Balancing Resources and Demand: A Bi-Objective Mixed-Integer Programming Approach of Healthcare Districts in Chile

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Abstract: In the search for equitable and efficient health service delivery, geographical partition into operational districts is a fundamental factor. This research delves into the intricate challenge of combinatorial optimization of healthcare districts, with an application to the Metropolitan region of Santiago, Chile, where growing population pressures exacerbate concerns about the distribution of healthcare resources. By emphasizing continuity from primary to secondary healthcare levels, we underline the importance of a good district plan, considering key parameters such as population homogeneity, compactness, and alignment between capacity and demand. By applying a mixed integer linear programming model with a bi-objective function, our findings indicate substantial scope for improving resource allocation, potentially cutting overages by up to 38.77% at the primary healthcare level and up to 15% at the secondary healthcare level.

1 INTRODUCTION

One of the foundations of healthcare service delivery is the partitioning of geographical regions into operational units or districts. This partitioning aims to optimize resource utilization and ensure equitable access to healthcare services across diverse demographic and socio-economic strata. With roots in various disciplines, including city planning, administrative jurisdiction allocation, and, notably, healthcare management, the concept of districting—also referred to as re-districting or territory design—emerges as a challenging combinatorial optimization problem (Validi and Buchanan, 2022).

Mathematical modeling of the healthcare districting problem entails formulating an objective function that drives towards creating districts that uphold geographic cohesion while synchronizing several key performance indicators, including but not limited to resource allocation, population balancing, and demand flux. Particularly in settings like Latin America, the diversity and disparity inherent in the population and regional geography fuel the complexity of this optimization problem, resulting in a challenging environment for applying districting models and strategies (Lin and Kao, 2008).

In this work, we focus on primary and secondary healthcare planning, where the implications of districting are prominently displayed. The pathway through healthcare often commences with a primary care encounter, typically with a General Practitioner (GP), and is subsequently integrated with secondary care depending on the patient's needs. A meticulously crafted districting plan becomes critical to orchestrating a hierarchical network that seamlessly guides patients along this journey, from primary care centers to advanced care centers, while balancing the load across the entire healthcare infrastructure (Ríos-Mercado and López-Pérez, 2013).

In-depth planning of healthcare systems involves a cascade of intertwined decisions, spanning the location of healthcare points, demarcation of service areas, capacity configuration, resource allocation, and staff scheduling, to name a few. The design of healthcare service regions, or in other words, the districting of healthcare services, surfaces as a cardinal dimension in this multifaceted planning landscape. Effective districting can pave the way towards minimizing costs, enhancing capacity utilization, elevating patient

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satisfaction, and, importantly, embedding equity into the accessibility of healthcare services across the society (Cortés et al., 2018; Ríos-Mercado, 2020). In general, several criteria are considered in the literature for planning of healthcare systems, including *Population Homogeneity*, which seeks to minimize deviations in population size, medical expenditure, or the proportion of elderly individuals in each district. *Population Homogeneity* or *balancing* is one of the most common objectives in healthcare districts as well as *Compactness*, which aims to minimize patients' travel time inside a district and *Contiguity* that is defined as the ability to move between territorial units within a district without leaving the district. Other criteria include, for example, *Capacity-Demand Match*, which imposes penalties for unmet demand or tries to minimize excess of demand over available capacity; or *Variety of Medical Procedures*, where each district must ensure a number of basic procedures.

This research spotlight focuses on unraveling the healthcare districting problem, specifically in the context of the City of Santiago, Chile. In recent years, this city has experienced a surge in population density, which has concurrently escalated the burden on healthcare centers. This increased load is notably evident in the lengthening waiting lists for surgical and medical consultations (del Gobierno de Chile, 2023). One hypothesis suggests that a suboptimal distribution of resources across healthcare centers may be a latent root cause. By re-evaluating and potentially re-engineering the allocation of communes within the City of Santiago relative to the current distribution, a more balanced and potentially more efficient distribution of resources, aligned with the demands of the growing population, might be achieved. Thus, addressing the Healthcare Districting problem emerges as a potential strategy to design districts that harmonize population and resource distributions, aiming for a seamless and equitable healthcare experience for all.

The article is organized as follows. In Section 2, a review of the relevant literature for the problem of Primary and Secondary Healthcare Districting is presented. The problem description and a mathematical formulation to solve it is introduced in Section 3. Section 4.1 showcases the case study along with its respective computational results and discussion. Conclusions and further research work are discussed in Section 5.

2 RELATED WORKS

Different approaches and techniques have been applied to the healthcare districting problem to meet desirable attributes of the geographical partition, such as population balance and resource distribution. Two pioneer works in the area are the one of Ghiggi et al. (1975) and Pezzella et al. (1981). The former laid down several key assumptions for districting the healthcare system, such as the idea that regions are composed of indissoluble communities with centralized populations, geographical connectivity of the districts, and balance of district populations. Their overarching goal was to devise a hybrid method for districting, ensuring each district's self-sufficiency concerning health systems while satisfying planners and citizens alike. Pezzella et al. (1981) presents a two-step methodology for partitioning a given geographical area, ensuring an optimal allocation of the available healthcare services. The first step consists of determining an optimal partition by taking into account the demand and capacity of each territorial unit. Then, in the second step, the first partition is aggregated into new partitions by considering the lower and upper bounds for the population in each district.

In addition, a crucial aspect that multiple studies have highlighted is the consideration of multiple objective functions. Datta et al. (2013) emphasizes geographical compactness, alignment with existing local authorities, and size and population homogeneity. On a similar note, Steiner et al. (2015) looked at optimizing population homogeneity, medical procedure variety per district, and intradistrict distances in a study in Brazil. Meanwhile, Yanık et al. (2020) introduced the innovative concept of gradual assignment. Here, demands could be split among neighboring districts, enriching their multi-period, multi-criteria model.

In terms of approaches employed to solve the healthcare districting problem, both exact and heuristic methods have been observed. However, recognizing the NP-Hard nature and computational intractability of the problem, several researchers have turned to heuristic and metaheuristic algorithms. Notably, Gray Wolf Optimizer and Antlion Optimizer algorithms were implemented in Farughi et al. (2019). These approaches find resonance in the works of Farughi et al. (2020) that also used the Gray Wolf Optimizer heuristic and compared its performance with an improved genetic algorithm. Additionally, genetic algorithms to solve single- and multi-objective healthcare districting problems have also been investigated in Datta et al. (2013) and Steiner et al. (2015).

The complexity of the problem and the introduction of newer dimensions like stochastic demand have led to innovative solutions. A robust optimization approach is investigated in Darmian et al. (2022), whereas Fan and Xie (2022) adopted a two-stage distributionally robust optimization approach considering the unpredictability of demand, especially relevant in the backdrop of the pandemic and vaccination drives.

The application of these methodologies and approaches is best understood through real-world case studies. Van Minh et al. (2014) assessed the capabilities of a primary healthcare system in a district, encompassing both physical and human resources, in a rural city in central Vietnam. Steiner et al. (2015) shows the application of a multi-objective optimization approach for dealing with a real-world spatial problem of aggregating the municipalities of the Parana State in Brazil into some microregions. Farughi et al. (2019) and Farughi et al. (2020) demonstrated their model in the South Khorasan Province of Iran, offering insights into practical implications. Yanık et al. (2020) applied its multi-criteria and multi-period formulation with the concept of gradual assignment to Istanbul, Turkey. Lastly, among the recent studies, Darmian et al. (2022) not only introduced a model considering contiguity and population balance but also factored in demand uncertainty, showcasing its practicality for a real-world case study of Iran's healthcare system.

This work is closely related to previous research on districting, as it considers several relevant features of the healthcare districting problem simultaneously. However, most studies focus on maximizing population homogeneity and/or compactness, treating other features such as capacity-demand match as constraints or service levels to be satisfied. In this work, we aim to minimize the largest excess of population across different healthcare levels over the available resources in the forming districts, while considering population homogeneity, compactness, and contiguity as constraints that must be met. Table 1 provides a summary of pertinent information from selected studies, including this work.

3 PROBLEM DESCRIPTION AND MATHEMATICAL FORMULATION

In this section, we provide a formal definition of the problem and mathematical formulation that consider several attributes discussed in the literature, such as population balance, compactness, contiguity, and capacity-demand match.

3.1 Problem Definition and Notation

We first formally describe the healthcare districting problem faced in this article.

The districting problems involve grouping a set of territorial units to form a district. This can be done for various purposes, such as administrative or political reasons. Territorial units represent individual parts of the territory that structure spatial organization. The main objective is to establish the assignment of territorial units to districts. These units have a certain allocation of resources. The objective of the districting problem in this article is to establish a territorial partition, which minimizes the maximum excess of demand over capacity among all districts for primary and secondary healthcare services individually.

To achieve this objective, we formulate this problem as a mixed-integer program based on the spanning tree formulation. This formulation allows us to impose the connectivity of territorial units into a district explicitly. Note that a spanning tree of an undirected graph is a subgraph in which any two vertices are connected by exactly one path and includes all of the vertices of the graph. Let $\mathcal{V}$ be the set of vertices, which represent the territorial units, defined as $\mathcal{V} = \{1,2,3,\ldots,|V|\}$. Let $\mathcal{E}$ be the set of edges, implying that if there exists an edge between vertex i and j, then $\{i,j\} \in E$.

The model requires an auxiliary graph $G(\mathcal{V}',\mathcal{E}')$, where the set of nodes corresponds to the territorial units, with an additional node $\{0\}$, given by $\mathcal{V}' = \{0\} \cup \mathcal{V}$. The set of edges represents border relationships, and the 0 node is adjacent to each territorial unit. Thus, $\mathcal{E}' = \mathcal{E} \cup \{\{0,i\} \mid \forall i \in \mathcal{V}\}$. Moreover, we define a set A of directed edges derived from $\mathcal{E}$, along with the directed edges emerging from node 0. This is given by $\mathcal{A} = \{(i,j),(j,i) \mid \{i,j\} \in \mathcal{E}\} \cup \{(0,i) \mid i \in \mathcal{V}\}$. The population for each territorial unit is denoted as p_j for all $j \in \mathcal{V}$. Similarly, the resources for each territorial unit j are denoted by r_j^p (resp. r_j^t) and represent the number of people served by the primary (resp. secondary) medical supplies within that unit. Meanwhile, the distance between a pair of territorial units (i,j) is represented by d_{ij}.

Additionally, we restrict the number of territorial units in each district to fall within an interval (s_1,s_2), preventing too many or too few territorial units in each district. In addition, a maximum distance L_{max} is imposed between any two territorial units that belong to the same district. This condition, know as compactness, seeks to avoid generating districts with a territory too large. Thus, the model aims to find K disjoint territory subsets that minimize the largest excess of population over the resources capacity among the K

Table 1: Selected publications on the healthcare districting problem in primary and secondary healthcare services.

		Attributes						
	Article	Population Homogeneity	Compactness	Contiguity	Accessibility	Capacity-demand Match	Variety of Medical Procedures Offered	Case study
1	(Ghiggi et al., 1975)	x		x		x		Province of Imperia Italy
2	(Pezzella et al., 1981)	x	x	x		x		Province of Cosenza Italy
3	(Datta et al., 2013)	x	x	x				East England
4	(Steiner et al., 2015)	x		x			x	Parana State, Brazil
5	(Farughi et al., 2019)	x						South Khorasan Province, Iran
6	(Farughi et al., 2020)	x	x	x		x		South Khorasan Province, Iran
7	(Yanık et al., 2020)	x	x	x	x	x		Istanbul, Turkey
8	(Fan and Xie, 2022)	x		x				-
9	(Darmian et al., 2022)	x	x	x	x			Iran
10	This work	x	x	x		x		Metropolitan Region of Santiago, Chile

districts.

It is worth noting that, within the general conditions of the healthcare districting problem for partitioning the geographical area, each district $d \in \mathcal{K} = \{1,\ldots,K\}$ forms a subgraph G_d. Together, they should cover the entire set $\mathcal{V}$. These conditions are expressed as follows:

$$\mathcal{V}_d \neq \emptyset \qquad d \in \mathcal{K} \tag{1}$$

$$\mathcal{V}_d \cap \mathcal{V}_{d'} \equiv \emptyset \qquad d,d' \in \mathcal{K}, d \not\equiv d' \tag{2}$$

$$\mathcal{V}_1 \cup \mathcal{V}_2 \cup \ldots \cup \mathcal{V}_K \equiv \mathcal{V} \tag{3}$$

3.2 Spanning Tree-Based MIP Model

In this section, we present the Mixed Integer Programming (MIP) formulation of the healthcare districting problem.

Firstly, we define the variables involved in the model, described as follows:

- Let $x_{ij} \in \{0,1\}$ be a binary variable where $x_{ij} = 1$ if edge $\{i,j\} \in \mathcal{E}'$ is part of the spanning tree of G that determines the solution, and $x_{ij} = 0$ otherwise.
- Let $y_{ijk} \in \{0,1\}$ be a binary vector such that $y_{ijk} = 1$ if arc $(i,j) \in \mathcal{A}$ is used to define the connectivity between the node 0 and territory k, where $k \in \mathcal{V}$, and $y_{ijk} = 0$ otherwise.
- Let η^p and η^t be the continuous variables that quantify the maximum excess of the population primary and secondary service capacity, respectively, over all districts.

The mathematical formulation is presented as follows:

$$\min \quad \beta\eta^p + (1-\beta)\eta^t \tag{4}$$

subject to:

$$\sum_{i\in\mathcal{V}} x_{0i} = K \tag{5}$$

$$\sum_{(i,j)\in\mathcal{E}'} x_{ij} = |\mathcal{V}'| - 1 \tag{6}$$

$$\sum_{(0,i)\in A} y_{0ik} = 1 \qquad \forall k \in \mathcal{V} \tag{7}$$

$$\sum_{(i,j)\in A} y_{ijk} - \sum_{(j,i)\in A} y_{jik} = 0 \qquad \forall j,k \in \mathcal{V} : i \neq j \tag{8}$$

$$\sum_{(j,k)\in A} y_{jkk} = 1 \qquad \forall k \in \mathcal{V} \tag{9}$$

$$x_{ij} \geq y_{ijk} + y_{jik} \qquad \forall (i,j) \in \mathcal{E}', \forall k \in \mathcal{V} \tag{10}$$

$$\sum_{k\in\mathcal{V}} (p_k - r_k^p) y_{0jk} \leq \eta^p \qquad \forall j \in \mathcal{V} \tag{11}$$

$$\sum_{k\in\mathcal{V}} (p_k - r_k^t) y_{0jk} \leq \eta^t \qquad \forall j \in \mathcal{V} \tag{12}$$

$$\bar{P}x_{0j} - \sum_{k\in\mathcal{V}} p_k y_{0jk} \leq \alpha\bar{P} \qquad \forall j \in \mathcal{V} \tag{13}$$

$$-\bar{P}x_{0j} + \sum_{k\in\mathcal{V}} p_k y_{0jk} \leq \alpha\bar{P} \qquad \forall j \in \mathcal{V} \tag{14}$$

$$s_1 x_{0j} \leq \sum_{k\in\mathcal{V}} y_{0jk} \leq s_2 x_{0j} \qquad \forall j \in \mathcal{V} \tag{15}$$

$$(y_{0jk} + y_{0ji})d_{ki} \leq L_{max} \qquad \forall j,i,k \in \mathcal{V} : i \neq k \tag{16}$$

$$x_{ij} \in \{0,1\} \qquad \forall \{i,j\} \in \mathcal{E}' \tag{17}$$

$$y_{ijk} \in \{0,1\} \qquad \forall (i,j) \in A, \forall\, k \in \mathcal{V} \tag{18}$$

The objective function (4) aims to minimize the maximum gap between resources and population among all districts. Note that the parameters β allow us to adjust the objective function to focus on minimizing the excess at the primary healthcare level (β = 1), at secondary healthcare level (β = 0) or at both levels at the same time (β ∈]1,0[). Constraint (5) establishes the number of districts to be formed. It is done by limiting the number of edges that can be connected to node 0. Constraints (6) ensure that the number of edges active in a solution defines a spanning tree in the auxiliary graph. Constraints (7)-(10) entails that a path must exist between each node $k \in \mathcal{V}$ and the node 0. Constraints (11)-(12) quantify the excess of the population over the available resources in each district for the primary and secondary healthcare services. Constraints (13)-(14) entail that the population assigned to each district can only deviate an α value from the mean population. Constraints (15) ensure that each district has a number of territorial units between the discrete interval (s_1, s_2). Constraints (16) impose that the distance between any two territorial units belonging to the same district must be at most L_{max}. Finally, the domain of the decision variables is represented in Constraints (17)-(18).

Figure 1 illustrates the method for deriving a solution to the healthcare districting problem via the spanning tree approach. Observing the figure from left to right, the initial step involves converting a given instance of the problem into a preliminary graph. This graph represents the territorial units as nodes and their adjacency as edges; that is, an edge is drawn between any two neighboring territorial units to signify their connection. Subsequently, in the central figure, an auxiliary graph is constructed by linking each node to an additional, artificial node. The constraints delineated by (7) to (10) are then applied to identify a path from each node $k \in \mathcal{V}$ to the artificial node 0. This step ensures that each territorial unit is allocated to a district. Moreover, the number of active edges connected to node 0 is restricted by the predetermined number of districts to be established. Consequently, a node k that does not serve as a district center must be connected to node 0 via a path that traverses through a sequence of neighboring nodes, which are also interconnected. This results in a cohesive cluster of nodes, thereby fulfilling the contiguity requirement for the districts.

In the next section, we apply the model (4)-(18) to the Metropolitan region of Santiago of Chile and seek to analyze how well the current geographical partition regarding the present population and available resources at each territorial unit.

4 COMPUTATIONAL RESULTS

In this section, we assess the solutions provided by the formulation (4)-(18) at solving a real instance based on the Metropolitan region of Santiago, Chile.

4.1 Case Study

The Metropolitan region of Santiago, Chile, as per the census conducted in 2017, has a total population of 7,112,808 residents spanning an area of 15,403 km^2. This equates to a density of 461.7 inhabitants per square kilometer. Demographically, the region is divided into 52 municipalities, which are grouped into six provinces: Chacabuco, Cordillera, Maipo, Melipilla, Santiago, and Talagante. However, when it comes to health administration, the structure is different. As stipulated by the Ministry of Health, the region is segmented into six distinct healthcare services, which are: North Metropolitan Health Service (North); Western Metropolitan Health Service (West);Central metropolitan health service (Central); Eastern Metropolitan Health Service (East); South Metropolitan Health Service (South); South-East Metropolitan Health Service (South-East). Various types of healthcare services are located within these health services. Consequently, different levels are established, leading to the following organizational structure for the establishments:

- **Primary Healthcare Centers:** They are characterized by being the first point of contact with the patient. Thus, they aim to provide high-quality initial outpatient care to identify the ailment. The facilities that offer these services include:
- **Secondary Healthcare Centers:** This pertains to emergency hospital care and inpatient hospitalization, which includes complex surgical procedures. Hospitals responsible for providing this service have high, medium, and low levels of complexity.

Based on the definition above, this research focuses on proposing new districts that take into account the capacities of both primary and secondary healthcare levels. The primary level, which includes Family Healthcare Centers (abbreviated as CESFAM in

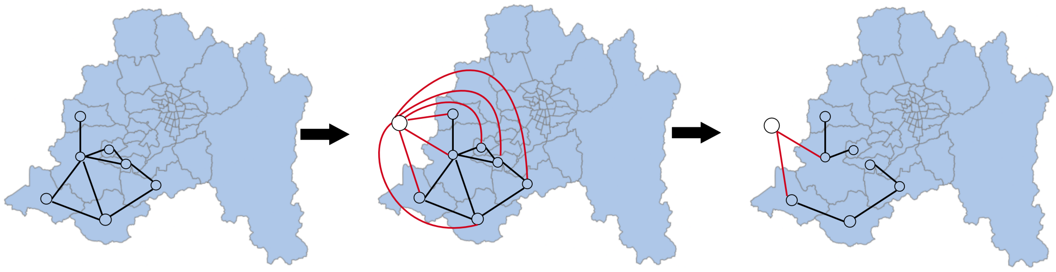

Figure 1: A solution of the healthcare districting problem represented as a spanning tree.

Spanish), plays a vital role in alleviating congestion in secondary service levels, such as hospitals.

We gathered the information for this case study from sources within the Ministry of Health and nationally certified surveys, such as the Census conducted in 2017 to determine the population in each municipality. The Metropolitan Region of Santiago currently has 22 secondary healthcare centers and 171 primary healthcare centers. The Ministry of Health mandates that each primary (resp., secondary) care center should be assigned at most 30,000 (resp., 100,000) people, representing its capacity to provide healthcare services. However, this number is only an approximation of the actual capacity of each center and does not consider the heterogeneity of the centers and their efficiency in providing services. To address this issue, we assume that the capacity of each center follows a discrete uniform distribution $U((1-\varepsilon)\bar{d},(1+\varepsilon)\bar{d})$, where $\bar{d}$ represents the assigned capacity and $\varepsilon = 0.1$ (resp., $\varepsilon = 0.2$) for primary (resp., secondary) care centers. The lower and upper bounds for the number of territorial units in each district, i.e., (s_1, s_2), are set to 4 and 15, respectively.

We select each parameter in the formulation by considering the current geographical partition and its characteristics. This ensures that the current solution is indeed a feasible solution to formulation (4)-(18). This consideration is crucial as the primary aim of this research is to ascertain whether, under the prevailing conditions and regulations of the Metropolitan region's healthcare system, there exists an improved geographical partition that can diminish the population excess over the current system's capacity. By doing so, this could partially alleviate the stress on the existing system and reduce both the waiting list and wait times.

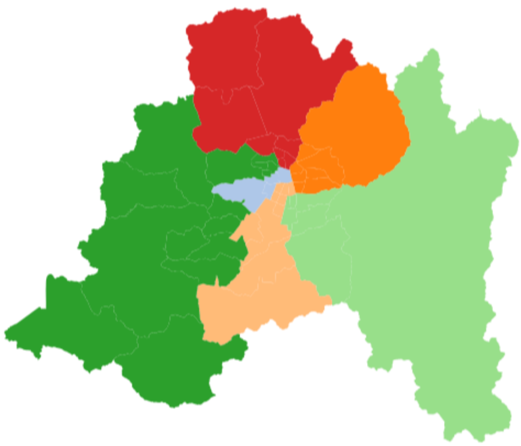

Figure 2: Current districts of the health care services in the Metropolitan region of Santiago.

Figure 2 displays the current districts stipulated by the Ministry of Health, and Table 2 shows the composition of population and resources of each district as well as their excess of population over their available capacity in the primary and secondary level. The MAPD line stands for Median Absolute Percentage Deviation, which is calculated as $(\sum_i^n |x_i - \bar{x}|/\bar{x})/n$, where x_i represents the ith data and $\bar{x}$ denotes the average value of the n data set.

Table 2: Current composition of healthcare services in the Metropolitan region of Santiago.

District	Population	Secondary Capacity	Primary Capacity	Secondary Excess	Primary Excess
Central	1,153,995	206,227	550,238	947,768	603,757
East	1,216,177	303,286	695,714	912,891	520,463
South	1,151,126	519,389	1,051,794	631,737	99,332
West	1,202,137	590,346	1,117,597	611,791	84,540
South East	1,356,538	363,570	974,520	992,968	382,018
North	961,721	224,287	766,596	737,434	195,125
MAPD	18.05%	39.03%	35.97%	23.23%	92.15%

4.2 Experimental Setup

The mixed-integer linear programming formulation (4)-(18) was implemented in Python 3.11 and solved using GUROBI 10.0.1 with the default settings. All tests were run on the computing infrastructure of the Universidad de Santiago de Chile, which consists of a Lenovo ThinkStation machine with 12th generation Intel Core i7-12700. We set the machine to use four 3.60GHz cores and 31GB RAM to solve each instance. We impose a time limit of 43,200 seconds (12 hours) to solve the same instance with three different values of the parameter $\beta \in \{0, 0.5, 1\}$.

4.3 Results

Tables 3-5 and Figure 4-6 display the results obtained for each value of $\beta \in \{0.0, 0.5, 1.0\}$. Recall that β determines which healthcare services are a priority to minimize the overall population surplus concerning its medical offer. When $\beta = 0.0$, the emphasis is solely on the primary healthcare level ($\beta = 1.0$). When $\beta = 1.0$, the focus is on the secondary level, Finally, with $\beta = 0.5$, there is a balanced consideration of both the total exceeding of both the primary and secondary healthcare level.

Table 3 ($\beta = 1$) shows a reduction of 25.07% in the largest excess of population over resources at the primary healthcare level, decreasing from 603,757 to 452,258 people. In addition to this reduction, Table 3 also indicates a decrease in the largest absolute deviation as a percentage among districts regarding the excess of population over resources at the primary healthcare level, from 92.15% to 83.13%. These positive results are also partially observed at the secondary healthcare level, where the largest excess of population over resources is reduced from 992,968 to 928,651 people (6.48%). However, the largest mean absolute deviations as a percentage at the secondary healthcare level increases from 23.23% to 32.31%, leading to a reduction in the homogeneity of resource distribution among districts.

In Table 5 ($\beta = 0$), we observe that the largest excess of the population over resources is reduced from 992,968 to 856,352 people (13.76%), and the largest mean absolute deviation as a percentage among districts is reduced from 23.23% to 13.38%. Nonetheless, these improvements come with a slight increase in the largest excess of population over resources at the primary healthcare level, rising from 603,753 to 612,519 people (1.45%), and an increase in the largest mean absolute deviation as a percentage of 2.79% (from 92.15% to 94.94%). This results in a deterioration of the homogeneity in the distribution of resources at the primary healthcare level.

Table 3: Composition of healthcare services in the Metropolitan region of Santiago with $\beta = 1$.

District	Population	Secondary Capacity	Primary Capacity	Secondary Excess	Primary Excess
Central	1,341,551	486,207	889,183	855,344	452,368
East	953,995	109,457	509,841	844,538	444,154
South	1,402,653	611,261	1,147,692	791,392	254,961
West	1,011,354	412,323	958,347	599,031	53,007
South-East	1,179,203	363,570	822,923	815,633	356,280
North	1,152,938	224,287	828,473	928,651	324,465
MAPD	19.52%	70.24%	40.68%	32.31%	83.13%

Table 4: Composition of healthcare services in the metropolitan region of Santiago with $\beta = 0.5$.

District	Population	Secondary Capacity	Primary Capacity	Secondary Excess	Primary Excess
Central	1,281,187	492,453	950,799	788,734	330,388
East	1,066,277	303,286	696,645	762,991	369,632
South	1,199,911	395,434	967,622	804,477	232,289
West	1,239,516	427,337	983,002	812,179	256,514
South-East	1,094,376	269,316	735,223	825,060	359,153
North	1,160,427	319,279	823,168	841,148	337,259
MAPD	9.17%	33.9%	18.94%	5.31%	26.07%

Table 5: Composition of healthcare services in the metropolitan region of Santiago with $\beta = 0$.

District	Population	Secondary Capacity	Primary Capacity	Secondary Excess	Primary Excess
Central	1,254,276	397,924	641,757	856,352	612,519
East	1,130,793	303,286	665,257	827,507	465,536
South	1,354,982	519,389	1,233,226	835,593	121,756
West	1,202,137	504,195	1,117,597	697,942	84,540
South-East	1,152,682	363,570	793,088	789,112	359,594
North	946,824	118,741	705,534	828,083	241,290
MAPD	19.3%	67.72%	43.50%	13.38%	94.94%

Results in Table 4 ($\beta = 0.5$) equate the importance of the primary and secondary healthcare level in the objective function. This results in the decrease of both maximum absolute deviations as a percentaje. For the case of the secondary healthcare level, this goes from 23.23% to 5.31%, while at the primary healthcare level, it decreases from 92.15% to 26.07%. On the other hand, considering the largest excess of population over available resources compared to the initial case, these decrease from 992,968 to 841,148, resulting in a reduction of 15.28% for secondary healthcare level; meanwhile, the excess for primary healthcare level goes from 603,757 to 369,632, decreasing by 38.77%.

Table 6 and Figure 3 summarize the results for each value of β. In the case that the decision maker is more interested in minimizing the largest excess in

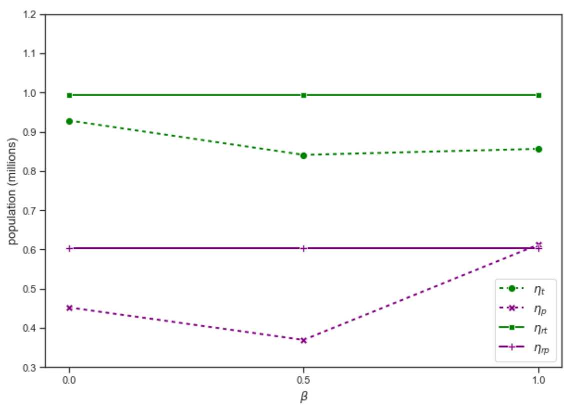

Figure 3: Capacity-demand match at primary and secondary levels according to different values of β.

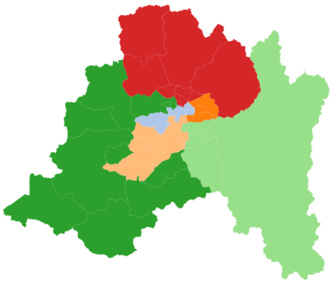

Figure 4: Obtained districts with $\beta = 1$.

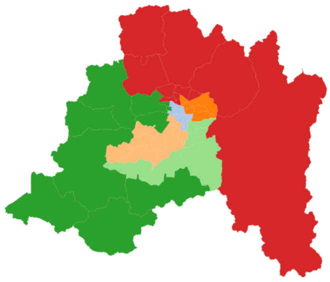

Figure 5: Obtained districts with $\beta = 0.5$.

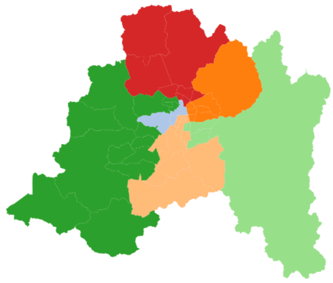

Figure 6: Obtained districts with $\beta = 0$.

Table 6: Summary of results for each value of β.

β	η_t	η_p	η_{rt}	η_{rp}	Gap_t	Gap_p
1	928,651	452,368	992,968	603,757	6,48%	25,07%
0.5	841,148	369,632	992,968	603,757	15.29%	38.78%
0	856,352	612,519	992,968	603,757	13,76%	-1,45%

the primary healthcare level ($\beta = 1$), the results suggest that it is possible to achieve an improvement of up to 25%, i.e., reduce up to 25% the largest excess of the population over the available resources. However, this will have an impact on the secondary healthcare level, which might increase their largest mean absolute deviation as a percentaje by 10%. In the opposite case, if the decision maker is more interested in minimizing the largest excess in the secondary healthcare level ($\beta = 0$), the results suggest that it is possible to achieve an improvement of almost up to 14%, however, as it is expected, an increase in the largest excess of the primary healthcare level of 1,45% is reported. In that case, both healthcare levels are considered simultaneously in the objective function, i.e., $\beta = 0.5$, the results suggest an improvement in both levels, primary and secondary healthcare levels. The former might reduce the largest excess up to 15%, while the latter might be reduced up to almost 39%. These results suggest that these decisions must not be taken independently, and considering both healthcare levels at the same time might provide a more efficient geographical partition of the Metropolitan region of Santiago in terms of balance between population and available resources. It is worth noticing that the difference between the improvements in the secondary (resp. primary) healthcare level using $\beta = 0.5$ and $\beta = 0$ (resp. $\beta = 1$) might be explained due to the formulation (4)-(18) could not find an optimal solution within the imposed time limit. More specifically, the optimality gaps reported after the 12 hours of computation were 58.5% for $\beta = 1$, 44.7% for $\beta = 0.5$ and 44.4% for $\beta = 0$.

5 CONCLUSIONS

The geographical partitioning of healthcare services is a fundamental pillar for ensuring efficient and equitable healthcare delivery. This research focuses on the City of Santiago, Chile, and sheds light on the complex issue of healthcare districting, a combinatorial optimization problem that, when addressed efficiently, can bring transformative improvements to healthcare service delivery.

Our study illustrates the intricate balancing act required between primary and secondary healthcare levels. Through mathematical modeling, we have demonstrated that a conscientiously designed districting plan can play a pivotal role in harmonizing the healthcare journey for patients, from their initial encounters with primary care to more advanced care needs. The complex interplay between population homogeneity, compactness, and capacity-demand match emerges as the cornerstone of this optimization problem.

Relying on a variety of methodologies, ranging from exact solutions to heuristic approaches, researchers have made commendable progress in addressing the healthcare districting problem. In our study, we consider a mixed-integer linear programming formulation with a bi-objective function assessing equity at two layers of the health service system (namely, primary and secondary). We represent and assess three scenarios and explore the implications of prioritizing primary, secondary, or both healthcare levels. Our findings underscore that considering both levels of healthcare simultaneously can lead to a more balanced geographic partitioning in terms of aligning population demand with available resources.

Results indicate significant potential for improvements. Specifically, for a balanced approach, the largest excesses of population over available resources could be reduced by up to 15.29% and 38.78% for secondary and primary healthcare levels, respectively. While optimizing for a particular level offers specific benefits, it can have consequential effects on the other level. Thus, decision-makers are tasked with judiciously evaluating trade-offs.

The intricate nature of this problem, coupled with the computational challenges encountered during optimization, implies that more efficient algorithms or hybrid methodologies could further improve the solution quality found in this work.

A logical extension of the current study would be to consider a range of medical specialties, which would offer a more comprehensive view of healthcare needs across the city. Alongside this, there is a clear need to refine and enrich the data concerning actual demand and the healthcare system's capacity. Enhancing data collection and analysis can provide more accurate insights and lead to better-informed decisions about resource allocation and distribution. One strong underlying assumption of this work is that primary healthcare resources are homogeneously distributed across districts; however, given the geographical location of primary centers and district sizes, it is unrealistic to assume that people can attend any center within their district. This issue should be addressed by considering a more realistic model, such as a two-level districting problem or an integrated approach that might consider decisions on the location and capacity of new primary centers. Furthermore, an area that warrants particular focus is determining the optimal number of districts required to meet healthcare demands. Pursuing this additional research would provide a critical perspective for long-term planning and expansion decisions.

Regarding the modeling approach, there is an evident requirement to embrace a more dynamic approach that factors in demand shifts over time and acknowledges the inherent variability in healthcare capacity. Such dynamic models would render a more realistic representation of healthcare system needs and behaviors. To further strengthen this analysis, it would be valuable to delve into advanced optimization techniques such as robust optimization and integrated approaches. These methodologies could generate more resilient solutions in the face of uncertainties and help identify optimal locations for future medical centers.

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