), and Factor Analysis (FA) produce dimensionally reduced data with the least loss of $\Delta F1_a$ using a synthetic dataset. Furthermore, dimension reduction with FA and FPCA yields a runtime advantage of up to 90 % over a non-reduced pattern search while losing only 18 % of $\Delta F1_a$. This result is validated with the real-world dataset *Commercial Vehicles Sensor Dataset*. However, the speed benefits must be weighed against potential loss in accuracy and tested in advance, especially in the case of sub-dimensional pattern recognition. Future studies can explore alternative pattern algorithms, increase dataset diversity, investigate sub-dimensional pattern recognition and variable pattern lengths, and consider the temporal offset of patterns.

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